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Second Order Statistics of NLOS Indoor MIMO Channels
Based on 5.2 GHz Measurements

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Abstract – Herein, results from measurements conducted by the
University of Bristol are presented. The channel characteristics of
Multiple Input Multiple Output (MIMO) indoor systems at 5.2 GHz
are studied. Our investigation shows that the envelope of the chan-
nel for non-line-of-sight (NLOS) indoor situations are approximately
Rayleigh distributed and consequently we focus on a statistical de-
scription of the first and second order moments of the narrowband
MIMO channel. Furthermore, it is shown that for NLOS indoor sce-
narios, the MIMO channel covariance matrix can be well approxi-
nated by a Kronecker product of the covariance matrices describing
the correlation at the transmitter and receiver side respectively. A sta-
tistical narrowband model for the NLOS indoor MIMO channel based
on this covariance structure is presented.

I. INTRODUCTION

It is well known that using antenna arrays at both the transmit-
ter and receiver over a MIMO channel can provide very high
channel capacity as long as the environment has sufficiently
rich scattering. Under these circumstances, the channel matrix
elements have low correlation leading to channel realizations
of high rank and consequently provide substantial channel ca-
pacity increases. In [1] and [2] the channel capacity for MIMO
systems has been investigated theoretically. Some experimen-
tal investigations have also been carried out trying to charac-
terize the MIMO channel and the corresponding capacity, see
[3, 4, 5, 6, 7].

There is of course great interest in modeling of MIMO
channels. A so-called one-ring model has been proposed and
investigated in [8]. In [9], a distributed scattering model has
been proposed in order to explain the pinhole phenomenon
that may appear in outdoor situations. In [10], a model based
on channel power correlation coefficients is presented. How-
ever, models based on experimental data are still rare. In [11],
the initial results for the NLOS MIMO channel measurements
have been reported.

This paper reports the results based on data measured in
University of Bristol as part of the SATURN project. An ini-
tial statistical model based on the experimental data is also pre-
sented. This paper is organized as follows: Section II gives a
brief description of the measurement setup including the test
environment and the test equipment. In Section III, a statistical
structure is proposed and a least squares rank one approxima-
tion method is described in order to separate a matrix into the
Kronecker product of two sub-matrices. Section IV presents
a statistical model based on this covariance structure and the
results from the model identification for different transmitter
locations. Finally we draw some conclusions in Section V.

II. MEASUREMENT SYSTEM

The test site is the Merchant Venture’s Building of the Uni-
versity of Bristol. The general layout of the test site includes
office rooms, computer labs, corridors and open spaces. The
entire measurements include 15 transmitter locations and 3 re-
ceiver locations. Both line-of-sight (LOS) and NLOS cases
were measured. Notice that in this paper, all the results are
from NLOS cases as shown in Fig. 1. Here, the arrow at each
transmitter location indicates the orientation of the transmit ar-
ray. The transmitter was located in a computer lab and the
receiver was located in a large modern office with cubicles.

The measurements were carried out using the Medav RUSK
BRI vector sounder, which has an 8-element omnidirectional

This work is conducted in part within SATURN (Smart Antenna Tech-

nology in Universal bRoadband wireless Networks) funded by the EU IST
program.
For a narrowband MIMO channel, the input-output relation assumes that there are $A$.

### Data Model and Second Order Statistics

The input-output relation includes 199 blocks with 16 MIMO snapshots within each block, therefore there are 3184 complete MIMO snapshots in total for each frequency subchannel. The time delay between two neighboring blocks is 26.624 ms. This means the total time for one complete measurement is 5.3 s. During the measurements, people were moving around both at the transmitter and receiver side.

### Measurement Analysis Method

#### A. Data Model and Second Order Statistics

Assume there are $m$ transmit elements and $n$ receive elements. For a narrowband MIMO channel, the input-output relation could be expressed in the baseband as

$$y(t) = H_m^H s(t) + n(t),$$

where $s(t)$ is the transmitted signal, $y(t)$ is the received signal and $n(t)$ is additive white Gaussian noise. The channel matrix $H_m^H$ here is an $n \times m$ matrix.

We define the transmitter, receiver and channel covariance matrices as

$$R^{T}\_i = E(h_i^* h_i) \quad \text{for} \quad i = 1, \ldots, n$$

$$R^{R}\_j = E(h_j^* h_j^\dagger) \quad \text{for} \quad j = 1, \ldots, m$$

$$R_{HH} = E[\text{vec}(H_m^m)\text{vec}(H_m^m)^\dagger],$$

where

$$\text{vec}(H_m^m) = [\text{vec}(h_1^1)^T, (h_2^2)^T, \ldots, (h_m^m)^T]^T,$$

$h_i$ is $i$th row of $H_m^m$, $h_j$ is $j$th columns of $H_m^m$, $(\cdot)^\dagger$ is complex conjugate transpose, $(\cdot)^T$ is transpose and $E[\cdot]$ denotes the expected value.

In [10], it is claimed that the correlation between the power of two subchannels could be modeled by the product of the correlations seen from the transmitter and receiver. Here we try to verify whether this structure could be extended to describe also the phase of the complex valued amplitudes. Notice that for the normalized channel matrix $H_m$, this structure can also be expressed as

$$R_{HH} = R^{T} \_i \otimes R^{R} \_j,$$

where $R_{HH}$ is the channel covariance matrix, $R^{T} \_i$ and $R^{R} \_j$ are the covariance matrices at the transmitter and receiver side defined in (4), (2), (3) respectively and ‘$\otimes$’ denotes the Kronecker product.

#### B. Normalization Method

It is well known that when the transmitted power is equally allocated to each transmit element, the channel capacity may be expressed as in [2]

$$C = \log_2 \det(I_n + \rho \frac{m}{m} H H^\dagger),$$

where $H$ is the normalized channel matrix and $\rho$ is the average signal-to-noise-ratio (SNR) at each receiver branch. To normalize the channel matrix from data, we use the same normalization factor for all realizations such that

$$\frac{1}{N} \sum_{i=1}^{N} \| H_i \|_F^2 = nm,$$

where $\| \cdot \|_F$ denotes the Frobenius norm.
C. Least Squares Kronecker Factorization

Fitting the channel covariance matrix $R_H$ optimally into the Kronecker product of two Hermitian matrices $X$ and $Y$ boils down to solving the following problem

$$\min_{X, Y} \| R_H - X \otimes Y \|_F.$$  \hspace{1cm} (9)

The least squares rank one approximation method in [12] can be used to solve this problem. The main idea is to rearrange the elements of the covariance matrix, $R_H$, and $X \otimes Y$ to get a least squares problem of the form

$$\min_k \| R_{\text{tran}} - x(y^*) \|_F,$$  \hspace{1cm} (10)

where

$$x = \text{vec}(X),$$  \hspace{1cm} (11)

$$y = \text{vec}(Y),$$  \hspace{1cm} (12)

$(\cdot)^c$ means complex conjugate and $R_{\text{tran}}$ is the transformed matrix. To obtain the transformed matrix $R_{\text{tran}}$, we use the permutation matrix $T$, defined such that

$$T \text{vec}(X \otimes Y) = \text{vec}(xy^T)$$ \hspace{1cm} (13)

for all matrices $X$ and $Y$. The matrix $R_{\text{tran}}$ is then defined by

$$\text{vec}(R_{\text{tran}}) = T \text{vec}(R_H).$$ \hspace{1cm} (14)

Note that (9) and (10) are equivalent since $T$ is orthonormal.

The solution to the least squares rank one approximation of $R_H$ in (10) is easily calculated using the singular value decomposition (SVD) [13]. Let $\lambda_{\text{max}}$ as the largest singular value of $R_{\text{tran}}$, and $u_{\text{max}}$ and $v_{\text{max}}$ as the left and right singular vector corresponding to $\lambda_{\text{max}}$. Then, $x$ and $y$ can be expressed as

$$x = \gamma u_{\text{max}},$$ \hspace{1cm} (15)

$$y^c = \gamma^{-1} \lambda_{\text{max}} v_{\text{max}},$$ \hspace{1cm} (16)

where $\gamma$ is an arbitrary scalar and its value depends on what $X$ and $Y$ should be. Then it is straightforward to transform the two vectors $x$ and $y^c$ into $X$ and $Y$ inversely.

It can be shown that the solution $X$ and $Y$ will always be Hermitian as long as $R_H$ is Hermitian, therefore it is not necessary to force that structure on the solution explicitly.

IV. Measurement Results

As stated in section II, the measurements use an 8-element transmitter and receiver. In this paper, pairs of 2 and 3 neighboring elements at both the transmitter and receiver have been used as an example to get 2x2 and 3x3 MIMO channel realizations that are used to analyze the second order statistics of the channel matrix. It is interesting to find that even though people were moving around during the measurements, the time variations on a single link of each narrowband subchannel are still not enough to give an average of zero, indicating a quite stationary scenario. Therefore both the frequency and spatial domain have been averaged, i.e. all the snapshots from different frequency subchannels and spatial 2x2/3x3 pairs (at the same Tx and Rx locations) have been seen as different channel realizations in order to get sufficient data to study both the channel distribution and second order statistics. Reasonably good results have been found for other setups with up to 5 elements at each side. The data are not enough to calculate setups with more elements due to the insufficient spatial average. In the following section, the data measured at Tx13 – Rx3 are used, see Fig. 1. Similar results have been found in the other four transmitter locations, see Fig. 1.

A. Distribution of the channel coefficient

The distribution of the channel coefficient has been investigated by plotting the histogram of the envelope of one channel coefficient and the cumulative density function (CDF) of the phase of one channel coefficient. Fig. 2 and Fig. 3 shows the...
result of the envelope distribution of one channel coefficient and the CDF of the phase of one channel coefficient comparing with the fitted Rayleigh distribution and uniform distribution on \([-\pi, \pi]\) respectively. We concluded that the channel coefficient for the NLOS indoor MIMO situation is approximately zero-mean complex Gaussian.

### B. Second order statistics

It is well known that a complex Gaussian distributed random variable is completely characterized by its first and second order moments. Although the individual elements of the channel matrix appear Gaussian distributed, we have not examined that they are jointly Gaussian. To investigate the second order statistics for this NLOS indoor scenario, we define the model error, \(\Psi\) to evaluate the difference between two matrices \(A\) and \(B\)

\[
\Psi(A, B) = \frac{\|A - B\|_F}{\|A\|_F}. 
\]

From the measured data, two model errors have been investigated, the results are listed in the first two rows of Table 1, where the \((\cdot)^{\frac{1}{2}}\) indicates a sample covariance estimate of the corresponding quantities in equations (2), (3), (4) respectively and the matrices \(X\) and \(Y\) are calculated using the least squares Kronecker factorization method. It is also interesting to compare the differences between the matrices \(X\), \(Y\) and the covariance matrices \(R_{xx}^{\gamma}, R_{yy}^{\gamma}\) in the last two rows of Table 1. Here since \(\gamma\) is an arbitrary scalar, the least squares method has been used to find \(\gamma\) which fits all elements between two matrices.

![Figure 3. CDF of the phase of one channel coefficient and the CDF of the uniform distribution within \([-180^\circ, 180^\circ]\)](image)

From Table 1, it is clearly shown that the channel covariance matrix could be well modeled as shown in equation (6), notice that this structure could explain above 95\% of the received signal power in both cases.

### C. Statistical Model and Simulations

If the channel coefficients are complex Gaussian, it is easy to show from equation (6), as in [8], that

\[
H = (R_{xx}^{\gamma})^{\frac{1}{2}}G[(R_{yy}^{\gamma})^{\frac{1}{2}}]^T, \tag{18}
\]

where \(G\) is a stochastic M by N matrix with independent and identically distributed (IID) \(CN(0,1)\) elements. Here \((\cdot)^{\frac{1}{2}}\) denotes any matrix square root such that \(R^{1/2}(R^{1/2})^* = R\). Note that this model is also a special case of the model suggested in [9].

Monte-Carlo computer simulations have been used to generate 1000 channel realizations and the CDF of the simulated channel capacity is compared with that from the measured data. The results for the 2x2 and 3x3 cases are given in Fig. 4. As a reference, the capacity for the IID channel is also included. From the figure, it is shown that the simulated CDF fits the CDF of measured data quite well, which agrees with our model.

### V. Conclusions

We access experimental MIMO channel measurements at 5.2 GHz. Our investigation shows that for NLOS indoor MIMO scenarios, the elements of the narrowband channel matrix are zero-mean complex Gaussian. Furthermore, it is shown that the channel covariance matrix can be well approximated by the Kronecker product of the covariance matrices at the transmitter and receiver side respectively. We also introduce a narrowband model for the NLOS indoor MIMO channel based on this second order statistical structure. Monte-Carlo simulations of the channel capacity have shown good agreement between the measured data and the statistical model.

### REFERENCES

Figure 4. Cumulative density function of channel capacity for measured data, statistical model and IID MIMO channel. The SNR at the receiver side is 20dB. Power is equally allocated to the transmit elements


