A Novel Relay-Based Interference Alignment Strategy for Multi-User Networks

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Abstract—Interference alignment (IA) is a transmission strategy that use channel state information (CSI) to improve the capacity of multiuser communication systems. However, the performance of IA is always limited by the accuracy of the CSI. In particular, this is a problem in time-varying channels, as the CSI at transmitter (CSIT) may become outdated when the channel is changing quickly. In this study, we propose a novel cooperative relay-based interference alignment (RIA) scheme, which aims to increase the number of Degrees of Freedom (DoF) for the multiuser networks, even when the required source-to-destination CSIT is completely outdated. With the RIA scheme, the DoF region is firstly derived under an ideal CSIT case. Then, the RIA scheme is respectively presented in non-ideal CSIT cases. Furthermore, the achievable DoF regions by RIA in these non-ideal cases are derived. In addition, the trade-off between relay-based and non-relay based IA schemes are analysed.

I. INTRODUCTION

Interference management is one of the main challenges in today’s wireless communication networks. As a revolutionary technique for interference management, interference alignment (IA) is proposed to improve the capacity of the multiuser network in recent years [1]-[7]. In general, the aim of the IA is to align the co-channel interfering signals in one signal space, which is separate from the desired signal space, so that to increase the multiplexing gain (which characterized the degrees of freedom (DoF) [1]) for the multiuser networks. In practice, however, it is difficult to achieve an ideal CSI in time-varying channels, particularly for the CSI on the transmitter side (CSIT), which needs to be fed back from the receivers. If the actual channel is changing quickly, it is possible for the CSIT to be fully obsolete (referred to as “completely outdated CSIT”).

To solve this problem, Maddah-Ali and Tse (MAT) have proposed a novel IA scheme (termed here as ‘MAT-IA’) [2] for the K-user multi-input single-output (MISO) broadcast channel (BC). The MAT-IA scheme shows that even the source-to-destination (S-D) CSIT is completely outdated, it is still useful for increasing the DoF for the multiuser network. Differing from the completely outdated CSIT assumption in [2], the “mixed CSIT” case is considered in [3], in which the transmitter has both non-outdated CSIT (referred to as “ideal CSIT”) and outdated CSIT. Under the “mixed CSIT” case, the zero-forcing (ZF) and MAT-IA is jointly implemented in the MISO BC [4]. In [5] and [6], a parameter α was introduced to IA schemes to describe the ratio of the time slots with ideal CSIT to the total number of time slots, so that a smooth DoF region connecting the perfect case (α = 0) with the completely outdated CSIT case (α = 1) could be achieved.

Most recently, a new space-time interference alignment (STIA) scheme, which take full advantage of the “mixed CSIT”, has been proposed in [7]. The STIA has been firstly implemented in the MISO BC with M transmitting antennas and K = M + 1 users, and the total number of DoF could reach M. In contrast to the MAT-IA schemes [2]-[6], the STIA scheme can be adaptively implemented according to the ideal part of the “mixed CSIT”. Though the STIA scheme shows a promising DoF optimization result, once the S-D CSIT is completely outdated in all time slots, the STIA scheme will totally lose its DoF gain comparing with that of MAT-IA.

In this study, we consider this issue and propose a transmission strategy for the outdated S-D CSIT scenario to facilitate IA with the help of the cooperative relays. Meanwhile, we show that the relay-based IA (RIA) will not lose its DoF gain even though the S-D CSIT is completely outdated. More specifically, the main contributions of this study are as follows:

- A system of K cooperative decode and forward (DF) relays for a K user MISO BC is proposed, and the RIA is firstly developed under an ideal CSIT case in a square channel (where K = M).
- Introduction of the parameters α and β to indicate the level of outdatedness in the CSIT in S-D, source-to-relay (S-R) and relay-to-destination (R-D) channels.
- The RIA is then presented under four non-ideal CSIT cases, and the region of DoF is derived in each specific case.
- The trade-off between RIA and MAT-IA are analysed.

II. SYSTEM MODEL

In this study, we consider a time division duplex (TDD) based multi-users (MUs) network, which includes a localised BS having K antennas and K MUs with each user having a single antenna. The S-D CSIT in BS is assumed to be totally outdated at all times. To help to facilitate IA strategy in such scenario, a cooperative relay system, which has K relays with each relay having K antennas, is proposed in the K MUs’ MISO BC (Fig.1). The K relays are connected by a relay control station (RCS) via a wired network, so that they can work cooperatively via RCS and will not cause interference in the wireless channels. The same frequency band, however, are reused in the S-R, R-D and S-D channels;
therefore if the channels work together, they will suffer co-channel interference from each other.

For the downlink propagation in Fig. 1, the channel matrices of BS to $i$th MU in the $a$th time slot is denoted as $h_{sd}^{(a)}[t(a)]$, where $h_{sd}^{(a)} \in \mathbb{C}^{K \times K}$. The channel matrices of BS to $i$th MU in the $b$th time slot is denoted as $h_{sr}^{(b)}[t(b)]$, where $h_{sr}^{(b)} \in \mathbb{C}^{1 \times K}$. The channel matrices of $u$th relay to $i$th MU in the $c$th time slot is denoted as $h_{rd}^{(c)}[t(c)]$, where $h_{rd}^{(c)} \in \mathbb{C}^{1 \times K}$. We assume that the signals are transmitted and received in a same unit time slot $\Delta t$, and define $\Delta t = |t(a)| = |t(b)| = |t(c)|$.

**III. RIA Scheme Under An Ideal CSIT Case**

In this section, we first introduce two parameters, $\alpha$ and $\beta$, to indicate the outdatedness level of the CSIT. The first parameter indicates the ratio of the time slots in which the CSIT is outdated ($T_d$) to the total number of time slots ($T$) : $\alpha = T_d/T$. The second parameter indicates the ratio of the MUs ($K_d$), which provide outdated CSI feedback to the transmitters, to the total number of MUs in the system ($K$) : $\beta = K_d/K$. Then, the points $d_{sd} = (\alpha_{sd}, \beta_{sd})$, $d_{sr} = (\alpha_{sr}, \beta_{sr})$ and $d_{rd} = (\alpha_{rd}, \beta_{rd})$ are proposed to indicate the outdatedness level of the CSIT in the S-D, S-R and R-D channels, respectively.

As the S-D CSIT is assumed to be completely outdated in this study, then:

$$d_{sd} = (\alpha_{sd} = 1, \beta_{sd} = 1)$$

(1)

This suggests that the MUs traditional interference-avoiding precoding schemes, such as ZF and block diagonalization (BD) precoding, do not work in the S-D channels.

Under assumption (1), the ideal CSIT case is now given by:

$$D = \{d_{sd}, d_{sr}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (\alpha_{sr} = 0, \beta_{sr} = 0), (\alpha_{rd} = 0, \beta_{rd} = 0)\}$$

(2)

which represents that the BS has ideal S-R CSIT and the selected relay has the ideal R-D CSIT.

Generally, the RIA strategy is implemented by two phases, which are processed one after another.

**Phase 1:** A $K$-length message, which is intended for the $i$th MU and transferred from the BS with $K$ antennas, is given by $x_i^{(i)} = [x_i^{(1)}, \ldots, x_i^{(K)}]$, where $x_i^{(i)} \in \mathbb{C}^{K \times 1}$.

Under the scenario (2), the BS has the ideal S-R CSIT; therefore, the interference-avoiding precoding can be used at the BS to send the $i$th message to the $i$th relay. As the transmitter (BS) and each receiver (each relay) have the same number of antennas, the SLNR precoding is considered in S-R channels, which is given by:

$$w_i^{(i)} = \arg \max_{w_i^{(i)} \in \mathbb{C}^{K}} \left( \frac{\|h_{sr}^{(i)}w_i^{(i)}\|^2}{\sum_{k=1,k \neq i}^K \|h_{sr}^{(i)}w_k^{(k)}\|^2} \right)$$

(3)

where $w_i^{(i)}$ is the precoding matrix for the $x_i^{(i)}$ and $(\sigma_i^{(i)})^2$ is the variance of identically distributed (i.i.d) white Gaussian noise, $z_i^{(i)}$, at the $i$th relay. The received signal at the $i$th relay at time $t(g)$ is expressed as follows:

$$r_i^{(i)}[t(g)] = h_{sr}^{(i)}[t(g)]w_i^{(i)}x_i^{(i)} + \sum_{k=1,k \neq i}^K w_k^{(k)}x_k^{(k)} + z_i^{(i)}$$

(4)

With the SLNR precoding under ideal S-R CSIT scenario, the interference term $\sum_{k=1,k \neq i}^K h_{sr}^{(i)}[t(g)]w_k^{(k)}x_k^{(k)}$ in (4) can be ideally reduce to zero. Therefore, with $K$ antennas, each relay gets a full rank to recover its desired message with the signal being received in time $t(g)$.

The propagation time through the air is ignored; therefore, when the BS transmits messages $s$ to the relays, all MUs can also hear them in time $t(g)$. The received signal at the $i$th MU is as follows:

$$y_{sd}^{(i)}[t(g)] = h_{sd}^{(i)}[t(g)]w_i^{(i)}x_i^{(i)} + \sum_{k=1,k \neq i}^K w_k^{(k)}x_k^{(k)} + z_i^{(i)}$$

(5)

where the interference term $\sum_{k=1,k \neq i}^K h_{sd}^{(i)}[t(g)]w_k^{(k)}x_k^{(k)} \neq 0$, and $z_i^{(i)}$ is the i.i.d white Gaussian noise received by the $i$th MU. The required time slots, $t_{p1}$, in Phase 1 of an ideal case is given by $t_{p1} = |t(g)| = \Delta t$, where $\Delta t$ is the unit propagation time slot.

**Phase 2:** After Phase 1, each MU will feed back a message, which contains the S-D CSI, to the $K$ relays via uplink channels. Meanwhile, through the orthogonal uplink reference pilots, the channel matrices of $K$ MUs to $u$th relay (D-R CSI) can be obtained, which is expressed as $H_{dr,(u)} = [h_{dr,(u)}^{(1)}, h_{dr,(u)}^{(2)}, \ldots, h_{dr,(u)}^{(K)}]$, where $h_{dr,(u)}^{(i)}$ is the $i$th MU to $u$th relay’s channel matrices. As the TDD system is considered in this study, the uplink and downlink channels are reciprocal which means that the required downlink R-D CSI can be
obtained from uplink D-R CSI. Let $(A)^T$ denote the transpose of the matrix $A$, then channel of $u^{th}$ relay to $K$ MUs is given as: $H_{rd,(u)} = (H_{dr,(u)})^T$. The uplink pilots, however, may be non-orthogonal if the $K$ MUs are not cooperative with each other; in this case, the maximum likelihood estimator, such as Bayesian Estimator [8], should be adopted in each relay to estimate the D-R CSI. The uplink estimation process will not be explained in detail in this study.

With the knowledge of $H_{dr,(u)}$, the uplink feedback messages, which is contained the S-D CSI information, can be recovered by $u^{th}$ relay. Once the relays obtain S-D CSI, the RCS will then select one of the relays with the ideal R-D CSIT to generate the RIA precoding matrix. The main idea of the RIA precoding is to make each MU ‘see’ the same linear combination of the interference signals, which are received in Phase 1. We assume that the signals received at the $j^{th}$ time slot, $t^{(j)}$, by the MUs are expressed as follows:

$$y_{rd}[t^{(j)}] = (H_{rd}[t^{(j)}])^T \begin{bmatrix} V^{(1)}[t^{(j)}]x^{(1)} \\ V^{(2)}[t^{(j)}]x^{(2)} \\ \vdots \\ V^{(K)}[t^{(j)}]x^{(K)} \end{bmatrix} + Z_d$$ \hspace{1cm} (6)

where $Z_d = [z_d^{(1)}, \ldots, z_d^{(K)}]$. In addition, the $H_{rd}[t^{(j)}]$ in (6) is the channel from the selected relay to all MUs, which is denoted by $H_{rd}[t^{(j)}] = [h_{rd}^{(1)}[t^{(j)}], \ldots, h_{rd}^{(K)}[t^{(j)}]]$, and the $V^{(i)}[t^{(j)}]$ is the RIA precoding matrix for the $i^{th}$ MU, which is given by:

$$V^{(i)}[t^{(j)}] = \begin{bmatrix} h_{sd}^{(1)}[t^{(j)}] \\ h_{sd}^{(i-1)}[t^{(j)}] \\ h_{sd}^{(i)}[t^{(j)}] \\ \vdots \\ h_{sd}^{(K)}[t^{(j)}] \end{bmatrix}^{-1} \begin{bmatrix} h_{sd}^{(1)}[t^{(j)}]w^{(1)} \\ h_{sd}^{(i-1)}[t^{(j)}]w^{(i-1)} \\ h_{sd}^{(i)}[t^{(j)}]w^{(i)} \\ \vdots \\ h_{sd}^{(K)}[t^{(j)}]w^{(K)} \end{bmatrix}$$ \hspace{1cm} (7)

Then, with the RIA matrix (7), the interference signals at the $i^{th}$ MU at time $t^{(j)}$ are identical to the interference term received in (5) in Phase 1, as $h_{rd}^{(i)}[t^{(j)}] \sum_{k=1, k \neq i}^{K} V^{(k)}x^{(k)} = h_{sd}^{(i)}[t^{(j)}] \sum_{k=1, k \neq i}^{K} w^{(k)}x^{(k)}$.

To achieve a full rank to recover the $K$-length message, the $i^{th}$ MU also needs to receive the signal from the relay in the following $K-1$ time slots, and the full set of received signals from $t^{(j)}$ to $t^{(j+K-1)}$ are expressed as follows:

$$\begin{bmatrix} y_{rd}^{(i)}[t^{(j)}] \\ \vdots \\ y_{rd}^{(i)}[t^{(j+K-1)}] \end{bmatrix} = \begin{bmatrix} \sum_{k=1, k \neq i}^{K} h_{sd}^{(i)}[t^{(j)}]w^{(k)}x^{(k)} + h_{rd}^{(i)}[t^{(j)}]w^{(i)}x^{(i)} + z_d^{(i)} \\ \vdots \\ \sum_{k=1, k \neq i}^{K} h_{sd}^{(i)}[t^{(j+K-1)}]w^{(k)}x^{(k)} + h_{rd}^{(i)}[t^{(j+K-1)}]w^{(i)}x^{(i)} + z_d^{(i)} \end{bmatrix}$$ \hspace{1cm} (8)

Then, for the $i^{th}$ MU, the Phase 2 received signals in the $K$ time slots (8) can be linearly combined with the received signal (5) from the BS in Phase 1:

$$\begin{bmatrix} y_{rd}^{(i)}[t^{(j)}] - y_{sd}^{(i)}[t^{(g)}] \\ \vdots \\ y_{rd}^{(i)}[t^{(j+K-1)}] - y_{sd}^{(i)}[t^{(g)}] \\ \left(h_{rd}^{(i)}[t^{(j)}]w^{(i)} - h_{rd}^{(i)}[t^{(g)}]w^{(i)}\right)x^{(i)} \\ \vdots \\ \left(h_{rd}^{(i)}[t^{(j+K-1)}]w^{(i)} - h_{rd}^{(i)}[t^{(g)}]w^{(i)}\right)x^{(i)} \end{bmatrix} = \begin{bmatrix} y_{rd}^{(i)}(t^{(j)}) \\ \vdots \\ y_{rd}^{(i)}(t^{(j+K-1)}) \\ \left(h_{rd}^{(i)}[t^{(j)}]w^{(i)} - h_{rd}^{(i)}[t^{(g)}]w^{(i)}\right)x^{(i)} \\ \vdots \\ \left(h_{rd}^{(i)}[t^{(j+K-1)}]w^{(i)} - h_{rd}^{(i)}[t^{(g)}]w^{(i)}\right)x^{(i)} \end{bmatrix}$$ \hspace{1cm} (9)

Thus, the required time slots $t_{p2}$ in Phase 2 of the ideal case is $t_{p2} = K \Delta t$. With result (10), the full rank matrix is achieved for the $i^{th}$ MU to decode the $K$-length message. During the two propagation phases, at least $t_{p1} + t_{p2}$ time slots are required, in other words, to decode the $K$-length message at each MU, the channels are used $(t_{p1} + t_{p2})/\Delta t$ times. Therefore, the sum of DoF (sum-DoF) of $K$ MUs network is obtained:

$$DoF_1(K) = K \cdot \frac{K}{(t_{p1} + t_{p2})/\Delta t} = \frac{K^2}{K + 1}$$ \hspace{1cm} (11)

where the CSIT outdated level satisfies (2). Note that the maximum number of DoF (11) under ideal case (2) is also the upper bound of achievable DoF by RIA under each non-ideal case in the following section.

IV. ACHIEVABLE DOF BY RIA AND TRADE-OFF UNDER NON-IDEAL CSIT CASES

A. Outdated S-R CSIT at the BS

1) All S-R CSIT is Outdated in Partial Time Slots: We first consider the case in which all relays provide outdated CSI feedback to the BS in the partial time slots: $d_{sr} = (0 < \alpha_{sr} \leq 1, \beta_{sr} = 1)$. The S-D CSIT and the R-D CSIT are the same as those in the ideal case (2), where $\{d_{sd}, d_{rd}\} = \{\alpha_{sd} = 1, \beta_{sd} = 1\}, (\alpha_{rd} = 0, \beta_{rd} = 0)\}.

For the propagation process in Phase 1, unlike the ideal case, the multiuser interference-avoiding precoding for the S-R channels can only prevent interference in the S-R channels during the partial time slots $T - T_d$. Therefore, to transfer $K$ desired messages to $K$ relays in Phase 1, the minimum required propagation time, $t_{p1}$, is as follows:

$$t_{p1} = \frac{\Delta t}{1 - \alpha_{sr}} = \frac{\Delta t}{1 - \alpha_{sr}}$$ \hspace{1cm} (12)

In Phase 2, the required time slots are the same as those in the ideal case (2), as there is no change in $d_{rd}$. The number of sum-DoF under the RIA scheme in this case is obtained as follows:

$$DoF_2(K) = K \cdot \frac{K}{(t_{p1} + t_{p2})/\Delta t} = \frac{K^2 (1 - \alpha_{sr})}{1 + K - K\alpha_{sr}}$$ \hspace{1cm} (13)

In comparison with the non-relay based MAT-IA scheme, the following Lemma can be proven:
Lemma 1. Under the non-ideal CSIT scenario:

\[ D_{n1} = \{d_{sd}, d_{sr}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (0 < \alpha_{sr} \leq 1, \beta_{sr} = 1), (\alpha_{rd} = 0, \beta_{rd} = 0)\} \]  

(14)

The RIA achieves more DoF than the non-relay-based MAT-IA scheme when \( \alpha_{sr} < 1 - \frac{1}{K(\sum_{k=1}^{K} (k-1))} \).

Proof: According to the MAT-IA in [5], the number of DoF for \( K \) users of a MISO BC with \( d_{sd} = (\alpha_{sd} = 1, \beta_{sd} = 1) \) is given by:

\[ DoF_{MAT}(K) = \frac{K}{\sum_{k=1}^{K} (k-1)} \]  

(15)

As an optimal IA scheme under this case, the achievable sum-DoF by RIA is bounded by (lower bound): \( DoF_2(K) > DoF_{MAT}(K) \), where

\[ K^2(1 - \alpha_{sr}) - 1 - K\alpha_{sr} > \frac{K}{\sum_{k=1}^{K} (k-1)} \]  

(16)

and this results in \( \alpha_{sr} < 1 - \frac{1}{K(\sum_{k=1}^{K} (k-1))} \).

2) Partial S-R CSIT is Outdated in All Time Slots: Let us now consider the case in which the partial relays providing outdated CSI feedback to the BS in all the time slots: \( d_{sr} = (\alpha_{sr} = 1, 0 < \beta_{sr} \leq 1) \). The S-D CSIT and the R-D CSIT are the same as those in the ideal case (2):

\[ \{d_{sd}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (\alpha_{rd} = 0, \beta_{rd} = 0)\} \]  

(17)

In Phase 1, the SLNR precoding for the S-R channels is only used for \((1 - \beta_{sr})K\) relays, which provide ideal CSI feedback to the BS. Next, the BS transfers the desired signal to the remaining \( \beta_{sr}K \) relays following the time division multiple access (TDMA) scheme. Therefore, in Phase 1, at least \( t_{p1} \) time slots are required for all the relays to achieve their desired messages with no interference, where \( t_{p1} = t_{p1}^{(2)} + \beta_{sr}K\Delta t \). The minimum number of required time slots in Phase 2 is the same as that in the ideal case, as there is no change in \( d_{rd} \).

Therefore, the achievable sum-DoF is obtained as follows:

\[ DoF_3(K) = K \cdot \frac{K}{(t_{p1} + t_{p2})/\Delta t} = \frac{K^2}{K(1 + \beta_{sr}) + 1} \]  

(18)

Furthermore, the following Lemma can be proven:

Lemma 2. Under the non-ideal CSIT scenario:

\[ D_{n2} = \{d_{sd}, d_{sr}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (0 < \alpha_{sr} \leq 1, \beta_{sr} = 1), (\alpha_{rd} = 0, \beta_{rd} = 0)\} \]  

(19)

The RIA scheme achieves more DoF than the non-relay-based MAT-IA scheme when \( \beta_{sr} < \frac{1}{K\sum_{k=1}^{K} (k-1)} \).

Proof: As an optimal IA scheme, the achievable sum-DoF by RIA is bounded by (lower bound): \( DoF_3(K) > DoF_{MAT}(K), \) where \( \frac{K^2}{K(1 + \beta_{sr}) + 1} > \frac{1}{\sum_{k=1}^{K} (k-1)}. \) This will result in

\[ \beta_{sr} < \frac{K}{\sum_{k=1}^{K} (k-1)} - \left(1 + \frac{1}{K}\right) = \sum_{k=2}^{K-1} (k-1) \]  

(20)

B. Outdated R-D CSIT at the Relays

1) All R-D CSIT is Outdated in Partial Time Slots: Once again, let us first consider the case in which all the MUs provide outdated CSI feedback to the \( K \) relays in the partial time slots: \( d_{rd} = (0 < \alpha_{rd} \leq 1, \beta_{rd} = 1) \). The S-D CSIT and the S-R CSIT at the BS are the same as those in the ideal case (2):

\[ \{d_{sd}, d_{sr}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (\alpha_{sr} = 0, \beta_{sr} = 0)\} \]  

(21)

In Phase 1, as the BS has ideal S-R CSIT, the propagation process from the BS to the relays and the MUs is the same as it was in the ideal case. In Phase 2, the RIA scheme is only implemented by the selected relay in \( \alpha_{rd}T \) time slots. Therefore, the minimum propagation time slots \( t_{p2} \) in this case is \( \frac{K\Delta t}{1 - \alpha_{rd}} \). Then, the achievable sum-DoF is obtained:

\[ DoF_4(K) = K \cdot \frac{K}{(t_{p1} + t_{p2})/\Delta t} = \frac{K^2(1 - \alpha_{rd})}{K + 1 - \alpha_{rd}} \]  

(22)

In addition, the following Lemma can be proven:

Lemma 3. Under the non-ideal CSIT scenario:

\[ D_{n3} = \{d_{sd}, d_{sr}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (0 < \alpha_{sr} \leq 1, \beta_{sr} = 1), (\alpha_{rd} = 0, \beta_{rd} = 0)\} \]  

(23)

The RIA scheme achieves more DoF than the non-relay-based MAT-IA scheme when \( \alpha_{rd} < 1 - \frac{K}{\sum_{k=1}^{K-1} (k-1)} \).

Proof: As an optimal IA scheme, the achievable sum-DoF by RIA is bounded by (lower bound): \( DoF_3(K) > DoF_{MAT}(K), \) then this will result in the inequality:

\[ \alpha_{rd} < 1 - \frac{K}{K\sum_{k=1}^{K} (k-1) - 1} = 1 - \frac{1}{\sum_{k=1}^{K-1} (k-1)} \]  

(24)

2) Partial R-D CSIT is Outdated in All Time Slots: Now, we consider the case in which the partial MUs provide outdated CSI feedback to relays in all time slots:

\[ d_{rd} = (\alpha_{rd} = 1, 0 < \beta_{rd} \leq 1) \]  

(25)

The S-D CSIT and the S-R CSIT at the BS are the same as those in the ideal case (2):

\[ \{d_{sd}, d_{sr}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), (\alpha_{sr} = 0, \beta_{sr} = 0)\} \]  

(26)

Similarly, in Phase 1, as the BS has ideal S-R CSIT, the propagation processes from the BS to the relays or to the MUs are the same as those in the ideal case. In Phase 2, the relay-based IA can be regarded as an RIA and MAT-IA combined scheme. Because of the outdated CSIT of the selected relay to \( \beta_{rd}K \) MUs, the RIA scheme is only implemented for the \((1 - \beta_{rd})K \) MUs. To achieve a full rank for recovering message of length \( K \), the minimum required time slots \( t_{p2} \) for the \((1 - \beta_{rd})K \) MUs is equal to \( K\Delta t \). After this, the MAT-IA scheme will be implemented for the \( \beta_{rd}K \) MUs which
provide completely outdated CSI to the relays. According to the partial-users MAT-IA in [6], the required time slots, \( t^2_{p2} \), for the \( \beta_{rd}K \) MUs to recover the \( K \)-length messages are given by:

\[
t^2_{p2} = \left[ \frac{1}{\beta_{rd}} \right] \beta_{rd}K \sum_{k=1}^{\beta_{rd}K} (k^{-1})
\]

(27)

Therefore, at least \( t_{p2} \) time slots is required in Phase 2, where \( t_{p2} = t_{p1} + t^2_{p2} \). The achievable sum-DoF is obtained:

\[
\text{DoF}_{\text{R}}(K) = \frac{K}{K-1 + \frac{1}{\beta_{rd}} \beta_{rd}K} \sum_{k=1}^{\beta_{rd}K} (k^{-1})
\]

(28)

In comparison with the full-users MAT-IA scheme, the following Lemma can be proven:

**Lemma 4.** Under non-ideal CSIT scenario:

\[
D_{n4} = \{d_{sd}, d_{sr}, d_{rd}\} = \{(\alpha_{sd} = 1, \beta_{sd} = 1), \left(\alpha_{sr} = 0, \beta_{sr} = 0\right), (\alpha_{rd} = 1, 0 < \beta_{rd} \leq 1)\}
\]

(29)

If \( \frac{1}{\beta_{rd}} \in Z \), the RIA scheme achieves more DoF than the non-relay-based MAT-IA scheme when \( \beta_{rd} < e^{1+K-1} \).

**Proof:** Assume that \( \gamma = \left[ \frac{1}{\beta_{rd}} \right] \beta_{rd} \); when \( \frac{1}{\beta_{rd}} \in Z \), there is \( \gamma = 1 \). If \( K \) is sufficiently large, then (15) is equal to \( \frac{K}{\ln K} \) [6]. Similarly, equation (28) is approximately given by:

\[
\text{DoF}_{\text{R}}(K) \approx \frac{K}{K-1 + \gamma \ln (\beta_{rd}K)}
\]

(30)

As an optimal scheme, the achievable sum-DoF by RIA is bounded by (lower bound):

\[
\frac{K}{K-1 + \gamma \ln (\beta_{rd}K)} \geq \frac{K}{\ln K}
\]

(31)

Equation (31) results in

\[
\beta_{rd} < K^{-1} \sqrt{K e^{-(1+K-1)}}
\]

(32)

When \( \frac{1}{\beta_{rd}} \in Z \), \( \gamma \) will be equal to 1, and (32) can be change to \( \beta_{rd} < e^{-(1+K-1)} \).

**C. Numerical Results**

The Fig.2 shows the achievable sum-DoF by RIA under the four non-ideal scenarios. The number of MUs in the numerical results is 50 (K=50). The upper blue curve is the upper bound of achievable sum-DoF by RIA (under ideal case (2)). The lower blue curve is the DoF region of MAT-IA. For the MAT-IA scheme, any non-ideal CSIT case, such as \( d = (0 \leq \alpha < 1, \beta = 1) \), will be regarded as a completely outdated CSIT case: \( d = (\alpha = 1, \beta = 1) \). Therefore the trend of the DoF of the MAT-IA does not adaptively change with \( \alpha \) or \( \beta \). Note that the MAT-IA is possible to achieve more DoF than the RIA in some cases, e.g. \( \alpha_{sr} = 0.98 \) (\( D_{n1} \)) and \( \beta_{rd} = 0.79 \) (\( D_{n3} \)), however, the DoF gap between MAT-IA and RIA is not significant. In general, the RIA can still be regarded as an optimal scheme in the large number multiuser networks.

**References**


