Performance Evaluation of a Data-Derived Iterative Channel Estimator for a COFDM System with Receive Diversity

Alex Dowler and Andrew Nix
Centre for Communications Research, University of Bristol, Merchant Venturers Bldg. Woodland Road, Bristol, BS8 1UB, United Kingdom.
E-mail: {Alex.Dowler, Andy.Nix}@bristol.ac.uk

Abstract—In this paper a low-complexity data-derived iterative channel estimator is evaluated in a receive diversity system for two and four receive antennas. The performance is shown to almost match the ideal (perfect channel knowledge) case in a low-Doppler environment. At higher Doppler frequencies performance is still good but deviates from the ideal case. A Wiener filter tracking algorithm is proposed that is advantageous at lower Eb/No ranges, however linear tracking is found to be better at higher Eb/No ratios and therefore further improves performance for higher order modulation schemes such as 16-QAM.

Keywords—Data derived, Iterative, Channel estimation, COFDM, Receive diversity, Maximal Ratio Combining.

I. INTRODUCTION

The ability of COFDM to efficiently exploit the wideband properties of the radio channel lies at the heart of its popularity. This feature has helped to establish COFDM as the physical layer of choice for broadband wireless communications systems [1-3]. For COFDM to operate in a Wide Area Network (WAN), efficient channel estimation techniques are required that operate in fast time varying channels. This is often achieved by inserting pilot symbols amongst the data symbols in the OFDM modulation grid. These pilots are distorted by the wideband properties of the radio channel; however knowledge of their original values enables the distortion at each pilot to be quantified. With suitable interpolation, a channel estimate at all intermediate symbols can be generated [4].

Previous work [5] has proposed an alternative to the scattered pilot scheme. Indoor WLAN systems, such as HIPERLAN/2 and IEEE 802.11a, use a training sequence at the start of each user frame to enable an estimate of the Channel State Information (CSI) vectors for all sub-carriers [2,3]. This method is viable for WLAN systems since the low mobile velocities associated with an indoor office environment result in a channel that is effectively time invariant over a single frame. Hence the initial CSI estimate can be assumed to be correct for the entire frame. In an outdoor WAN environment, mobile velocities are typically much higher and hence the channel cannot be assumed to remain constant but must be tracked in time. Our proposed system requires no additional pilots and instead uses received decoded data to estimate the transmitted data and hence estimate the channel. By using the last known channel estimate on the next decoded packet, a recursive system is implemented. Two noise reduction techniques and two channel tracking/prediction algorithms are also suggested. In this contribution, the viability of this channel estimation technique is investigated for use with multiple antenna systems. The performance of the resulting system is then evaluated.

Multiple antenna systems can be used to increase throughput (e.g. Spatial Multiplexing [6]) and/or provide spatial diversity gain (Space-Time Block/Trellis Codes [7-9]). A Multiple-Input Multiple-Output (MIMO) system with \( M \) transmit antennas and \( N \) receive antennas can have up to \( M \) independent data streams transmitted simultaneously. There will be \( M \times N \) independent channels between the transmitter and receiver. \( N \) independent equations can be generated, relating the transmitted and received data. It can therefore be seen that if the transmitted and received user data is to be used to estimate the channels, \( N \) equations must be solved, with \( N \times M \) unknowns. It is clear that the solution is only viable if there are an equal number of equations and unknowns, i.e. \( N \times M = N \) only when \( M = 1 \). This is a system with a single transmit antenna and multiple receive antennas, i.e. a receive diversity scheme. The technique can be applied to full MIMO systems if the channel can be considered to be constant across two or more sub-carrriers or subsequent COFDM symbols; however this is the subject of future work. In this paper spatial diversity is considered through multiple antennas in the receiver, together with a maximal ratio combining scheme to improve performance.

The paper is organised as follows: In Section II, the channel environments considered are presented. The COFDM system parameters for the test system are described in Section III. In Section IV the iterative estimator is described in detail, along with two noise reduction techniques to improve performance. Simulation results are presented in Section V. Section VI proposes two channel tracking/prediction algorithms to further improve performance in high mobility channels. Finally Section VII discusses the results and concludes the paper.

II. CHANNEL ENVIRONMENTS

In order to study the performance of wide area COFDM systems it is essential to satisfactorily characterize the transmit channels. The radio link performance in a mobile environment is primarily limited by Doppler and delay spread.

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In order to establish some assessment scenarios, the channels proposed in the evaluation report for ETSI UMTS Terrestrial Radio Access (UTRA) [10] are considered (Table 1). RMS delay spreads range from 65ns for indoor environments up to 4000ns for WANs. The channel is time-variant with maximum Doppler rates, \( f_D \), of 5.55Hz in pedestrian environments and up to 222Hz in vehicular environments when operating in the 2GHz band.

### III. THE OFDM SYSTEM

Table II, shows the values that were chosen for the OFDM parameters.

#### TABLE II. OFDM PARAMETERS FOR 4G.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Operating Frequency</td>
<td>2GHz</td>
</tr>
<tr>
<td>FFT Size, ( K )</td>
<td>512</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>4096 kHz</td>
</tr>
<tr>
<td>Sample period</td>
<td>244 ns</td>
</tr>
<tr>
<td>Useable Symbol Duration</td>
<td>( T ) = 125 \mu s</td>
</tr>
<tr>
<td>Guard Interval Duration</td>
<td>( T_g = T/6 = 20.8 \mu s )</td>
</tr>
<tr>
<td>Total Symbol Duration</td>
<td>( T_{total} = 144.2 \mu s )</td>
</tr>
<tr>
<td>Channel Coding</td>
<td>Punctured 1/2 rate convolution code, ( K^{133, 171} ) octal</td>
</tr>
<tr>
<td>Sub-carrier spacing ( \Delta f )</td>
<td>8 kHz</td>
</tr>
</tbody>
</table>

For the simulations in this paper, QPSK and 16-QAM, both with 1/2 rate coding, will be used. The guard interval is fixed at \( -T/6 \), although efficiency could be further improved by employing an adaptive guard interval. A frame is defined as a number of COFDM symbols sent by one user. The first OFDM symbol of a frame will consist entirely of training symbols; all subsequent COFDM symbols contain user data. This user data is divided into “coded packets” where a coded packet can be made up of \( L_c \) OFDM symbols. Note that a “user packet” (consisting of header and payload) can comprise a number of coded packets, so the length of each encoded section does not place limits on the length of a user packet. A bitwise random interleaver is used across all data bits within an encoded packet in order to mitigate the effects of error bursts, which degrade the performance of the Viterbi decoder. The channel model assumes Rayleigh fading with a classical Doppler spectrum. It is assumed that the channel remains constant within each COFDM symbol, but changes between symbols. The channel is correlated within a frame of \( L_c \) COFDM symbols and the frames are uncorrelated between one another. Simulations in this paper use a frame length of 30 COFDM symbols. The different channels between the transmit and receive antennas are uncorrelated.

### IV. DATA-DERIVED ITERATIVE CHANNEL ESTIMATION

The data at the receiver is given by:

\[
Y_n(k, t) = H_n(k, t)X(k, t) + V_n(k, t)
\]

where \( H_n(k, t) \) is the CSI vector of channel \( n \) at the \( k \)-th sub-band at time slot \( t \), \( X \) is the transmitted symbol, \( V_n \) is the corrupting noise, and capital letters denote the frequency domain. It is clear that an estimate of \( X \) can be achieved given an estimate of \( H \), or \( H \) estimated given \( X \), and that these estimates will be corrupted by noise.

Fig. 1 shows a block diagram of the receiver. In its most basic form, the receiver demodulates and decodes the data using the channel estimate from the previous COFDM symbol. The first channel estimate in a frame \((t=1)\) is made from an initial uncoded training sequence. In order to demodulate the receive symbols efficiently, the signals from the \( N \) receive antennas must be combined. In maximal ratio receive combining (MRRC) the signals are co-phased, summed, and then normalized, using the last available channel estimate:

\[
\tilde{S}(k, t) = \frac{\sum_{n=1}^{N} H_n^*(k, t-1)Y_n(k, t)}{\sum_{n=1}^{N} |H_n(k, t-1)|^2}
\]

All coded data packets \((t>1)\) are demodulated and decoded using the channel estimate from the previous packet:

**Fig. 1: Block Diagram of Receiver Structure**
\[
\tilde{X}(k,t) = \frac{\tilde{S}(k,t)}{\tilde{H}(k,t-1)}
\]  
(3)

The decoded data is then re-encoded and re-modulated to construct an estimate of the transmitted data (\(\tilde{X}\)). This can then be used to estimate the channel for the current coded packet, which is then used as an estimate for the following packet.

\[
\tilde{H}(k,t) = \frac{\tilde{S}(k,t)}{\tilde{X}(k,t)}
\]  
(4)

The channel estimate will be corrupted by noise, which will obviously degrade system performance. To mitigate the effects of noise, two methods are proposed:

1) In a low-Doppler environment, a coded packet can span several COFDM symbols, since the channel will only change slowly and the distance between estimates can therefore be increased. If the channel estimate is averaged across \(L_c\) COFDM symbols, the effect of noise will be reduced. Clearly this method is not viable if \(L_c = 1\).

2) The spectrum of the superimposed noise is typically white, whereas the actual channel response is strictly band-limited due to the maximum excess delay of the channel. If this maximum excess delay is known, out-of-band components can be eliminated and hence the corrupting effect of noise is reduced. The maximum excess delay can be assumed to be no longer than the guard interval, since a longer delay than this will cause severe performance degradation regardless of the channel estimation algorithm used. The maximum delay can also be measured more accurately using the method described in [11]. Simulation results presented in this paper assume that the maximum excess delay is known (See Table I). Out-of-band components are eliminated by performing an IFFT on the channel estimate (to transform the waveform into the time domain), windowing the taps within the known delay spread (i.e. reducing all other taps to zero), and then transforming back into the frequency domain by means of an FFT.

Accurate knowledge of the delay spread will result in enhanced noise suppression since the optimal number of FFT bins will be nulled. The computational complexity of this process can be simplified since the filter matrix can be pre-calculated:

\[
\tilde{H}_n' = \tilde{H}_n \cdot G \cdot T \cdot F
\]  
(5)

\(\tilde{H}_n'\) is the filtered channel estimate for receive antenna \(n\), and \(\tilde{H}_n\) is a row vector containing the original channel estimate for one COFDM symbol. \(G\) is the \(K\)-point IFFT matrix, \(L\) is the channel length, \(T\) is the truncation matrix (a \(KxK\) zero matrix with the top-left \(LxL\) values equal to the \(LxL\) identity matrix) and \(F\) is the \(K\)-point FFT matrix.

Since \(G\), \(T\), and \(F\) are known, and due to the associative nature of matrix multiplication, the product \(G \cdot T \cdot F\) can be pre-calculated, thus reducing the amount of computation required for each channel estimate.

One or both of these noise reduction methods can be employed, where appropriate, in order to reduce the effect of noise on the channel estimate.

The overall complexity of this algorithm can be considered to be relatively low: Encoding and modulation of the received data are computationally inexpensive when compared, for example, to Viterbi decoding. Modulation in particular can be performed very fast since many symbols can be modulated in parallel since this process is not sequential. An alternative system using scattered pilots would have to de-multiplex the data and pilots from the received stream, and perform interpolation on the known channel estimates. A 2D Wiener filter interpolator is particularly computationally intensive, so this algorithm represents a far cheaper alternative. In many cases this algorithm will use fewer pilots than a scattered pilot system, so also offers a throughput benefit.

V. PERFORMANCE RESULTS

System performance is first analyzed in ETSI UTRA environment 1 (see Table I). The relatively low Doppler frequency results in a channel that is highly correlated in time, so a packet length of 3 COFDM symbols was chosen, thus allowing channel averaging to be utilized as a method of noise reduction.

Fig. 2 shows the results for this system for 2 and 4 receive antennas. It is clear that FFT noise reduction improves performance more than simply averaging, and that there is negligible advantage to be gained from both techniques in this channel. The iterative estimator combined with FFT noise reduction results in a system performance that is only 0.4dB from the case of perfect channel knowledge in both the 2 and 4 antenna cases. A larger diversity gain can be seen for 4 receive antennas, as expected.

Since channels with high Doppler spreads will be significantly less correlated in time, simulations for environment 6 use a coded packet of only one COFDM symbol.
symbol. In [5] it was found that system performance suffered greatly if longer coded packets were used.

Fig. 3 shows the performance of the system in environment 6 for 2 and 4 receive antennas. Again the expected diversity gain can be seen, but even with FFT noise reduction the performance is 2dB poorer than the case of perfect channel knowledge. The receiver structure uses the CSI of the previous coded packet to receive and decode the current coded packet. Even with perfect channel knowledge of the previous COFDM symbol (i.e. assuming the use of a perfect but delayed channel estimate) it can be seen that system performance is Doppler limited.

VI. CHANNEL TRACKING

In [5] it was suggested that channel tracking could be used to further improve performance. In particular, a linear tracker was proposed and for convenience the method is described below. This paper also suggests the use of a Wiener filter-based tracker, which is expected to offer superior performance to the earlier linear approach.

A. Linear Tracking

The simplest channel tracking algorithm will take a linear extrapolation from the two previous channel samples:

\[
\hat{H}_n(k, t+1) = 2\hat{H}_n(k, t) - \hat{H}_n(k, t-1)
\]  

(6)

This is clearly not valid for the channel \(\hat{H}_n(k, 2)\), since only one channel estimate will be available. In this case there is no choice but to use the estimate for \(\hat{H}_n(k, 1)\). While the linear tracker could be expanded to use more than two previous samples (as extra samples become available), this will introduce further error since the channel is not linear but follows a Rayleigh distribution in this particular case.

B. Wiener Filter Tracking

Wiener filters exploit the correlation of the channel in time and frequency in order to reduce the corrupting effect of noise on the channel estimate. We propose the use of a 1-D Wiener filter to predict the channel estimate from previous channel knowledge. A 2-D filter could be used but at the cost of increased computational requirements. This approach would compromise the low complexity of our algorithm.

The channel estimate is given by:

\[
\hat{h} = R_{hp}(R_{pp} + \frac{1}{SNR} I)^{-1} \hat{p}
\]  

(7)

where \(\hat{h}\) is a column vector of filtered channel estimates, \(\hat{p}\) is a column vector of known channel samples, \(R_{hp}\) is the cross-covariance matrix between \(h\) and the noisy channel estimates \(\hat{p}\), and \(R_{pp}\) is the auto-covariance matrix of the known channel samples [12]. As we are only using a 1D filter, \(R_{hp}\), and \(R_{pp}\) are given by the correlation function in time only, since the frequency correlation will be equal to 1. The elements of \(R_{hp}\) and \(R_{pp}\) are therefore given by:

\[
E[\hat{p}_l \hat{p}^*_l] = E[p_l \hat{p}_l^*] = J_0(2\pi f_{max} T_s (l - l'))
\]  

(8)

where \(f_{max}\) is the maximum Doppler frequency, \(T_s\) is the extended COFDM symbol period, and \(J_0(x)\) is the zeroth order Bessel function of the first kind.

The Wiener filter uses two previous channel samples (as does the linear interpolator). Since the Wiener filter acts to reduce noise, it would be expected to have superior performance to the linear filter.

Fig. 4 shows results for a system with 2 receive antennas for both tracking algorithms in environment 6. In [5] we saw that the linear tracker degrades performance over the non-tracked system at low Eb/No, but improves performance at higher Eb/No. For the multiple antenna system, the operating Eb/No range is below the level at which the linear tracker becomes advantageous (found to be approximately 15dB in [5]), and therefore the linear tracker offers no performance advantage. The Wiener tracker performs better as a tracking algorithm as it does not degrade performance at low Eb/No. However, the method can be seen to offer only a...
very small performance increase. Furthermore, it is still unable to offer a performance improvement at the Eb/No levels of interest in a MRCR system.

Fig. 5 shows the system performance for 16-QAM modulation. There is a greater performance degradation over the perfect CSI case than for QPSK due to the higher number of bits per symbol. In the QPSK case, one incorrect bit in the decoded data stream could result in a reconstructed $\hat{X}(k,t)$ symbol that is $90^\circ$ out of phase from the actual transmitted $X(k,t)$. This occurs due to the Gray-coding of symbols in the modulation map, which ensures that the minimum Euclidean distance occurs between code-words having a Hamming distance of 1. In the case for 16-QAM, the phase between symbols having a Hamming distance of 1 can differ by up to $143^\circ$, and it is also possible for the magnitude to be in error. This will lead to a greater error in the channel estimate, hence the noticeable degradation in performance.

This graph shows two other interesting results: Firstly, due to the higher Eb/No operating range of 16-QAM, the linear channel tracker can be seen to become beneficial at around 16dB. Secondly, an error-floor exists when the Wiener tracker is employed. While the Wiener tracker out-performs the linear tracker at lower Eb/No due to its noise reducing properties, as the relative noise level decreases the benefits of the tracker also diminish. This is because the Wiener filter is being used to extrapolate a channel estimate from existing values. In [12] it is shown that the Wiener filter is good at interpolating channel values, but performs significantly worse when extrapolating channel values, hence it limits the performance of this tracking algorithm.

VII. Conclusions

A low-complexity channel estimation technique has been tested in a receive-diversity system, and has been shown to perform very well in a low-Doppler environment when aided by an FFT noise reduction technique. In fact at pedestrian velocities the performance is within 0.5 dB of the ideal perfect channel knowledge case, without the complexity of scattered pilots and associated interpolation requirements. At vehicular velocities performance is not as close to the perfect CSI case, but suffers from only 2dB of degradation. The technique used in this paper can be applied to full MIMO systems if the channel can be considered to be constant across two or more sub-carriers or COFDM symbols. Future work will investigate the use of this technique in a full MIMO system as well as investigating its performance when integrated in current WLAN standards such as IEEE 802.11a.

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References