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Sediment transport by runoff on debris-mantled dryland hillslopes

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[1] Hillslopes supply sediment to river channels, and therefore impact drainage basin functioning and evolution. The relationship between hillslope attributes and sediment flux forms the basis of geomorphic transport laws used to model the long-term topographic evolution of drainage basins, but their specific interactions during individual storm events are not well understood. Runoff-driven erosion of coarse particles, prevalent in dryland environments, presents a particular set of conditions for sediment transport that is poorly resolved in current models. In order to address this gap, we developed a particle-based, force-balance model for sheetwash sediment transport on coarse, debris-mantled hillslopes within a rainfall-runoff model. We use the model to examine how the interplay between hillslope attributes (gradient, length and grain size distribution) and runoff characteristics affects sediment transport, grain-size changes on the hillslope, and sediment supply to the slope base. The relationship between sediment flux and hillslope gradient was found to transition from linear above a threshold to sigmoidal depending on hillslope length, initial grain sizes, and runoff characteristics. Grain sizes supplied to the slope base vary in a complex manner with hillslope attributes but an overall coarsening of the hillslopes is found to occur with increasing gradient, corroborating previous findings from field measurements. Intense, short duration storms result in within-hillslope sediment redistribution and equifinality in sediment supply for different hillslope characteristics, which explain the lack of field evidence for any systematic relationships. Our model findings provide insights into hillslope responses to climatic forcing and have theoretical implications for modeling hillslope evolution in dry lands.


1. Introduction

[2] Processes of hillslope erosion fundamentally impact landscape morphology and evolution. They regulate landscape relief and, through sediment supply to river channels, they influence rates of incision [e.g., Sklar and Dietrich, 2001] and river long profile development [e.g., Solyom and Tucker, 2004]. Hillslope grain sizes, particularly the coarse sediment fractions (>2 mm), affect the sediment classes delivered to valley floors and the amount of bed material [Wolcott, 1988] thereby modifying river channel sedimentary characteristics and affecting long-term rates of aggradation and degradation. Sediment supply from hillslopes to valley floors is poorly constrained because it is episodic and spatially variable within basins [Korup, 2009], and is controlled by the interplay between hillslope characteristics and climatic forcing [Tucker and Bras, 2000]. Within different hydroclimatic settings, hillslopes erode by different dominant mechanisms including concentrated mass movement, rill and gully erosion, sheetwash erosion, soil creep, bioturbation and gravitational processes [Carson and Kirkby, 1972]. The relationship between hillslope attributes and sediment flux forms the basis of geomorphic transport laws used to model the topographic evolution of drainage basins [Dietrich et al., 2003]. Therefore, gaining functional insights into the controls on hillslope sediment flux for different erosional mechanisms is necessary to constrain both basin-scale sediment supply and to determine the limits of applicability of generalized transport equations.

[3] Theoretical understanding of the relationship between sediment flux and hillslope characteristics is largely based on ‘disturbance-driven sediment transport’ regimes [Roering et al., 2007] – sediment transport processes occurring in the absence of overland flow. Under these conditions, nonlinear relationships between sediment flux and gradient have been theoretically derived and experimentally documented for creep and landsliding [Kirkby, 1967; Roering et al., 1999, 2001a], bioturbation [Gabet, 2000], dry ravel [Gabet, 2003] and tree throw [Gabet et al., 2003]. This nonlinearity arises because sediment transport increases more rapidly as slopes reach a threshold gradient equivalent to the angle of repose. While it is now widely acknowledged that slope gradient exerts a first order and nonlinear control on sediment flux induced by different mechanisms, there remains a theoretical...
use the terms ‘overland flow’ and ‘hillslope runoff’ interchangeably to denote surface water flow on hillslopes and the term ‘sheetwash erosion’ or ‘surface wash’ to denote the erosion induced by interrill surface water flow.

2. Debris-Mantled Hillslopes in Dryland Environments

[6] Debris-mantled hillslopes are common in arid and semiarid environments (Figure 1) where low rates of chemical weathering give rise to thin, non-cohesive soils mantled with a layer of coarse rock fragments derived from weathered bedrock that can reach boulder size [Abrahams et al., 1994; Carson and Kirkby, 1972]. Mean annual rainfall in such settings is low, but is delivered as infrequent and short-lived yet high intensity and spatially variable storm events that generate surface infiltration-excess runoff from sparsely vegetated hillslopes. Brief rainstorms coupled with spatially varying surface characteristics often result in patchy and short-lived runoff generation [Yair et al., 1978], which leads to localized sediment transport on hillslopes and episodic sediment delivery to the slope base. However, the material supplied to the hillslope surface by weathering must be in equilibrium with the downslope transport processes because there is no significant accumulation of debris on these slopes [Carson and Kirkby, 1972]. Nevertheless, numerous field experiments in dryland environments are inconclusive with respect to the functional relationships between hillslope attributes, rainfall rates and the resultant runoff and sediment transport rates. For example, some studies find an inverse relationship between sediment transport and hillslope gradient [Yair and Klein, 1973], a curvilinear relationship with a peak at 12° [Abrahams and Parsons, 1991] or no relationship at all [Bracken and Kirkby, 2005]. Experimentally derived relationships between hillslope gradient, particle sizes and surface runoff rates have been similarly varied due to the complex interaction between coarse debris cover, infiltration and runoff dynamics [Lavee and Poesen, 1991; Poesen et al., 1990; Yair and Lavee, 1976]. The diversity of these findings is in part explained by the use of small-scale rainfall simulation plots that isolate particular sections of the hillslope and limit the development of hydraulic flow, which would be expected to exert a significant control on longer slope segments.

[7] While many runoff-based erosion models have been developed [Laflen et al., 1991; Morgan et al., 1998; Smith et al., 1995], they tend to be either empirical (lacking representation of physical mechanisms) or over parameterized for the useful exploration of functional relationships between hillslope attributes and sediment transport. Such models, generally developed for soil-mantled catchments, have narrow limits of applicability in terms of soil grain sizes and are thus inappropriate for debris-mantled, steep dryland hillslopes [Abrahams et al., 1994]. Similarly, bed load transport models developed for river channels have limitations when applied to hillslopes because of variations in critical entrainment thresholds arising due to the different flow depth/particle diameter ratios on hillslopes compared to channels [Abrahams et al., 2001; Torri et al., 1990]. Studies have shown that the Shields relation, although widely applicable for representing incipient motion of sediment in rivers [Buffington and Montgomery, 1997], does not apply to

Figure 1. Photograph of a debris-mantled hillslope in the Negev, Israel, illustrating the very coarse rock fragments over the hillslope surface and the absence of vegetation. The slope length is ~150 m, the gradient is ~15° and the largest rock fragments are ~128 mm. The fine matrix underlying the rock fragments is primarily medium to coarse sand.
Table 1. Range of Model Variables Used in the Model Experiments

<table>
<thead>
<tr>
<th>Model Variables</th>
<th>Default Values</th>
<th>Range Explored</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hydrological</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rainfall intensity (mm h(^{-1}))</td>
<td>50</td>
<td>37.5, 50, 75, 100, 150</td>
</tr>
<tr>
<td>Rainfall duration (h)</td>
<td>1</td>
<td>0.1, 0.2, 0.4, 1</td>
</tr>
<tr>
<td>Initial infiltration rate (mm)</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>Final infiltration rate (mm h(^{-1}))</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Manning’s friction (n)</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td><strong>Hillslope Characteristics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (m)</td>
<td>50</td>
<td>10–350</td>
</tr>
<tr>
<td>Width (m)</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Gradient ((^{\circ}))</td>
<td>20</td>
<td>10–50</td>
</tr>
<tr>
<td>Profile shape</td>
<td>Straight</td>
<td></td>
</tr>
<tr>
<td><strong>Sediment Characteristics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean bed roughness length ((k_s, \text{mm}))</td>
<td>2.5</td>
<td>1.5–5.0</td>
</tr>
<tr>
<td>Initial hillslope GSD (denoted here by the (d_{50}, \text{mm}))</td>
<td>32</td>
<td>32–128</td>
</tr>
<tr>
<td>Grain size uniformity</td>
<td>mixed, (d_{50} = 32 \text{ mm})</td>
<td>single-class ((d = 4–32 \text{ mm})), multi-class ((d_{50} = 32–128 \text{ mm}))</td>
</tr>
<tr>
<td>Total number of particles</td>
<td>3000 p/m(^2)</td>
<td></td>
</tr>
<tr>
<td>Particle density (g cm(^{-3}))</td>
<td>2.5–2.9</td>
<td></td>
</tr>
<tr>
<td>Lift coefficient ratio</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td><strong>Other</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grid cell size, (d_x (\text{m}))</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Timestep, (dt (\text{s}))</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

runoff-driven sediment transport on debris-mantled hillslopes due to the shallow flow depth in relation to particle diameter [Abrahams et al., 1988a; Guy and Dickinson, 1990]. Sediment transport on steep slopes covered in coarse rock fragments is governed by particular hydraulic and gravitational processes which require new modeling approaches.

In this paper we develop a physics-based sediment transport model for debris-mantled hillslopes which we use to investigate functional relationships between hillslope characteristics and rainfall properties in terms of sediment flux and grain size distribution (GSD) sediment supplied to the hillslope base during individual runoff events.

### 3. Model Description

An event-driven, particle-based model for sheetwash sediment transport on debris-mantled hillslopes is developed within a pre-existing rainfall-runoff model. The rainfall-runoff model produces spatial values of flow depth and velocity which are used to drive a particle-by-particle force-balance model derived from first principles for grain sizes >1 mm. In other words, this model is designed to transport coarse particles and ignores fine sediments that are more prevalent on soil-mantled hillslopes. Particles on the hillslope surface can be composed of mixed sizes of any distribution or of single grain sizes of any diameter greater than 1 mm. No assumptions are made about incipient motion (e.g., using Shields criterion) because the model resolves all the forces on each particle at each time and space step based on the flow hydraulics acting on them. In constructing the model, we make the following simplifying assumptions: No distinction is made between the mode of transport (rolling, sliding, saltating) – particles are only assumed to be in transport. Particle hiding and particle-on-particle collisions are not represented because they are considered stochastic processes that would obfuscate understanding of the primary controls on sediment flux. The model operates over timescales of individual rainfall events and therefore topographic evolution of the hillslope is not simulated.

#### 3.1. Runoff Simulation

Dynamic overland-flow generation is modeled using the hillslope component of COUP2D, a distributed model of rainfall-runoff processes for dryland catchments that simulates infiltration-excess and saturation-excess runoff as a result of filling a fixed soil moisture store [Michaelides and Wainwright, 2002; Michaelides and Wilson, 2007; Michaelides and Wainwright, 2008]. In brief, COUP2D represents infiltration on the hillslopes using the modified Green and Ampt [1911] infiltration model to determine overland flow, which is routed on a 2D rectangular grid using the kinematic wave approximation, rated using the Manning’s \(n\) friction factor, with flow routing from cell to cell defined by a steepest descent algorithm. COUP2D produces spatial values of runoff flow depths and velocities which constitute the driving force for the particle transport model.

#### 3.2. Sediment Transport by Runoff

The sediment-transport component is based on a force-balance approach applied to discrete, coarse particles of multiple sizes. The model explicitly represents surface GSD on the hillslope and calculates particle fluxes and transport distances driven by dynamic drag and lift forces induced by fully or partially submerged flow conditions derived from the computed flow hydraulics. Particles of various sizes are generated from a user-defined distribution of grain diameters and the corresponding particle masses are derived from the input diameters combined with a narrow range of particle densities for a given lithology (Table 1).
Varying particle density within a defined range ensures that not all particles of a given diameter have exactly the same masses and introduces some variability that compensates for using the same shape for all particles in the model, assumed here to be spherical. Particles are scattered randomly on the hillslope grid at the start of each simulation ensuring that there are no biases in transport based on initial positions on the grid. The number of particles applied to the grid can vary, but in this study it was scaled by the hillslope size based on 3000 particles per m$^2$ – we assume 1 m$^2$ is fully covered by $D_{50}$ (= 32 mm) sized particles. The total number of particles is defined as an initial condition within each model run and the boundary condition of particle flux both at the hillslope crest and over the whole hillslope profile, is zero, based on the assumption that during a single rainfall event no new sediment is being delivered to the slope via weathering or other processes. This is a reasonable assumption given that dryland hillslopes are typically weathering-limited [Carson and Kirkby, 1972], although it could be relaxed in subsequent versions of the model running over longer timescales.

[12] On debris-mantled hillslopes, the most common condition during runoff events is shallow, rapid flows with flow depths generally smaller than the mean particle diameter [Carson and Kirkby, 1972]. Currently there is no consensus theoretical approach for dealing with sediment transport under partially submerged flow conditions, especially on steep slopes, where transport of coarse particles occurs in the absence of deep flows. Conventional theory on initiation of motion states that under fully submerged conditions, movement of a particle is caused by lift and drag forces that rotate particles out of pockets typical of a water-worked bed [Middleton and Southard, 1978; Wiberg and Smith, 1987]. However, there is no developed theory for sediment transport under partially submerged conditions, where particles are not tightly packed and are primarily exposed to drag forces, but where the interaction between the submerged cross-sectional area of a particle and the flow is poorly understood [Abrahams et al., 1988a; Dwivedi et al., 2010; Gregoretti, 2008]. Several field and experimental studies have found that coarse particles selectively move faster and farther than fine ones on alluvial fans [Stock et al., 2008] and steep shallow rivers [Aguirre-Pe et al., 2003; Dey, 2003; Solari and Parker, 2000] further supporting that shear stress based theory of incipient motion may not be applicable on hillslopes.

[13] Here we represent sediment transport by shallow flows on debris-mantled hillslopes using a force-balance approach. This scenario differs from coarse grained river beds (Figure 2) because: a) coarse debris on dryland hillslopes tend to overlie matrix-supported regolith [Abrahams et al., 1994], whereas water-worked river beds are clast-supported; b) hillslope gradients are significantly steeper than river channel gradients; and c) hillslope overland flow is shallow relative to grain diameter, compared to river flow. Therefore, on hillslopes the diameter of coarse grains is large relative to the underlying bed roughness length and grain protrusion into the flow is high (Figure 2a) compared to rivers, where the underlying bed roughness is high and grain protrusion into the flow is relatively low (Figure 2b). Both relative roughness of the underlying substrate and grain protrusion into the flow affect the angle of repose of individual grains and hence initiation of motion [Li and Komar, 1986; Miller and Byrne, 1966; Wiberg and Smith, 1987]. We calculate the angle of repose for each individual grain using the formulation developed in Wiberg and Smith [1987] based on data in Miller and Byrne [1966] expressed as a ratio of the grain diameter to bed roughness length:

$$\phi_o = \cos^{-1}\left(\frac{D}{k_s} + \frac{z^*}{k_s + T}\right),$$

where $\phi_o$ is the particle angle of repose, $D$ is particle diameter (m), $k_s$ is the mean bed roughness length (m) and $z^*$ is the average level of the bottom of the almost moving grain (and depends on particle sphericity and roundness). A large ratio of $D/k_s$ represents a coarser particle on a finer bed and vice versa. We use a value of $z^* = -0.02$ representing natural sand with sphericity of 0.7 (Zingg shape index) and roundness of 0.5 (Russell and Taylor roundness grades) [Wiberg and Smith, 1987].

[14] Under fully submerged flows, we account for lift and drag forces acting on the particles (Figure 3a). Under partially submerged conditions (when flow depth, $h$ (m), is less than particle diameter, $D$ (m)), we represent the drag force...
acting on the submerged area of the particle (Figure 3b). The lift force, $F_l$ (N) is calculated by [Middleton and Southard, 1978]:

$$F_l = c_l \left( \frac{\rho V^2_{(x,y)}}{2} \right) \pi D^2,$$

where $c_l$ is the lift coefficient, $V$ is flow velocity upstream of the particle (m s$^{-1}$) and $\rho$ is water density (kg m$^{-3}$). The subscripts $x$, $y$, $t$ denote the spatial and temporal dimensions over which velocity changes. The drag force, $F_d$ (N) is calculated by:

$$F_d = c_d \left( \frac{\rho V^2_{(x,y)}}{2} \right) A_w,$$

where $A_w$ is the particle submerged area (m$^2$), calculated as:

$$A_w = \frac{\pi D^2}{4} \text{ for } h \geq D,$$

$$A_w = \frac{\pi D^2}{4} \left( \frac{h_{x,y}}{D} \right) \text{ for } h < D,$$

and $c_d$ is the drag coefficient. The gravity force, $F_g$ (N), acting on a particle is calculated by:

$$F_g = \left( \frac{\pi}{6} \right) D^3 (\rho_s - \rho) g,$$

where $\rho_s$ is particle density (kg m$^{-3}$). On each particle and at each time step, the relative particle submergence is evaluated and the respective forces (lift, drag and gravity) are calculated on each particle based on the computed runoff hydraulics. The forces acting on each particle are resolved and used to determine if it moves:

$$F_r = F_d + F_l + F_g \frac{\sin \theta}{F_g \cos \theta} \tan \phi_s,$$

where $\phi_s$ is the coefficient of static surface friction acting on a particle resisting movement (assumed to approximate the dynamic friction coefficient acting on moving particles [Bagnold, 1966]) and $\theta$ is the hillslope gradient. If $F_r \geq 1$ then transport occurs, and travel distance ($dx$, m) for a particle of mass $m$ is calculated in its time derivative form as:

$$d^2 x = \frac{F}{m} dt^2,$$

where $F$ (N) is the detaching force acting on the particle (i.e., the drag and/or lift force). Changes in the drag coefficient ($c_d$) due to varying particle velocity are calculated using the Chow approximations [Chow, 1979], derived from the experimentally determined relationship between drag coefficient and particle Reynolds number ($Re_p$) for spherical particles:

$$Re_p = \frac{V_p D}{\nu},$$

where $V_p$ is the particle velocity (m s$^{-1}$) and $\nu$ is kinematic viscosity (m$^2$ s$^{-1}$). Incorporating dynamic variations in the drag force is more appropriate for mixed grain sizes and allows for more representative responses of sediment transport to changes in topographic, hydrological, hydraulic and GSD parameters. The lift coefficient is also dynamic because it is calculated with direct reference to the drag coefficient:

$$c_l = n_l c_d.$$

A value of 0.8 is used for the lift coefficient ratio ($n_l$), based on experimental studies [Chepil, 1958] that have found that the lift coefficient is commonly smaller than the drag coefficient. However, this can be varied in the model.

[15] At each time step the balance of forces is resolved on each particle on the hillslope and as overland flow evolves the model routes sediment down the hillslope. Particles

\[\text{Figure 3.} \text{ Diagram showing the forces acting on a particle with mass M that are represented in the model in (a) fully submerged flows, and (b) partially submerged flows where } \theta \text{ is the hillslope gradient, } h \text{ is the flow depth, } \varphi \text{ is the angle of repose and } \tan \varphi \text{ is the coefficient of static friction of each particle.}\]
move independently of the grid cells based on the computed travel distance at each time step, but the grid is used to assign the flow velocity and depth from the nearest cell to a particle at each time step for use in the force balance calculation. When particles reach the base of the hillslope grid they are output to a GSD of hillslope sediment supply to the channel (Figure 4). Throughout the paper we use the term ‘sediment flux’ to represent the sediment supplied to base of the hillslope. This flux is expressed as a normalized ratio of the number of particles arriving at the base of the slope to the initial number of particles seeded on the hillslope. GSDs for sediment data were analyzed using GRADISTAT software [Blott and Pye, 2001] to obtain characteristic grain sizes ($D_{10}$, $D_{50}$, $D_{90}$) by logarithmic method of moments.

4. Model Experiments

[16] In a series of model experiments we address the following questions in the context of debris-mantled hillslopes: (1) What is the functional relationship between hillslope attributes (gradient and length) and sediment flux for individual rainfall events? (2) How is the relationship between sediment flux and slope attributes affected by the initial hillslope GSD? (3) How do rainfall intensity and duration affect the runoff characteristics that control sediment flux?

[17] Prior to conducting the model experiments, we investigated the sensitivity of sediment flux to mean bed roughness length ($k_r$, mm) for slope gradients ranging from 10° to 50° on a 50-m hillslope. The investigated values of $k_r$ ranged from 1.5 to 5.0 mm, representing surfaces of coarse sand through to fine and medium gravel, respectively. The initial hillslope grain size distribution has a $D_{50}$ of 32 mm and particles ranging in diameter from 1 to 64 mm. The aim is to represent a relatively finer bed overlain by coarse debris (Figures 1 and 2a) representative of many dryland hillslopes.

[18] In the first set of simulations, we systematically investigated the relationship between gradient and sediment flux for different slope lengths under equilibrium runoff, defined as the condition whereby at all points on the slope runoff discharge is in steady state. Equilibrium runoff typically arises during long rainfall events. Two rainfall intensities (50 and 100 mm h⁻¹) were applied, each with duration of one hour that results in equilibrium runoff conditions. The rainfall rates chosen are typical of dry lands [Nicholson, 2011] and although infrequent, high intensity long duration rainstorms do occur [Bracken et al., 2008; Nicholson, 2011; Yair and Kossovsky, 2002] and may result in equilibrium runoff on dryland hillslopes where infiltration rates are relatively low or the infiltration capacity has been reached. Hillslope lengths were varied from 10 m to 350 m and gradients from 10° to 50° in order to represent a range of topographic conditions from steep, mountainous landscapes to lowland environments. Each hillslope length/gradient combination was simulated explicitly in separate model runs. We simulate the condition of equilibrium runoff on a range of slopes in order to: a) consistently compare different slope lengths and gradients; b) investigate the phase space of model behavior; and c) compare against the more common scenario of non-equilibrium runoff.

[19] In the second set of simulations, we investigated the effect of the initial hillslope GSD on sediment flux with equilibrium runoff. The initial hillslope GSD was systematically varied using both multi- and single-class grain size compositions on a 50-m slope for two rainfall intensities (50 and 100 mm h⁻¹). For the multiclass sediments, we varied the $D_{50}$ from 32 to 128 mm using six classes of grain size; for the single-class sediments we varied the $D$ from 4 to 32 mm using one grain size class.

[20] The third set of simulations investigated the relationship between runoff dynamics and sediment flux on slopes with different gradient and length. Rainfall intensity and duration were co-varied in order to produce different runoff conditions that are more typical of the short-lived storms experienced in dry lands. The experiments are based on the rationale that higher intensity rainstorms are shorter lived than lower intensity rainstorms. We kept the total depth of rainfall constant (15 mm) between each set of simulations, but in all cases rainfall duration was too short to generate equilibrium runoff. The rainfall rates and durations applied were 150 mm h⁻¹ for 6 min, 75 mm h⁻¹ for 12 min and 37.5 mm h⁻¹ for 24 min. These rainfall rates and durations are typical for many dryland environments [Bracken et al., 2008; Nicholson, 2011; Yair and Kossovsky, 2002].

[21] In the model the hillslope grain size distribution is continuous and binned in phi classes ranging from 1–2 mm to 128–256 mm. The range of bins used depends on the model run and the initial hillslope GSD represented. Table 1 outlines the model variables and their ranges explored.

5. Results

5.1. Sensitivity of Sediment Flux to Hillslope Surface Roughness

[22] Figure 5 presents the relationship between sediment flux and hillslope gradient for different values of mean bed roughness length ($k_r$). The change in $k_r$ has a significant impact on the relationship between sediment flux and gradient because it affects the angle of repose of all grain size classes. Figure 5 shows how as $k_r$ decreases, the sediment flux increases and changes in functional form from linear above a threshold to sigmoidal. This occurs because the
angles of repose decrease as the mean bed roughness length decreases relative to the median grain size of the overlying coarse fraction. We selected \(k_s = 2.5\) mm as the default value used in the remaining modeling experiments to represent a fine gravel bed, but acknowledge that this should be changed in the model depending on the field setting as it has a significant impact on modeled sediment flux rates on hillslopes.

5.2. Sensitivity of Sediment Flux to Gradient and Length Under Equilibrium Runoff

[23] Changes in slope length and gradient exert a significant control on the runoff dynamics, which is the primary driving force of sediment transport. Runoff discharge increases in the downslope direction during rainfall events. Equilibrium runoff increases linearly with slope length as the flow accumulation area increases (Figure 6) and for any hillslope it is, therefore, highest at the slope base. Slope gradient does not affect equilibrium runoff discharge for each slope length, but it does affect flow depth and velocity – flow depth varies inversely, and velocity varies proportionally, with gradient (Figure 7).

[24] The relationship between sediment flux and slope gradient is generally nonlinear under the conditions modeled, with the form changing according to slope length and runoff magnitude (Figure 8). At low effective rainfalls on short slopes (10 and 50 m), the relationship is linear above a threshold gradient (Figure 8a). However, as slope length and effective rainfall increase, the relationship becomes sigmoidal due to the progressively rapid inception of transport of more grain-size classes as gradient increases. Longer slopes produce higher sediment flux at lower gradients but as gradient increases, sediment flux converges for all slope lengths. At high effective rainfall, the relationship between sediment flux and gradient becomes more distinct between different slope lengths and the inflection points of the sigmoidal curves tend to move to the left (toward lower gradients) (Figure 8b). Sediment transport of different grain sizes is dependent on both flow velocity and depth because flow depth determines relative particle submergence and flow velocity affects the magnitude of the drag or lift force. Therefore, both slope length and gradient are important controls of sediment transport. In the case of Figure 8, the effect of higher runoff on longer slopes is evident, since it leads to higher sediment flux at lower gradients compared to shorter slopes. The invariance of sediment flux with length at high gradients suggests that gradient effects dominate over length effects (Figure 8) as slopes approach a threshold angle.

[25] The effect of the interaction between runoff dynamics and hillslope GSD on sediment flux can be better understood by examining the GSD of the sediment supplied to the base of the slope. Figure 9 shows the change in grain size enrichment ratios with hillslope gradient for two rainfall rates and different slope lengths. The enrichment ratios (ER = ratio of supplied grain size at slope base to initial hillslope grain size) for different hillslope lengths vary in a complex, nonlinear way with gradient. The ER typically starts out low (<0.25) and then abruptly increases before decreasing to below 1 at high slope gradients. For shorter slopes (10–100 m), supplied grain sizes to the base of the slope become enriched in material much coarser than the initial (ER = \(C_24\)), as slope gradient reaches the angle of repose for coarse grain size classes (Figures 9a and 9c). As gradient increases further, the grain sizes supplied from shorter slopes become relatively enriched in the finer material while for the longer slopes the supplied sediment becomes progressively enriched in coarser material. An increase in rainfall rate from 50 to 100 mm h\(^{-1}\) results in a systematic coarsening of the supplied sediment for longer slopes and in a relative fining for the shorter slopes, at
lower gradients (Figures 9b and 9d) compared to the lower rainfall rate.

[26] This nonlinear behavior of supplied GSD with changes in slope characteristics is a result of the interaction between several concurrent processes: 1) As gradient increases, the angle of repose of different grain sizes is reached. Larger grains have lower angles of repose than smaller ones and are therefore more prone to moving at lower gradients if a detaching force of the appropriate magnitude acts on them. 2) As gradient increases, flow depth decreases and flow velocity increases for the same runoff discharge. 3) The changes in flow hydraulics with an increase in gradient result in a relative decrease in particle submergence (as depth decreases) but in a potential change in drag force as velocity increases. 4) Longer slopes produce higher equilibrium discharge than shorter slopes, so for the same gradient long slopes have higher flow depths and velocities. Therefore, the interplay between relative submergence, flow velocity and angles of repose for different grain size classes is affected. Both slope length and gradient changes produce complex, nonlinear patterns in sediment supply GSD.

5.3. Sensitivity of Sediment Flux to Initial GSD

[27] Figure 10a shows that the relationship between sediment flux and gradient for single-class GSDs is nonlinear, but the form varies significantly with diameter (from 4 to 32 mm). This arises due to the interaction between the runoff hydraulics, angle of repose and grain size, which affect the particle submergence and drag/lift force relative to particle stability. For the range of diameters examined here, the 16 mm grain size produces the highest overall fluxes over the range of gradients (Figure 10a). Although the coarser grains (32 mm) move at lower gradients than the finer grains due to lower angles of repose, low runoff depths at high slope gradients result in low particle submergence, which is insufficient to sustain high sediment flux rates. Conversely, although the smaller grains (4 mm) are more likely to be fully submerged in the shallow flows, their high angle of repose maintains low sediment flux rates unless runoff rates increase; hence there is a marked difference in the 50 and 100 mm h\(^{-1}\) curves for this grain size (Figure 10a). The intermediate size grains (16 mm) are large enough to have low angles of repose and small enough to be entrained in shallow flows, thus resulting in the highest overall fluxes over all slope gradients. Single-class GSDs in general yield abrupt changes in the relationship between sediment flux and gradient (Figure 10a) due to arising thresholds in transport as the flow hydraulics change.

[28] Figure 10b shows the relationship between sediment flux and gradient for multiclass GSDs. The relationship is nonlinear but it does not exhibit the abrupt transitions in

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**Figure 7.** (a) Equilibrium flow depth and (b) equilibrium flow velocity changes with hillslope gradient, for different hillslope lengths for 50 and 100 mm h\(^{-1}\) rainfall intensities (one hour duration).

**Figure 8.** Relationship between normalized sediment flux and hillslope gradient for different hillslope lengths under equilibrium runoff conditions: (a) 50 mm h\(^{-1}\) rainfall for 1 h and (b) 100 mm h\(^{-1}\) rainfall for 1 h.
sediment flux shown for single-class GSDs. For coarse GSDs or low runoff magnitudes, sediment flux increases linearly with gradient above a threshold (Figure 10b). However, at higher magnitude runoff, the relationship becomes sigmoidal as sediment flux becomes progressively invariant with gradient at higher slopes. Multiclass GSDs containing mixtures of particle sizes produce smoother curves because of the availability of sediment for transport at a variety of slope and flow conditions. Figure 10c illustrates the difference in the flux-gradient relationship for a single-class and multiclass GSD with the same $D_{50}$.

5.4. Sensitivity of Sediment Flux to Gradient and Length Under Non-equilibrium Runoff

In this final results section we examine the effects of high intensity, short duration rainstorms which are characteristic of dry lands [Nicholson, 2011]. Equilibrium runoff requires relatively long rainfall duration for the infiltration capacity to be reached. If a rainfall event is shorter than the time it takes runoff to flow from slope crest to slope base, runoff will increase only part way down the slope. Downslope from this point the runoff discharge during the rainfall event is constant at all locations on the slope. In the cases of the rainfall events applied in these simulations, the downslope runoff discharge increases until 50–150 m (depending on the event), and then levels off with distance (Figure 6b), in contrast to the equilibrium runoff events in which discharge increases linearly with slope length (Figure 6a).

Rainfall intensity and duration determine downslope runoff dynamics which, in turn, exert a strong control on grain-size selective transport, particle transport distances and within hillslope sediment redistribution. For the three

![Figure 9](image1.png)

**Figure 9.** Relationship between grain size enrichment ratio (ER) and hillslope gradient for different hillslope lengths. Enrichment ratios for (a) $D_{50}$ for 50 mm h$^{-1}$, (b) $D_{90}$ for 50 mm h$^{-1}$, (c) $D_{50}$ for 100 mm h$^{-1}$, and (d) $D_{90}$ for 100 mm h$^{-1}$ (all rainfall events are one hour duration). The gray dashed lines indicate ER = 1 where the supplied sediment has the same characteristics as the initial hillslope sediment.

![Figure 10](image2.png)

**Figure 10.** Relationship between sediment flux and gradient for different hillslope grain size characteristics. (a) Effect of decreasing the diameter ($D$) of a single-class GSD. (b) Effect of increasing the $D_{50}$ of a multiclass GSD. (c) Comparison of the effect of single- and multiclass GSDs on the sediment flux – gradient relationship, where the $D_{50}$ of multiclass GSD = $D$ of single-class GSD.
rainstorms applied here, the relationship between sediment flux and hillslope length varies as a function of runoff intensity and duration. Figure 11 shows how for each slope gradient, sediment flux peaks at particular slope lengths that correspond to the distances over which runoff increases during the rainstorm. This critical distance varies with slope gradient and rainfall duration because runoff can travel farther at higher velocities and over longer storm lengths. The family of curves corresponding to the three rainfall rates and durations also highlights equifinality [see, e.g., Beven, 2006] in slope responses to different rainfall characteristics across a variety of slope lengths and gradients. This suggests that data collection in the field may yield confusing information because of the high sensitivity of the system to both external forcings and inherent attributes. These simulations exclude the effects of GSD variations, which would further compound complexities in the relationship between sediment flux and slope gradient.

The effect of critical runoff distance on sediment transport is further illustrated in Figure 12, which shows changes in the particle numbers and grain sizes with distance downslope following the runoff events. Maximum sediment transport occurred in the zone with the highest rate of runoff increase and manifests as a pulse of particles moving from the top half of the slope to the bottom half. The amplitude and location of the sediment pulse is controlled by the intensity and duration of the runoff and the slope gradient. In the location where the peak sediment transport occurs, a post-event increase in surface grain size is seen. The magnitude of the coarsening is dependent on the runoff intensity. In the examples presented here, the changes in grain size following a runoff event vary within the slope. Overall coarsening for the entire slope occurs, but the degree of coarsening is more strongly controlled by the gradient than by the magnitude of the runoff event. After the 6-min, 150 mm h\(^{-1}\) event, the 40\(^{\circ}\) within-hillslope mean grain size increased by 1.8 mm overall, relative to the initial (19.97 mm), with the top half of the slope coarsening to a mean grain size of 22.3 mm and the bottom half to 21.3 mm. The 12-min, 75 mm h\(^{-1}\) event on the 40\(^{\circ}\) slope resulted in a similar overall coarsening (1.8 mm), but the within-hillslope variation was lower, with the top half at 21.9 mm and the lower half at 21.6 mm. The 24-min, 37.5 mm h\(^{-1}\) event actually resulted in an overall fining of the hillslope GSD by 0.3 mm with a fairly uniform decrease on the top half of the slope (19.65 mm) and the bottom (19.70 mm). The amount of coarsening and the within-hillslope variations in grain size decreased as the gradient and runoff intensity decreased.

6. Discussion

[32] This study set out to investigate the functional controls on sediment flux on coarse debris-mantled hillslopes, characteristic of many arid and semiarid environments, through the process of sheetwash transport. Through a series of model experiments we explored the response of sediment flux and GSD of fluxed sediment to slope gradient and length, hillslope GSD and rainfall intensity and duration. The results from our model experiments show that the relationship between sediment flux and gradient is complex and varies depending on a number of hillslope and rainfall characteristics. Although we found that the relationship between sediment flux and gradient is generally nonlinear and thus consistent with studies of other transport mechanisms [Gabet, 2000, 2003; Roering et al., 1999, 2001b], the form of the relationship varied from linear above a threshold to sigmoidal, depending on the interplay between hillslope attributes, rainfall characteristics and surface GSD relative to a mean bed roughness. This form is similar to the one found experimentally by Gabet and Mendoza [2012] for the percentage of dry ravel flux reaching the end of their flume.
Even for the same hillslope length, gradient and hydrological conditions, a change in the initial GSD was sufficient to change significantly the form of the relationship (Figure 10). Changes in the initial slope GSD affect entrainment thresholds and the grain size classes transported in the flow, which directly control sediment flux for each hillslope gradient. Our results show that the spread of the GSD exerts a strong control on sediment flux. Increases in the $D_{50}$ of a wide GSD (multiclass) results in a decrease in sediment flux for a given slope angle. Changes in the $D_{50}$ of a narrow GSD (single-class) induced a more extreme change in sediment flux than did analogous changes in $D_{50}$ in multiclass grain sizes because uniform distributions are much more sensitive to entrainment thresholds and therefore result in abrupt changes in transport rates with slope gradient as runoff hydraulics change. The importance of the spread of GSD on flux rates has been demonstrated in a range of fluvial environments [Buffington et al., 1992; Church and Hassan, 2002; Reid and Laronne, 1995; Singer, 2010; Wilcock, 1998], but to the best of our knowledge, has thus far not been explicitly shown on hillslopes.

Our model experiments demonstrate the importance of runoff characteristics on sediment flux and, in particular, they highlight different slope responses under equilibrium and non-equilibrium runoff discharge. Long duration rainfall events lead to hydrological steady state at all locations on the hillslope and runoff increases with distance downslope. Therefore, under equilibrium runoff conditions sediment flux generally increased with slope length for a given slope angle. The relationship between sediment flux and gradient under these conditions was nonlinear (sigmoidal) with the flux rates converging at the highest slope gradients for all slope lengths. The inflection points of the sigmoidal relationship were found to shift toward lower gradients when the effective rainfall increased, leading to higher runoff especially on longer slopes. However, a more realistic scenario for dryland hillslopes is high intensity, low duration rainstorms that result in partial area contribution of runoff [cf. Yair et al., 1978], instead of equilibrium runoff. Short duration rainfall events result in runoff increasing only partway down the slope so, in contrast to the equilibrium runoff scenario, our results show that sediment flux does not increase with slope length. Instead, sediment flux peaked at a particular slope length, which is a function of the critical distance over which runoff increased downslope. It is determined by the rainfall intensity and duration, and decreases over longer slope distances. Over the long-term, short runoff distances relative to slope length have important implications for the topographic evolution of runoff-dominated hillslopes [Ahnert, 1988] and for downslope sorting of grain sizes [Carson and Kirkby, 1972].

 figure 12. Changes in the number of particles and the associated grain size changes within a 200 m hillslope following 3 different rainfall events for a 40° and 20° slope, relative to the initial conditions (black dots). The high intensity rainfall produces pulses in sediment transport at the location corresponding to maximum runoff. For each gradient, the amplitude of the pulse relates to the rainfall intensity, whereas the location of the pulse is determined by the rainfall duration. The solid and dashed black curves for the 20° and 40° slopes respectively, are fitted using LOWESS (locally weighted scatterplot smoothing) with a span = 100 points.
Our results demonstrate equipollatility in sediment flux under various runoff conditions for different slope attributes and we believe, they partly explain the lack of conclusive evidence on the relationship between sediment flux and slope gradient from field-based plot studies [Abrahams and Parsons, 1991; Bracken and Kirkby, 2005; Yair and Klein, 1973]. Considering that the stochasticity of rainstorms and the variability in hillslope properties in arid and semiarid environments may lead to large variations in sediment transport, it is unsurprising that poor theoretical relationships have been derived from field-based studies. Field evidence from the Negev desert in Israel [Yair and Raz-Yassif, 2004] supports these assertions and confirms discontinuous sediment transport due to low duration flow on debris-mantled hillslopes, as well as a lack of relationship between hillslope length and runoff and erosion.

Moreover, our modeling results provide a physical basis for understanding nonlocal effects on hillslope sediment transport [Foufoula-Georgiou et al., 2010] under conditions of constant local slope which nonetheless result in varying sediment transport distances (as indicated by the fraction of particles reaching the slope base). In particular, in our runoff-driven sediment transport model, hillslope length exerts a strong control on the spatial extent and magnitude of runoff contribution, which affects the transport distances of varying particle sizes downslope. Hillslope GSD can have a similarly strong impact on sediment transport distances under conditions of constant hillslope gradient, length, and hydrology. The form of the relationship between sediment flux and gradient and the associated fluxed GSDs produced by our model (Figures 8 and 9) also suggest that nonlocal effects arise from the interaction between runoff hydraulics and grain sizes and their gradual translation downslope.

Obtaining field or laboratory data for testing the range of functional relationships produced by the modeling presented here is challenging, due to the number of interacting variables that cannot be systematically controlled in the field or represented in experiments at the appropriate scale. However, a number of model results concur with and help explain published field data and observations. Several studies have found a systematic coarsening of slopes as gradient increases [Abrahams et al., 1985; Kirkby and Kirkby, 1974].

Our model reproduces this systematic increase in the mean hillslope grain size at steep slope gradients following runoff events (Figure 13), and captures spatial variations in hillslope GSD changes after short duration runoff events and differences in the degree of coarsening, depending on hillslope and runoff attributes. Although some experimental studies have suggested that higher stone cover leads to increased infiltration rates [Abrahams et al., 1988b; Poesen et al., 1990], thus contradicting the presence of coarser grains on steeper slopes, other studies on semiarid hillslopes have shown that runoff increases with stone cover and can produce runoff coefficients >50% [Yair and Lavee, 1976]. Regardless of any possible feedbacks between infiltration and stone cover, the range of rainfall and infiltration rates recorded for debris-mantled hillslopes in arid and semiarid areas suggests that rainfall thresholds for runoff generation are still very low on these slopes [Yair and Kossovsky, 2002] and are thus capable of transporting coarse sediment.

Our model results illustrate the possible dynamics that may explain this coarsening. In particular, Figure 9 shows the complex changes in grain size enrichment ratios as slope length, gradient and runoff magnitude change. They show how, overall, the supplied grain sizes to the base of the slope gradually increase with gradient for all slope lengths, although longer slopes supply finer material than shorter slopes. This phenomenon is a result of the interaction of flow depth (which affects submergence of smaller particles) and slope length (which influences particle transport distances). Therefore, our model results present a realistic mechanism for the observed increased coarsening with steeper gradients, even for non-equilibrium runoff conditions which result in selective, partial sediment transport and sediment redistribution within the hillslope.

Sediment flux and changes in GSD within the hillslope were significantly affected by the duration and intensity of the rainfall event and the spatial extent of runoff. Debris-mantled dryland hillslopes are therefore highly sensitive to climate, primarily due to the surface characteristics which lead to runoff production even during relatively low rainfall intensities. Runoff generation can thus be viewed as an integrator of climatic and surface properties [Yair and Kossovsky, 2002], with a significant control on sediment transport within dryland hillslopes and on the supply of coarse sediment fraction to river channels. Importantly, this climatic sensitivity of dryland hillslopes suggests that for simulating long-term topographic evolution of drainage basins, discrete rainfall events are important and individual geomorphic transport laws will not adequately represent the relationship between sediment flux and hillslope attributes for the full range of events and surface conditions. The representation of landscape evolution as a continuous process shaped by an averaged, ‘geomorphically effective’ rainfall rate may thus be inappropriate [Tucker and Bras, 2000], but particularly so in environments with high inter-annual rainfall variability and high climatic sensitivity. The implication of these results is that coarse hillslope sediment supply to valley floors in debris-mantled, dryland drainage basins is a stochastic process, driven by discrete, random, runoff-producing rainfall events.

7. Conclusion

There is a notable absence of theoretical understanding of runoff-driven sediment transport on debris-mantled
hillslopes, which has important implications for coarse sediment supply to river channels. This lack of theory for runoff-driven systems contrasts with a growing body of work on other modes of hillslope sediment transport induced by disturbances other than overland flow (e.g., landsliding, bioturbation, tree throw, soil creep) that occur in soil-man
teed landscapes. However, runoff is arguably one of the most important drivers of landscape change globally which is directly coupled to climate, so understanding the functional relationships between rainfall characteristics and hillslope attributes provides the basis for exploring long-term changes and geomorphic evolution of drainage basins. In this paper, we address this gap and present a new physics-based model of coarse sediment transport on hillslopes. The model uses a force-balance approach on discrete particles of multiple sizes >1 mm driven by a rainfall-runoff model that simulates spatial fields of flow depth and velocity. We use a series of model experiments to explore the relationships between hillslope attributes (gradient, length, GSD) and rainfall characteristics (intensity, duration) for conditions typical of arid and semiarid environments.

[41] Our results highlight that sediment supply to the base of the hillslope is significantly affected by interactions between hillslope and rainfall characteristics. Specifically, we found that: 1) the functional form of the relationship between sediment flux and gradient varied from linear above a threshold to sigmoidal depending on hillslope, grain-size and rainfall characteristics; 2) the GSD supplied to the base of the hillslope varies in a complex way with gradient and length; 3) rainfall events of varying intensity and duration may result in equifinal sediment supply to the slope base on very different hillslopes; 4) short-lived rainfall events result in sediment redistribution within the hillslope and grain-size changes on different parts of the slope; 5) coarsening of hillslope GSD emerged as slope gradient increased.

[42] Our results suggest that sediment transport by runoff on debris-mantled hillslopes cannot be simply represented as a function of slope gradient. Instead, the characteristics of the rainfall event and surface GSD exert a first order control on the process of downslope redistribution of sediment and coarse material supply to river channels. These findings have implications for understanding nonlocal controls on sedi
ment supply by debris-mantled hillslopes and for the limits of applicability of geomorphic transport laws in runoff-dominated settings.

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