On the Performance of Opportunistic NOMA in Downlink CoMP Networks

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Abstract—In this letter, an opportunistic NOMA (ONOMA) scheme is proposed in Coordinated Multi-Point (CoMP) system. Compared with the conventional joint-transmission (JT) NOMA in CoMP, the main purpose of the ONOMA scheme is to advance the capability of the CoMP system and the effectiveness of the successive-interference-cancellation (SIC) decoding process. The relationship between the topology of the ONOMA cells and the sum-rate of CoMP network is analysed. Meanwhile, the outage probability of ONOMA system is derived and evaluated in ideal case and non-ideal case respectively. In the numerical results, the sum-rate and the outage performance of the proposed ONOMA CoMP are compared with the JT-NOMA CoMP and it is shown that the proposed ONOMA scheme outperforms the conventional JT-NOMA scheme in CoMP system.

Index Terms—Non-orthogonal multiple access, coordinated multi-point system, joint transmission, opportunistic transmission.

I. INTRODUCTION

As one of the key technology enhancements for LTE-A, the coordinated multi-point (CoMP) transmission scheme is an area of intense research [1]. For the downlink of the CoMP system, the access points (AP) allocate the same channel to a cell-edge user and this channel can not be allocated to other users at the same time. Thus, the spectral effectiveness of the CoMP system degrades when the cell-edge users increase in number if the orthogonal multiple access is employed. Recently, a promising non-orthogonal multiple access (NOMA) scheme has been proposed for CoMP in an endeavour to tackle this difficulty [2].

The NOMA technique is acknowledged as a potential candidate air interface technique for the fifth generation (5G) mobile networks [3]-[7]. By using NOMA, which merges the superposition coding (SC) scheme at transmitters with the successive interference cancellation (SIC) scheme at receivers [3], users are capable of decoding their own signal even though they are using the same frequency channel simultaneously. For the downlink of the cellular network, the system level performance along with the user fairness of NOMA is studied in [4]. In [5], a cooperative NOMA with simultaneous wireless information and power transfer scheme is investigated. In [6] and [7], the ergodic sum-rate and outage behavior of NOMA is studied in the normal cellular network and a relay-based heterogeneous network respectively.

The NOMA strategy is able to offer extensively improved spectrum efficiency, however, the complexity of NOMA scales with the number of users alongside the level of multi-user interference [3]. To overcome this issue alongside providing additional performance enhancements, an opportunistic NOMA (ONOMA) scheme is proposed in this research. More specifically, the main contributions of this letter are as follows: a) we first consider a joint-transmission NOMA (JT-NOMA) scheme in CoMP, and then propose a novel ONOMA scheme; b) by analyzing the topology of ONOMA CoMP, we describe the procedure of the ONOMA scheme in ideal and non-ideal scenarios respectively; c) the achievable sum-rate and the outage performance of the proposed scheme are derived for different ONOMA topologies; d) the comparisons of the sum-rate and outage probability of JT-NOMA with ONOMA are computed and presented.

II. NOMA in CoMP

In this study, we consider the downlink of a CoMP network which includes $B$ APs and $K$ users. The architecture under consideration here is a single antenna AP and user platform, thus beamforming is not considered at this stage. Let $B$ and $K$ denote the set of APs and users, respectively, where $B = \{1, 2, ..., B\}$ and $K = \{1, 2, ..., K\}$. The signal which is broadcasted by AP $b$ ($b \in B$) to all the $K$ users is denoted as $\sum_{i \in K} \sqrt{a_{i,b}} \hat{s}_i$, where $P_b$ is the normalized transmit power at AP $b$; $s_i$ is the desired signal of the user $i$ ($i \in K$); the power allocation coefficient to $s_i$ is denoted as $a_i$. The Rayleigh fading channel from the AP $b$ to the user $i$ is denoted as $h_{i,b}$, where $h_{i,b} = \sqrt{n_{i,b} g_{i,b}}$; the factor $g_{i,b}$ is the independent and identically distributed circular symmetric Gaussian random variables (RV) with zero mean and variance $\sigma^2_{g,b}$, representing fast fading; the factor $n_{i,b}$ denotes the slow fading. The additive white Gaussian noise (AWGN) at the user $i$ is denoted as $n_i$ (with variance $\sigma^2_n$).

A. JT-NOMA in CoMP

For the JT-NOMA in CoMP, the transmit power that is allocated to $s_i$ is denoted by $a_iP$ for all APs, where $P$ is the normalised power. Then the observation at the user $k$ is given by

$$r_k = \sum_{i \in K} \hat{h}_{i,k} \sqrt{a_iP} s_i + n_k,$$  \hspace{1cm} (1)

where $\hat{h}_{i,k}$ denotes the equivalent channel coefficient to the user $k$ and $\hat{h}_{i,k} = \sum_{b=1}^{B} h_{i,b}$. Here we assume that: $|\hat{h}_1|^2 \leq \cdots \leq |\hat{h}_K|^2$, then in agreement with the SC in NOMA, the power allocation values to the $K$ users are sorted as: $a_1 \geq \cdots \geq a_K$. Subsequently, every receiver makes use of the SIC technique and is capable of decoding the signals of the users in the weaker channels faultlessly [3], [4]. Consequently, the rate of detecting $s_k$ at user $k$ can be expressed as

$$R_k = \log \left(1 + \frac{|\hat{h}_{i,k}|^2 a_k}{I_k + \sigma^2_n P^{-1}}\right),$$  \hspace{1cm} (2)

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where $I_k = \left| \hat{h}_k \right|^2 \sum_{m \in K} a_m$ along with $a_m < a_k$; note that when $m = K$, $I_k = 0$.

The sum-rate of $K$ users in conventional JT-NOMA CoMP is expressed as

$$R_{\text{sum}} = \sum_{k \in K} \left\{ \log \left( 1 + \frac{\left| \hat{h}_k \right|^2 a_k}{I_k + \sigma_k^2 P^{-1}} \right) \right\}. \quad (3)$$

### B. Opportunistic NOMA Strategy in CoMP

We assume that the range of the transmit power at each AP is $[P_{\text{min}}, P_{\text{max}}]$. The transmit power allocated to $s_k$ at the AP $b$ is given by $P_{k,b} = a_k P$, then $P_{k,b} \in [P_{\text{min}}, P_{\text{max}}]$. The ONOMA strategy is implemented via the following procedure:

1) **Initialization** (Reference Broadcast): The $B$ APs separately broadcast a normalized reference signal $s_r$ to each $K$ users, with reference transmit power $P_r$. Via the $B$ reference signals, each user creates a reference power set. The user $k$’s ($k \in K$) reference power set is expressed as $D_k = \{P_r | \hat{h}_{k,1}|^2, \ldots, P_r |\hat{h}_{k,B}|^2\}$.

2) **Scheduling** (AP Selection and ONOMA Cells Generation): By using the opportunistic AP selection algorithm, each user generates a set for its preferred APs. The user $k$’s preferred AP set is denoted by $S_k$. The required channel state information (for SC precoding in NOMA as [2]-[6]) along with the AP selection results are fed back from $K$ users to $B$ APs. Then based on the feedback, the CoMP system generates $B$ ONOMA cells. The ONOMA cell $b$ contains the AP $b$ and the users who select the AP $b$ in their preferred AP sets, therefore the set $B$ can also indicates the $B$ ONOMA cells. The set of the users in ONOMA cell $b$ is denoted by $W_b$.

**Algorithm 1** Opportunistic AP Selection (OAPS) Scheme

1) **Input** $B$, $D_k$, $P_r$, $P_{\text{max}}$ and $P_{\text{min}}$.
2) **Set** $b = 0$, $S_k = \emptyset$ and $B = \text{card}(B)$.
3) **Define** $\vec{D}_k = P_r^{-1} \cdot D_k$ and normalized $\hat{D}_k$, such that $\forall b \in B$, $\hat{D}_k[b] \in [0,1]$.
4) **Let** $e_k$ denote an AP selection threshold value at user $k$.
5) **Randomly** select the value of $e_k$ from an interference range. (e.g. the interference range is set to $[0,0.1]$ in simulation results).
6) **Define** $\delta_k = P_{\text{min}} e_k$. For all $b \in B$, compare $P_{\text{max}} D_k[b]$ with $\delta_k$; if $P_{\text{max}} D_k[b] \geq \delta_k$ , add the AP’s index $b$ to $S_k$.

For the user $k$, the interference from the APs which are selected by $S_k$ is defined as the intra-ONOMA cell interference (this can be canceled by SIC); the interference from the APs which are not selected by $S_k$ is defined as the inter-ONOMA cell interference (this cannot be canceled by SIC). Let $\Psi_k$ denote the inter-ONOMA cell interference to the user $k$; the observation at user $k$ is given by

$$r_k = \sum_{b \in S_k} h_{k,b} \sum_{i \in W_b} a_i P_s i + \Psi_k + n_k, \quad (4)$$

where $\Psi_k = \sum_{j \in \{B \setminus S_k\}} \sum_{n \in W_j} h_{k,j} \sqrt{a_n P_s} n_k$.

By implementing the **Algorithm 1**, the threshold value $\epsilon_k$ at user $k$ ensures that for all $j \in \{B \setminus S_k\}$ and $b \in S_k$, there exist $h_{k,j} \sqrt{a_n P_s} n_k < h_{k,b} \sqrt{a_i P_s} i$, where $n \in W_j$ and $i \in W_b$. Therefore, the complexity of SIC is reduced as the SIC is only charge of the intra-ONOMA cell interference. Note that some ONOMA cells may have an overlapping area (by using Algorithm 1), which means the users could be selected by different ONOMA cells at the same time. Based on whether the ONOMA cells have overlapping area, the ONOMA CoMP can be divided into ideal case and non-ideal case.

1) **Ideal Case of ONOMA CoMP:** For the ideal case, each user only selects one AP in its preferred AP set, which means for all $k \in K$, $\text{card}(S_k) = 1$, where $\text{card}(S_k)$ denote the cardinality of a set $S_k$. Therefore, there is no overlapping area between the different ONOMA cells. Assume that $a_m < a_k$ for all $m \in \{W_b; b \in S_k\}$, by using SIC, the rate of detecting $s_k$ at user $k$ can be expressed as follows

$$R_{k,b} = \log \left( 1 + \frac{a_k |h_{k,b}|^2}{I_k + \Psi_k + \sigma_k^2 P^{-1}} \right), \quad (5)$$

where $b \in S_k$, $I_k = |h_{k,b} \sum_{n \in \{W_b \cap S_k\}} a_n|^2$ and $\Psi_k = |\sum_{j \in \{B \setminus S_k\}} h_{k,j} \sum_{n \in W_j} a_n|^2$. Note that for all $m \in \{W_b; b \in S_k\}$, if there is no $a_m < a_k$ then $I_k = 0$. The sum-rate of $K$ users for the ideal case ONOMA is given by

$$R_{\text{sum}} = \sum_{b \in B} \sum_{k \in W_b} \log \left( 1 + \frac{a_k |h_{k,b}|^2}{I_k + \Psi_k + \sigma_k^2 P^{-1}} \right). \quad (6)$$

2) **Non-ideal Case of ONOMA CoMP:** For the non-ideal case, two or more ONOMA cells may have the overlapping area, and that is because some users may select multiple APs. Let $O_b$ denote the set of APs whose ONOMA cells exist the overlapping area with the ONOMA cell $b$, then an optimized ONOMA strategy can be implemented via the **Algorithm 2**.

**Algorithm 2** Non-ideal Cases ONOMA Implementation

1) **Input** $S_k$ and $W_b$; compute the cardinality of $S_k$ for all $\forall k \in K$.
2) If $\text{card}(S_1) \geq \cdots \geq \text{card}(S_K)$, the power allocation values to the $K$ users will be sorted as $a_1 \geq \cdots \geq a_K$.
3) For $M = \{1, 2, \ldots M\}$ users whose preferred AP sets have the same cardinality, the power allocation values will be sorted based on their equivalent channels. e.g. Let $\hat{h}_m$ denote the equivalent channel of user $m (m \in M)$, where $\hat{h}_m = \sum_{k \in S_m} h_{m,b}$; if $\text{card}(S_1) = \cdots = \text{card}(S_M)$, but $|\hat{h}_1|^2 \leq \cdots \leq |\hat{h}_M|^2$, then $a_1 \geq \cdots \geq a_M$.
4) For all $b \in B$, let $S^*_b$ denote the element who has the largest cardinality in $\{S_i; i \in W_b\}$ and define a null set $U_b$; then $\forall o \in \{W_b \setminus \cap S^*_b\}$, compare $\text{card}(S^*_b)$ with $\text{card}(S_o)$; if $\text{card}(S_o) \geq \text{card}(S^*_b)$, add the user’s index $o$ to $U_b$.
5) For all $b \in B$ and $k \in K$, AP $b$ broadcasts signal $\sum_{i \in \{W_b \cup U_b}\} a_i P_s i$ and user $k$ decodes its observations.
Assume that \(a_m < a_k\) \((m \in \{W_b; b \in S_k\})\), the rate of detecting \(s_k\) at user \(k\) is given by

\[
R_{k,k} = \log \left( 1 + \frac{|\sum_{b \in S_k} h_{k,b}|^2}{I_k + \Psi_k + \sigma_k^2 P^{-1}} \right),
\]

where \(I_k = |\sum_{b \in S_k} \sqrt{\sum_{m \in W_b} a_m h_{k,b}^2}|^2\) and \(\Psi_k = |\sum_{j \in \psi(\mathcal{B}, S_k)} \left( \frac{h_{k,j}}{\sum_{l \in \{W_i, l \neq k\}} a_l} \right)^2\). If there is no \(a_m < a_k\) for all \(m \in \{W_b; b \in S_k\}\), then \(I_k = 0\). The sum-rate of \(K\) users for the non-ideal case ONOMA is given by

\[
R_{\text{sum}} = \sum_{k \in K} \log \left( 1 + \frac{|\sum_{b \in S_k} h_{k,b}|^2}{I_k + \Psi_k + \sigma_k^2 P^{-1}} \right).
\]

III. OUTAGE PROBABILITY OF ONOMA

Define \(R^R_{k,n}\) and \(R_{k,n}\) as user \(k\)'s targeted and real data rate to decode the signal \(s_n\), respectively. Then outage will happen when \(R_{k,n} < R^R_{k,n}\), which means the user \(k\) cannot detect the user \(n\)'s signal \(s_n\) before detecting its desired signal \(s_k\). Let us define this outage event as \(E_{k,n} = \{R_{k,n} < R^R_{k,n}\}\). For the ideal ONOMA CoMP, by substituting (5) to \(E_{k,n}\), then

\[
E_{k,n} = \left\{ \frac{a_n}{\sum_{m \in \{W_b; b \in S_k\}} a_m + (\rho_k |h_{k,b}|^2)^{-1}} < 2^{R^R_{k,n} - 1} \right\},
\]

where \(\rho_k = \frac{P}{\Psi_k + \sigma_k^2}\). The complementary set of the outage event can be derived by

\[
E_{k,n}^c = \left\{ |h_{k,b}|^2 > \frac{\theta_n \rho_k^{-1}}{a_n - \theta_n \sum_{m \in \{W_b; b \in S_k\}} a_m} \right\},
\]

where \(\theta_n = 2^{R^R_{k,n} - 1}\) and step (a) follows the condition that \(a_n > \theta_n \sum_{m \in \{W_b; b \in S_k\}} a_m\).

Define \(\nu_n = \frac{\theta_n}{\sum_{m \in \{W_b; b \in S_k\}} a_m}\) and \(\phi = \max\{\nu_1, \ldots, \nu_k\}\), the outage probability at user \(k\) is shown as

\[\mathbb{P}_k = 1 - \mathbb{P}(E_{k,1} \cap \cdots \cap E_{k,k}) = 1 - \mathbb{P}(|h_{k,b}|^2 > \phi).\]

The cumulative distribution function (CDF) and probability density function (pdf) of the unordered Rayleigh channel \(|h_{k,b}|^2\) is given by \(F(x) = 1 - e^{-x/\sigma_k^2}\) and \(f(x) = \sigma_k^2 e^{-x/\sigma_k^2}\) respectively. Define \(N_b = \text{card}\ \{W_b; b \in S_k\}\) and assume that user \(k\)'s decoding order in ONOMA cell \(b\) is \(n_b(k)\) (where order \(n_b(k)\) is higher than \(n_b(k+1)\)). Based on the high order statistics in [8], (11) can be derived as

\[\mathbb{P}_k^{\text{out}} = \int_0^\phi N_b f(x) (F(x))^{n_b(k)-1} (1 - F(x))^{N_b - n_b(k)} \frac{dx}{(n_b(k) - 1)!(N_b - n_b(k))!} \]

\[= \frac{N_b - n_b(k) (-1)^i (1 - \exp(\sigma_k^2 n_b(k) + i))}{\Delta_{n_b(k), k}^i \sigma_k^2 n_b(k) + i)\}
\]

where \(\Delta_{n_b(k), k} = \frac{(N_b - n_b(k))!(n_b(k) - 1)!!}{(n_b(k))!}\) and (b) follows from the power series of exponential functions.

For the non-ideal ONOMA, \(E_{k,n}^c\) is expressed as

\[
E_{k,n}^c = \left\{ \frac{\rho_k \sum_{b \in S_k} |h_{k,b}|^2 a_n}{\sum_{m \in W_b} a_m + 1} > \theta_n \right\}
\]

\[
\sum_{b \in S_k} |\sqrt{\gamma_{k,b} u_{k,b} g_{k,b}}|^2 > \theta_n (\rho_k a_n)^{-1}
\]

\[
= \sum_{b \in S_k} |H_{k,b}|^2 > \hat{v}_n,
\]

where (14) is derived by substituting (7) to \(\{R_{k,n} > R^R_{k,n}\}\); step (c) follows the condition that \(\forall b \in S_k; \gamma_{k,b} > 0\), where \(\gamma_{k,b} = (1 - \theta_n \sum_{m \in W_b} a_m)\); in (16), \(\hat{v}_n = \theta_n (\rho_k a_n)^{-1}\) and \(|H_{k,b}|^2 = |\sqrt{\gamma_{k,b} u_{k,b} g_{k,b}}|^2\). According to [9], \(|H_{k,b}|^2\) can be regarded as the generalized chi-square distribution with variance \(\hat{\nu}_k = \gamma_{k,b} \sigma_k^2\).

Define \(\phi = \max\{\hat{v}_1, \ldots, \hat{v}_k\}\) and \(M_k = \text{card}(S_k)\). The pdf of the unordered generalized chi-square RV \(|H_{k,b}|^2\) is given by

\[f_k(x) = \frac{\gamma_{k,b} x^{M_k-1} \exp(-\nu_{k,b} x)}{\Gamma(M_k)}\]

where \(\Gamma(.)\) denotes the gamma function. Define \(N_b = \text{card}\ \{W_b \cup U_k; b \in S_k\}\); then assume that the user \(k\)'s decoding order in ONOMA cell \(b\) is \(n_b(k)\) agian. With the aid of order statistics and following the similar process in (12)-(13), the outage probability can be derived from integrating the pdf of the ordered variable \(|H_{k,b}|^2\) as

\[\mathbb{P}_k^{\text{out}} = \int_0^{\phi} N_b f(x) (F(x))^{n_b(k)-1} (1 - F(x))^{N_b - n_b(k)} \frac{dx}{(n_b(k) - 1)!(N_b - n_b(k))!} \]

\[= \frac{N_b - n_b(k) (-1)^i (F_k(\hat{\nu}_k))^{n_b(k)+i}}{\Delta_{n_b(k), k}^i \sigma_k^2 n_b(k) + i)\}
\]

The unordered CDF \(F_k(\hat{\nu}_k)\) in (19) can be derived from the integration of the unordered pdf \(f_k(\hat{\nu}_k)\) as

\[F_k(\hat{\nu}_k) = \int_0^{\phi} \frac{\gamma_{k,b} x^{M_k-1} \exp(-\nu_{k,b} x)}{\Gamma(M_k)} \frac{dx}{\Gamma(m + 1)}
\]

where (d) follows the power series of exponential functions.

IV. NUMERICAL RESULTS

In this section, a time-invariant CoMP system which includes two single-antenna APs and five single-antenna users is considered and a Monte Carlo simulation is used to evaluate the performance of JT-NOMA and ONOMA. The set of the two APs and five users are denoted by \(B_s\) and \(K_o\), respectively.
In order to obtain the statistical fading distribution, all the five users are randomly distributed in the two ONOMA cells in each simulation loop; if a user $k$ is allocated in the ONOMA cell $b$ but not in the overlapping area, then $|g_{k,b} \sqrt{\mu_{k,b}}|^2 > \epsilon_k$ and $|g_{k,b} \sqrt{\mu_{k,b}}|^2 < \epsilon_k$ (where $b \in \{B, b\}$); if a user $k$ is in the overlapping area of the $B_o$ ONOMA cells, then $\forall b \in B_o$, $|g_{k,b} \sqrt{\mu_{k,b}}|^2 > \epsilon_k$.

The performance of JT-NOMA is evaluated under the same opportunistic conditions, i.e. the same distribution topologies and channel fendings are employed by JT-NOMA in each simulation loops. The power allocation coefficients for both JT-NOMA and ONOMA are sorted based on the decoding priority of the $K_o$ users, e.g. assume that $\forall i \in K_o$, the decoding priority of user $i$ is higher than user $i+1$, then power allocation coefficient of user $i$ is set as $a_i = \frac{\text{card}(K_o)-i+1}{\mu}$. The channels in the following simulation results are normalized such that $|g_{k,b} \sqrt{\mu_{k,b}}|^2 \in [0,1]$. For the ideal case in ONOMA, the $\epsilon_k$ tends to 0 ($\epsilon_k \rightarrow 0$) such that $S_k$ can exclude the APs only with extremely poor channel gain ($|g_{k,b} \sqrt{\mu_{k,b}}|^2 \rightarrow 0$). However in practice, more APs with slight poor channel gain also need to be excluded by $S_k$, therefore in the following simulation results, the maximum range of $\epsilon_k$ in simulations is set as $0 < \epsilon_k < 0.1$.

In Fig.1, we compare the sum-rate performance between JT-NOMA CoMP and ONOMA CoMP with different inter-cell interference threshold levels: the ideal case ($\epsilon_k \rightarrow 0$) and two practical non-ideal cases where $\epsilon_k \in [0,0.01]$ and $\epsilon_k \in [0,0.1]$. Compared with JT-NOMA, the performance gain of ONOMA becomes significant when the signal to noise ratio (SNR) is larger than 6 dB. It should also be noticed that as the inter-ONOMA cell interference is proportional to the transmit power, for the cases that $\epsilon_k > 0$, the ONOMA sum-rate curves tend to flat with the increase of SNR. The Fig.2 provides a comparison of outage probability between JT-NOMA and ONOMA CoMP under different target data rates. Along with the simulation results, the theoretical curves are plotted at $\epsilon_k \rightarrow 0$ and $\epsilon_k \rightarrow 0.1$. Compared with the outage probability curves at $\epsilon_k \rightarrow 0$, it is shown that for each curve in which $\epsilon_k > 0$, the performance loss is proportional to the value of SNR. Moreover, the increase of $\epsilon_k$ results in a higher outage probability under the same SNR level.

V. CONCLUSIONS

An ONOMA scheme is proposed to reduce the complexity of SIC and in the meantime advance the performance of NOMA-CoMP system. The relationship between the topology of the ONOMA cells and the sum-rate of CoMP network has been analysed. Further, the outage performance of ONOMA system has been discussed in ideal case and non-ideal case respectively. Comparing with the conventional JT-NOMA CoMP, the numerical results show that the proposed ONOMA CoMP can achieve better sum-rate and outage probability performance.

REFERENCES


Figure 1. Sum-rate vs. SNR with different ranges of $\epsilon_k$

Figure 2. Outage probability vs. SNR with different target rates