Efficient Modeling of Correlated Shadow Fading in Dense Wireless Multi-Hop Networks

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Abstract—Correlated shadow fading has a detrimental effect on the performance of wireless systems. Neglecting shadowing correlations could lead to inaccurate simulation results and unreliable wireless system design. In this paper, we propose and analyze a correlated shadow fading model based on Gaussian random fields. The model enables the generation of spatially correlated shadow fading for all meshed links in wireless multi-hop networks. Both analytical and numerical results show that the proposed model is in good agreement with the literature in terms of the statistical properties and correlation coefficients. Furthermore, the Circulant Embedding method of the proposed simulation model significantly reduces the computational cost.

Keywords—wireless multi-hop networks; shadow fading; spatial correlation; Gaussian random field; Circulant Embedding method

I. INTRODUCTION

There is a growing need for the performance evaluation of dense wireless multi-hop networks, including mesh, ad-hoc, and sensor networks. System-level simulations are generally considered as the most convenient approach to investigate the behavior of such networks. Nevertheless, the complexity of the channel model introduces a clear tradeoff between accuracy and computational cost in wireless network simulations.

A wireless channel is typically modeled as a combination of three components: path loss, slow fading (also known as shadow fading or shadowing) and fast fading. The mean path loss is mostly determined by the distance between the transmitter and the receiver. Fast fading is caused by multi-path propagation and its statistical properties have been studied extensively in the literature [1]. Shadow fading is caused by obstacles in the communication path and is defined as the fluctuation in the received power averaged over a few tens of wavelengths. It plays an important role in wireless system design, including coverage prediction in network planning and performance evaluation of network algorithms. Therefore we focus on shadow fading in this paper. Shadow fading on a decibel (dB) scale is commonly modeled by independent and identically distributed (i.i.d.) Gaussian random variables in system-level simulations. This achieves low computational complexity. However, it fails to capture the spatial correlation of shadow fading. Several experimental studies have shown that shadowing is significantly correlated in various scenarios [2]-[5]. Correlated shadowing has a negative effect on the connectivity of wireless multi-hop networks [5]. In dense multi-hop networks, a large number of nearby meshed links magnify the detrimental effects of correlated shadow fading on network performance. Neglecting shadowing correlations in dense multi-hop networks could lead to inaccurate network simulation results and unreliable wireless system design. Taking the correlation into account usually means high computational costs. Therefore an important goal is to develop a realistic and computationally effective correlated shadowing model for use in dense wireless multi-hop networks.

Most of the existing correlated shadowing models are based on cellular networks, where correlations are considered either between a base station (BS) and several mobile stations (MSs) or between a BS and several MSs. The model of Gudmundson [2] describes the autocorrelation function of shadowing and is widely used to predict received power correlations for the MS-BS links as MS moves. Gudmundson’s model considers each MS to experience shadowing independently, thus resulting in uncorrelated shadow fading for neighboring MSs. This lack of correlation does not happen in the real world since nearby MSs have similar shadowing environment, thus experiencing correlated shadow fading. To overcome this limitation, a two-dimensional shadow fading model has been proposed to generate correlated shadowing values for neighboring MSs [6]. A unique shadow fading map is generated for each BS to represent shadowing losses for geographic locations. On the other hand, the cross-correlation of shadow fading between two transmitting BSs to a common MS was considered in [7]. Recent work [8][9] took a step further by incorporating both spatial auto-correlation and site-to-site cross-correlation into system-level shadow fading models.

For a multi-hop (ad-hoc, or sensor) network, it is assumed that all nodes have the same low antenna height. This is very different compared to a conventional cellular network, where the radio link is established between a high-antenna BS and a low-antenna MS. Moreover, the propagation distance in an ad-hoc or sensor network is much shorter than that in a cellular network. Therefore none of the previously discussed correlated shadowing models can be applied directly to a multi-hop network. To fill this gap, Wang, Tameh, and Nix [9] extended Gudmundson’s model [2] to predict the shadowing correlation on a peer-to-peer (P2P) link when there is mobility on both ends of the link. Nevertheless, it has the same limitation as [2], resulting in uncorrelated shadow fading losses for MS-MS links that are in close vicinity to each other. Patwari and
Agrawal [5] investigated the spatially correlated link shadowing in multi-hop networks and developed a joint path loss model (also known as the NeSh model) for arbitrary pairs of links in a wireless multi-hop network. However, we show in Section IV that the link shadowing loss model in [5] is only valid when the link distance is much greater than the decorrelation distance. For dense networks, the NeSh model may produce inaccurate results due to the large number of short links involved. In the NeSh simulation model, the correlated path losses are generated by first generating i.i.d. Gaussian vectors and then multiplying them by the square root of an appropriate covariance matrix. This approach is impractical when applied to dense multi-hop networks due to the large size of the covariance matrix. It is particularly the case when considering node mobility, which requires the generation of a new covariance matrix for each node position change.

In this paper, we take Patwari and Agrawal’s [5] empirical observations about the correlation structure in wireless multi-hop networks as valid, and hence we do not present any additional experimental data. Instead, we focus on the realistic and efficient modeling of correlated shadow fading for system-level simulation. Rather than calculating the covariance matrix as in [5], we directly generate link shadowing losses that have the desired statistical distribution and correlation properties. We propose a correlated shadowing model with two parts: a shadowing map and a link shadowing loss function. The random shadowing map models the non-site-specific shadowing environment. By connecting the link shadowing loss with the underlying shadowing environment, we preserve the physical relationships which exist between links in the real world. We also propose the use of the Circulant Embedding method [11] to efficiently generate a Gaussian field as part of the simulation model. Both analytical and numerical results show that the proposed shadow fading model agrees well with the empirically-observed link shadowing properties. The link shadowing variance induced by the proposed model provides a better fit to the empirical results than the NeSh model. Furthermore, the proposed model significantly reduces the computational complexity on system-level simulations.

The overview of the process of the proposed shadow fading model is given in Fig. 1. Section II describes the two-part correlated shadowing model and the assumptions made in this paper. Section III presents a step-by-step implementation guide for the model, together with suggestions on parameter settings. The evaluation of the proposed model is given in Section IV. Finally, the paper is concluded in Section V.

II. MODEL DESCRIPTION AND ASSUMPTIONS

The main assumption in this work is that the shadow fading losses experienced on links in a wireless multi-hop network are a result of signals passing through an underlying shadowing map. The physical model of a pair of links in a shadowing map is illustrated in Fig. 2. For an ad hoc, or sensor network, it is also assumed that all nodes have the same configuration (e.g., antenna height, transmission power). Therefore shadowing losses seen by the nodes at both ends of the radio link are identical, i.e., \( X_{i,j} = X_{j,i} \), where \( X_{i,j} \) denotes the shadowing in link \((i, j)\). It is reasonable to consider that if the shadowing environments on two close links are highly correlated, the link shadowing losses may also be highly correlated.

This section presents a correlated shadowing model which calculates the link shadowing loss deterministically from the shadowing map. We first outline some known properties of the link shadowing loss. Then we describe the proposed two-part shadow fading model that agrees with these known properties. Finally we show that our model leads to a positive correlation between shadowing losses on a pair of links.

A. Shadowing Properties

Several existing studies [1][12] have shown that the link shadowing loss in dB can be modeled by a zero-mean Gaussian random variable with environment-dependent variance.

\[
L_{\text{Shadow}} \sim N(0, \sigma_{\text{Shadow}}^2) \tag{1}
\]
It is reported in [12] that the variability in shadowing process for low mounted wireless links in urban environments tends to increase with increasing distance. The shadowing standard deviation reaches a peak at a certain critical distance and then starts to fall. It can be modeled using the following function [12]

$$
\sigma_{\text{shadow}}(d) = S \cdot \left(1 - \exp\left(-\frac{d - d_a}{D_a}\right)\right)
$$

(2)

where \(d\) is the distance between the transmitter and the receiver, \(S\) is the maximum standard deviation, \(D_a\) is the growth distance factor in meters, and \(d_a = 10m\) [12].

We now seek a correlated shadowing model which has the following properties:

Prop.1. Regardless of which end is the transmitter, the link shadowing loss is the same.

Prop.2. The shadowing loss in dB is a zero-mean Gaussian random variable with a standard deviation of \(\sigma_{\text{shadow}}\).

Prop.3. The standard deviation \(\sigma_{\text{shadow}}\) of link shadowing loss is distance dependent as in (2).

B. Shadowing Map

As in prior literature [5][6], we start with the assumption that the underlying shadowing map is a stationary and isotropic Gaussian random field with zero-mean and exponentially-decaying spatial correlation. A review of the Gaussian random fields and correlation functions is provided in [13]. Here we briefly describe some characteristics and properties of the Gaussian random field for completeness.

A random field is a spatial stochastic process on the two-dimensional Euclidean space \(\mathbb{R}^2\). A stochastic process \(\{X_t, t \in \mathcal{S}\}\) is said to be Gaussian if all its distributions are Gaussian, i.e., for any choice of \(n\) and \(t_1, \ldots, t_n \in \mathcal{S}\), we have

$$
X = (X_1, \ldots, X_n) = (X_{t_1}, \ldots, X_{t_n}) \sim N(\mu, \Sigma)
$$

(3)

where \(\mu\) is the expectation vector and \(\Sigma\) is the covariance matrix. A Gaussian process is determined completely by its expectation function \(\mu = E(X)\) and covariance function \(\Sigma_{ij} = \text{cov}(X_i, X_j)\). A Gaussian random field is called stationary if the expectation function is constant, and the covariance function is invariant under translations, i.e., \(\text{cov}(X_{t+\delta}, X_{t}) = \text{cov}(X_{t}, X_{t+\delta})\). If the distribution remains the same under rotations, the field is said to be isotropic with a covariance function of the form

$$
\text{cov}(X_s, X_t) = \text{cov}([s-t])
$$

(4)

where \([s-t]\) is the Euclidean distance between \(s\) and \(t\).

In this paper, we simulate a Gaussian random field \(f(x)\) on a rectangular grid of size \(n \times m\) as the shadowing map. Let \(\Delta d\) denote the spacing along the grid. The shadowing map will cover a simulation area of size \(L \times W = n \Delta d \times m \Delta d\). In particular, we will generate a zero-mean Gaussian random process on each of the grid points \(\{(i \Delta d, j \Delta d)\}^m_{i=0} \times^n_{j=0}\) corresponding to a covariance function given by

$$
\text{cov}(f(s), f(t)) = \sigma^2 \cdot \exp\left(-\frac{[s-t]^2}{\delta^2}\right)
$$

(5)

where \(\sigma^2\) is the variance of the shadowing map, \(\delta\) is the decorrelation distance, and \([s-t]\) is the Euclidean distance between \(s\) and \(t\). A realization of the shadowing map is visualized in Fig. 2.

C. Link Shadowing Losses

From [5], each link’s shadowing loss \(X_{n,a}\) is calculated by a weighted integral of the spatial field \(p(x)\) as

$$
X_{n,a} = \frac{1}{\|X_n - X_a\|_2^2} \int_{X_n} p(x)dx
$$

(6)

The variance of the link shadowing loss \(X_{n,a}\) is given by

$$
\text{var}(X_{n,a}) = \sigma^2 \cdot \left[1 + \frac{\delta}{\|X_n - X_A\|} \left(e^{-\delta^2 \|X_n - X_A\|^2} - 1\right)\right]
$$

(7)

Although it is intuitively correct to approximate the link shadowing loss as the weighted sum of individual shadowing values along the communication path, the weighting coefficients need to be determined carefully. As can be seen in (6), all obstacles are given an equal weight of \(1/\|X_n - X_A\|^2\). In other words, the NeSh model [5] assumes that all obstacles in the communication path have the same effect on the link shadowing loss. However, in the real world, obstacles that are close to the antenna have higher impacts on link shadowing. This is because that the relative loss of diffracting or scattering over or around the object is more for the obstacles near the antenna. Therefore, the weighting coefficients must be distance-dependent to reduce the impact of obstacles in the middle of a link on shadow fading.

We further abstract this empirical observation by assuming that the shadow fading loss is dominated by the shadowing values in the near field at both ends of the link. A similar assumption has been made in [9]. The following function is therefore proposed for the shadowing loss \(X_{ab}\) of Link AB as

$$
X_{ab} = \frac{1 - \exp\left(-\frac{d_{ab}}{\delta}\right)}{\sqrt{2}} \left(f(A) + f(B)\right)
$$

(8)

where \(f(A)\) and \(f(B)\) represent the shadowing values in the near field of nodes A and B respectively, and \(d_{ab}\) is the Euclidean distance between the near field of nodes A and B.
The function in (8) clearly satisfies Prop.1 (symmetry). Next, we prove that it also satisfies Prop.2 and 3 as follows.

It is well known that the sum of two correlated Gaussian random variables (e.g. $A$ and $B$) is Gaussian such that

$$A + B \sim N(\mu_A + \mu_B, \sigma_A^2 + \sigma_B^2 + 2\sigma_A\sigma_B\rho)$$

The proposed model can be seen to have Prop. 2 since (8) is simply a scaled sum of $f(A)$ and $f(B)$. Thus, the covariance of Link $AB$ and Link $CD$ is defined as

$$\text{cov}(X_{AB}, X_{CD}) = E(X_{AB}X_{CD}) - E(X_{AB})E(X_{CD})$$

and the correlation coefficient is given by

$$\text{corr}(X_{AB}, X_{CD}) = \frac{\text{cov}(X_{AB}, X_{CD})}{\sqrt{\text{var}(X_{AB})\text{var}(X_{CD})}}$$

Using (8) as the link shadowing loss, (12) can be rewritten as

$$\text{cov}(X_{AB}, X_{CD}) = 2\left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)\left(1 - \exp\left(-\frac{d_{CD}}{\delta}\right)\right)(f(A) + f(B))$$

where $f(A)$, $f(B)$, $f(C)$ and $f(D)$ denote the shadowing value in the near field of nodes $A$, $B$, $C$ and $D$, and $d_{ij}$ stands for the distance between the near field of nodes $i$ and $j$.

Substituting (10) and (14) into (13), the correlation function is

$$\text{corr}(X_{AB}, X_{CD}) = \left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)\left(1 - \exp\left(-\frac{d_{CD}}{\delta}\right)\right)$$

and the standard deviation of the link shadowing is given by

$$\text{std}(X_{AB}) = \sigma\left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)$$

It can be observed that (11) has the same form as (2), thus satisfying Prop.3.

### D. Link Pair Correlation

Consider a pair of links Link $AB$ and Link $CD$ as shown in Fig. 2, with link shadowing loss $X_{AB}$ and $X_{CD}$ respectively. The correlation of $X_{AB}$ and $X_{CD}$ is defined as

$$\text{cov}(X_{AB}, X_{CD}) = E(X_{AB}X_{CD}) - E(X_{AB})E(X_{CD})$$

and the correlation coefficient is given by

$$\text{corr}(X_{AB}, X_{CD}) = \frac{\text{cov}(X_{AB}, X_{CD})}{\sqrt{\text{var}(X_{AB})\text{var}(X_{CD})}}$$

Using (8) as the link shadowing loss, (12) can be rewritten as

$$\text{cov}(X_{AB}, X_{CD}) = 2\left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)\left(1 - \exp\left(-\frac{d_{CD}}{\delta}\right)\right)(f(A) + f(B))$$

where $f(A)$, $f(B)$, $f(C)$ and $f(D)$ denote the shadowing value in the near field of nodes $A, B, C$ and $D$, and $d_{ij}$ stands for the distance between the near field of nodes $i$ and $j$.

Substituting (10) and (14) into (13), the correlation function is

$$\text{corr}(X_{AB}, X_{CD}) = \left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)\left(1 - \exp\left(-\frac{d_{CD}}{\delta}\right)\right)$$

and the standard deviation of the link shadowing is given by

$$\text{std}(X_{AB}) = \sigma\left(1 - \exp\left(-\frac{d_{AB}}{\delta}\right)\right)$$

This section, we show how the proposed model may be applied in wireless multi-hop network simulations to generate correlated shadow fading. As previously mentioned, the link shadowing losses are calculated deterministically from the underlying shadowing map. The accurate and efficient generation of the shadowing maps is a prerequisite for correlated shadow fading.

### A. Circulant Embedding Method

We propose to use the Circulant Embedding method [11] to generate the shadowing map as part of the simulation process. The Circulant Embedding method [11] allows the accurate and efficient generation of a Gaussian random field via the Fast Fourier Transform (FFT). The idea is to embed the covariance matrix into a block circulant matrix with each block being circulant itself. Then the matrix square root of the block circulant matrix is constructed using FFT techniques. Finally, the marginal distribution of appropriate sub-blocks of this Gaussian field has the desired covariance structure. The algorithm can be briefly described as follows.

- Building and storing the covariance matrix.
- Embedding in block circulant matrix (BCM).
- Computing the square root of the BCM.
- Extracting the appropriate sub-block.

The computation of the covariance matrix is the most time-consuming step. However, for more realizations of the Gaussian random field, we can store the results of the square root of the BCM and repeat the final step only. For further details on how to implement the Circulant Embedding method, the reader is referred to [14].

### B. Simulation Process and Parameter Settings

As shown in Fig. 1, the simulation process of the correlated shadowing model is explained as follows.

Step.1. Set the parameters of the underlying shadowing map, which include: dimensions of the simulated region $L$ and $W$, spatial resolution $\Delta d$, shadowing standard deviation $\sigma$, and the de-correlation distance $\delta$.

Step.2. Generate shadowing maps with the covariance given in (3) using the Circulant Embedding method.

Step.3. Set node locations, establish links and determine the near field of nodes.

Step.4. Calculate the link shadowing losses from the pre-generated shadowing maps using (8).

The shadowing map values are only generated on the spatial grid with resolution $\Delta d$. For any node off those grid points, the shadowing value has to be spatially interpolated from the values at nearby grid points. A linear interpolating method has been proposed in [9] where $\Delta d$ is set equal to $\delta$. In this paper, we use the simplest interpolating method to approximate the shadowing value by returning the value of the
nearest grid point. In this case $\Delta d$ must be much less than the distance up to which the shadowing value remains approximately constant. The empirically accepted bound is a few tens of wavelengths [9]. The shadowing standard deviation is often estimated by empirical measurements. Commonly accepted values for $\sigma_e$ are between 8-12dBi [9]. The de-correlation distance describes the size of the obstructions in the environment. For a typical European city $\delta$ is 20m [9]. The physical location of each node can be set manually or generated randomly using a mobility model. The locations are then transformed into their discrete forms using $d_{\Delta}$. The near field of a node is initially set to be $\delta$ from the node location. However, this parameter can either be tuned to match existing models or estimated from measurements.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, a range of numerical results are presented using the proposed correlated shadowing model. The performance of the proposed model is studied with different parameter settings and compared against the existing models in terms of the statistical properties and correlation coefficients.

For validation propose, we generate 10000 samples of the underlying shadowing map within a square area of 200×200 m$^2$ with a spatial resolution equal to 2×2 m$^2$. The de-correlation distance and the maximal standard deviation of link shadowing are set to $\delta = 20m$ and $\sigma_e = 8dBi$. We then set node locations and calculate link shadowing losses from the pre-generated shadowing maps using (8).

A. Single-Link Properties

By varying the length of the link from $\delta$ to 10$\delta$, a set of shadow fading losses are obtained. Fig. 3 illustrates the generated shadowing values at a link distance of 10$\delta$ using the proposed model. The simulated data shows a good agreement with the Gaussian distribution assumed for the link shadowing loss. Furthermore, Fig. 4 depicts the normalized link shadowing variance $\text{var}(X_w)/\sigma^2$ as a function of the normalized link distance $d_{\Delta}/\delta$. For the purpose of comparison, the plot also includes the function in the NeSh model [5]. It can be observed that the link shadowing variance of the proposed model increases with the increasing link distance. It then converges at a normalized distance between 4 and 6, and the value satisfies the desired property, i.e., $\sigma_e^2$. However, the NeSh model will only converge to the empirical model when the link distance is much greater than $\delta$. For short links in a dense multi-hop network, our model is able to generate link shadowing losses that fit better to the empirical results than the NeSh model.

B. Link Correlation Properties

Let two links of the same length $l$ share a common end, and gradually increase the angle $\theta$ between them. We define the near field of a node as $a\delta$ from the node location, and study the impact of parameter $a$ on the correlation between two links. Fig. 5 shows a plot of the correlation coefficient as a function of the angle between links using the proposed model. For both short links ($l = 5\delta$) and long links ($l = 15\delta$), the cross-link correlation coefficient decreases as the increasing angle. This is expected because when the angle becomes larger, the shadowing environment at both ends of the two radio links become less correlated. Moreover, we observe that for the same link distance and separation angle, the greater the value of $a$, the smaller the correlation coefficient. We also compare the predicted correlation coefficients using our model with the results using the NeSh model. As shown in Fig. 6, a fair fit between the proposed model and the NeSh model is achieved by tuning parameter $a$. It means that the proposed model allows link shadowing losses to be generated which have correlation properties that are consistent with the NeSh model. In addition, we observe that the value of $a$ is distance dependent and is greater for longer links.

The results provided in this section are not intended as a full comparison but make some particular predictions. Analysis of the channel data between all meshed links in the network will allow the model to be enhanced.

C. Simulation Complexity

As mentioned in Section I, the NeSh simulation model is impractical for dense multi-hop network simulations due to the large number of meshed links. We consider a network of 100 nodes deployed in a square area of 200×200 m$^2$ with a spatial resolution of 2×2 m$^2$. In the NeSh model, the size of the
covariance matrix is $4950 \times 4950$. The calculation of such a large matrix ($2.45\times10^7$ numerical double integrations) takes around 90 hours on a 2.8GHz Intel Core i7 processor. Moreover, in the case of node mobility, the calculation of a new covariance matrix is needed for every node position change. Assuming that the nodes are mobile, we generate 100 random topologies to represent snapshots of the locations of the mobile nodes. It will take the NeSh model around 9000 hours to calculate the 100 covariance matrices. Our model significantly reduces the computational complexity by generating shadowing losses directly from the shadowing map without calculating the covariance matrix. The proposed model only needs 20 seconds to generate 10000 shadowing maps. TABLE I. shows the computational requirements for the generation of correlated shadowing losses using both the NeSh model and the proposed model. The reader is referred to [15] for the details of computational complexity. The low computational complexity of the proposed modeling method enables an efficient use of the correlated shadow fading model in dense multi-hop network simulations.

| TABLE I. COMPUTATIONAL REQUIREMENTS FOR THE NESh MODEL AND THE PROPOSED MODEL |
|---------------------------------|---------------------------------|
| NeSh model                      | Proposed model                  |
| Diagonalization of the          |                                 |
| covariance matrix               | 2.02$\times10^8$                |
|                                 | multiplications                 |
|                                 |                                 |
| Generation of one               | 1.23$\times10^7$                |
| realization                     | multiplications                 |
|                                 |                                 |
| Storage                         | 2.45$\times10^7$                |
|                                 | values                          |
|                                 |                                 |

V. CONCLUSIONS

This paper presents a correlated shadow fading model that agrees well with the literature in terms of the statistical properties and correlation coefficients. The proposed model enables the generation of spatially correlated shadow fading for all meshed links in wireless multi-hop networks. With the dense deployment of wireless nodes in a multi-hop network, this model is crucial for the realistic performance evaluation of the network algorithms. Furthermore, the proposed simulation model significantly reduces the computational cost in dense multi-hop network simulations. Future work will explore the effect of the proposed correlated shadowing model on system-level performance evaluation of wireless multi-hop networks.

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