Measuring incompatible observables by exploiting sequential weak values

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Abstract
One of the most intriguing aspects of quantum mechanics is the impossibility of measuring at the same time observables corresponding to non-commuting operators, because of the wavefunction collapse. This impossibility can be partially relaxed when considering joint or sequential weak values evaluation. Indeed, weak value measurements have been a real breakthrough in the quantum measurement framework that is of the utmost interest from both a fundamental and an applicative point of view. In this paper, we show how we realized for the first time a sequential weak value evaluation of two incompatible observables using a genuine single-photon experiment. These (sometimes anomalous) sequential weak values revealed the single-operator weak values, as well as the local correlation between them.
Measurements are the very basis of Physics. In Quantum Mechanics they assume even a more fundamental role, since observables can have undetermined values that “collapse” on a specific one only when a strong measurement (described by a projection operator) is performed. Furthermore, a crucial feature of quantum measurement is that measuring one observable completely erases the information on its conjugate one (e.g. measurement of position erases information about momentum). This impossibility can be partially relaxed when considering joint or sequential weak values evaluation [1–5]. Weak values, introduced in [1] and firstly realized in [6–8], represent a new quantum measurement paradigm, where only a small amount of information is extracted from a single measurement, so that the state basically does not collapse. They can have anomalous values (imaginary, unbounded values) and, while their real part is usually interpreted as a conditional average of the observable in the limit of zero disturbance [9], their imaginary part is related to the disturbance (or backaction) of the measuring pointer during the measurement process [10]. Weak values have been used for addressing fundamental questions [11] such as contextuality [12, 13], but can also be seen as a groundbreaking tool for quantum metrology allowing high-precision measurements (at least in presence of specific noises [14]), as the tiny spin Hall effect [8] or small beam deflections [15] and characterization of wavefunction [16–18].

Nevertheless, up to now only WMs on a single observable (eventually followed by a strong measurement) or joint WMs performed on commuting observables and on different particles (or optical modes) have been realised experimentally [6–8, 11, 12, 14–27]. However, sequential weak values, which are more sensitive to the system’s dynamics and whose time order is crucial, have not been performed yet. One of the most intriguing properties of sequential weak values is that they allow the simultaneous measurement of non-commuting observables, challenging “one of the canonical dicta of quantum mechanics” [4] (i.e. the impossibility of measuring two non-commuting observables at the same time because of the wave function collapse). This result has not been reached in any previous experiment, since none of them allowed simultaneous measurement (also of weak values) of non-commuting observables on the same particle [28]. Here we achieve for the first time this result by experimentally demonstrating the peculiar predictions regarding single and sequential weak values, measuring at the same time non-compatible polarizations using real single-photons.

Specifically, the weak value of an observable \( \hat{A} \) is defined as \( \langle \hat{A} \rangle_w = \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \), where a
key role is symmetrically played by the pre-selected ($|\psi_i\rangle$) and post-selected ($|\psi_f\rangle$) quantum states. When the pre- and post-selected states are equal, the weak value is just the expectation value of $\hat{A}$.

Weak values are usually obtained taking advantage of the coupling between the observable $\hat{A}$ and the pointer observable $\hat{P}$, according to the unitary transformation $\hat{U} = \exp(-ig\hat{A} \otimes \hat{P})$. When the weak interaction regime is assumed, one can describe the evolution of this system, prepared in the pre-selected state and projected on the post-selected state, as

$$\langle \psi_f | e^{-ig\hat{A} \otimes \hat{P}} | \psi_i \rangle \simeq \langle \psi_f | \psi_i \rangle (1 - ig\langle \hat{A} \rangle_w \hat{P}).$$

By measuring the observable $\hat{X}$—canonically conjugated to $\hat{P}$—one can extract, in general, the real part of the weak value $\langle \hat{A} \rangle_w$ from the relation $\langle \hat{X} \rangle = \text{Re}[g\langle \hat{A} \rangle_w]$ (and the weak value itself if $\text{Re}[\langle \hat{A} \rangle_w] = \langle \hat{A} \rangle_w$), given that $g$ is independently estimated.

Measurements of joint [3] or sequential [4] weak values of two observables $\hat{A}$ and $\hat{B}$ are obtained when two different couplings ($g_x$ and $g_y$) to two distinct pointer observables (in our experiment the two transverse momenta $\hat{P}_x$ and $\hat{P}_y$) are realised between the pre- and post-selection of the state. In particular, if the measurement is performed exploiting simultaneous interactions, we are dealing with measurement of the joint weak value, and by measuring the covariance of the position observables $\hat{X}$ and $\hat{Y}$ ($\langle \hat{X}\hat{Y} \rangle$) one obtains [3]

$$\langle \hat{X}\hat{Y} \rangle = \frac{1}{4}g_x g_y \text{Re} \left[ (\hat{A}\hat{B} + \hat{B}\hat{A})_w + 2\langle \hat{A} \rangle_w^*\langle \hat{B} \rangle_w \right],$$

while if we have a sequence of two weak interactions, e.g. the first interaction is described by the unitary transformation $\hat{U}_x = \exp(-ig_x\hat{A} \otimes \hat{P}_x)$ and the second by $\hat{U}_y = \exp(-ig_y\hat{B} \otimes \hat{P}_y)$, when measuring $\langle \hat{X}\hat{Y} \rangle$ one obtains [4]

$$\langle \hat{X}\hat{Y} \rangle = \frac{1}{2}g_x g_y \text{Re} \left[ (\hat{A}\hat{B})_w + \langle \hat{A} \rangle_w^*\langle \hat{B} \rangle_w \right].$$

We can already see that the procedure for estimating the sequential weak value $\langle \hat{A}\hat{B} \rangle_w$ is strictly different from the usual procedure of estimating the single weak value of the product operator $\hat{A}\hat{B}$, which corresponds to a single displacement of some measuring pointer. Here, the result is proportional to the correlation between two pointers’ displacements $\hat{X}$ and $\hat{Y}$. It thus corresponds to the weak values of the operators $\hat{A}$ and $\hat{B}$, as well as the temporal correlation between them. In addition, when $\hat{A}$ and $\hat{B}$ are non-commuting, the product $\hat{A}\hat{B}$
is non-Hermitian, hence the weak coupling to it leads to a non-unitary evolution in time, while in our approach the two separate weak couplings to \( \hat{A} \) and \( \hat{B} \) lead to unitary evolution in time. Intriguing schemes exploiting sequential weak averages for the direct measurement of density function is discussed in [5] (where, indeed, it is shown that sequential weak values are necessary, specifically a weak average obtained from a sequence of two weak interactions plus a strong measurement).

Thus, the real part of sequential (Re\( [(\hat{A}\hat{B})_w] \)) or joint (Re\( [(\hat{A}\hat{B} + \hat{B}\hat{A})_w] \)) weak values can be evaluated by measuring \( \langle \hat{X}\hat{Y} \rangle \) and by evaluating each weak value independently, i.e. \( \langle \hat{A} \rangle_w \) and \( \langle \hat{B} \rangle_w \) (these can be obtained by measuring the mean values of the positions and momenta independently, namely \( \langle \hat{X} \rangle, \langle \hat{Y} \rangle, \langle \hat{P}_x \rangle \) and \( \langle \hat{P}_y \rangle \) [3, 4]).

In our experiment, we focus on the case of sequential weak values measurement, where the operators \( \hat{A} \) and \( \hat{B} \) are the linear projectors \( \hat{\Pi}_V = |V\rangle\langle V| \) and \( \hat{\Pi}_\psi = |\psi\rangle\langle \psi| \) (with \( |\psi\rangle = \cos \theta |H\rangle + \sin \theta |V\rangle \)). The considered quantum system is a (heralded) single photon prepared (pre-selected) in the initial state \( |\phi_i\rangle = |\psi_i\rangle \otimes |f_x\rangle \otimes |f_y\rangle \), with \( |\psi_i\rangle = \cos \theta |H\rangle + \sin \theta |V\rangle \) and \( |f_\xi\rangle = \int d\xi |F_\xi(\zeta)\rangle |\zeta\rangle \), where \( |F_\xi(\zeta)\rangle^2 \) is the probability density function of detecting the photon in the position \( \xi \) (with \( \xi = x, y \)) of the transverse spatial plane. \( |F_\xi(\zeta)\rangle^2 \) in our experiment is reasonably Gaussian, since the single photon guided in a single-mode optical fiber is collimated with a telescopic optical system. By experimental evidence, we can assume that the (unperturbed) \( |F_\xi(\zeta)\rangle^2 \) is centered around zero and has the same width \( \sigma \) both for \( \xi = x \) and for \( \xi = y \).

The single photons undergo two sequential weak interactions inducing displacements in two orthogonal directions according to the two unitary transformations \( \hat{U}_y = \exp(-ig_y\hat{\Pi}_V \otimes \hat{P}_y) \) and \( \hat{U}_x = \exp(-ig_x\hat{\Pi}_\psi \otimes \hat{P}_x) \). This spatial displacement - due to the polarisation-sensitive spatial walk-off of the Poynting vector of the single photon induced by its propagation into a birefringent medium - realises in practice the weak interaction (see Fig. 1 for details).

Then, the single-photon is projected on the post-selected linear polarization state \( |\psi_f\rangle \) and detected by a spatial-resolving detector. Thus, the post-selected single-photon state is \( |\phi_f\rangle = \langle \psi_f|\hat{U}_x\hat{U}_y|\psi_i\rangle \). Since we are focusing on linear polarisations only, it is possible to evaluate the sequential weak value of the (in general) non-commuting projectors \( \langle \hat{\Pi}_\psi\hat{\Pi}_V \rangle_w \), as well as the single weak values \( \langle \hat{\Pi}_\psi \rangle_w \) and \( \langle \hat{\Pi}_V \rangle_w \). In fact, according to Eq. (3), we have \( \langle \hat{X}\hat{Y} \rangle = \frac{1}{2} g_xg_y (\langle \hat{\Pi}_\psi\hat{\Pi}_V \rangle_w + \langle \hat{\Pi}_\psi \rangle_w \langle \hat{\Pi}_V \rangle_w), \langle \hat{X} \rangle = g_x\langle \hat{\Pi}_\psi \rangle_w, \langle \hat{Y} \rangle = g_y\langle \hat{\Pi}_V \rangle_w [34]. \)
inverting these relations, it is possible to obtain the weak values of the two non-commuting observables \( \langle \hat{\Pi}_V \rangle_w \) and \( \langle \hat{\Pi}_\psi \rangle_w \), as well as the sequential weak value of the two non-commuting observables \( \langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w \). Note that this relation between position mean values and polarisation weak values holds only in the case of weak interaction, i.e. only for \( g/\sigma \ll 1 \). In our case we have evaluated \( g_x/\sigma \sim g_y/\sigma \sim 0.15 \).

The experimental setup is presented in Fig.1: it hosts a heralded single-photon source based on pulsed parametric down-conversion (PDC), exploiting a 796 nm mode-locked Ti:Sapphire laser (repetition rate: 76 MHz) whose second harmonic emission pumps a \( 10 \times 10 \times 5 \) mm \( \text{LiIO}_3 \) nonlinear crystal, producing Type-I PDC.

The idler photon (\( \lambda_i = 920 \) nm) is coupled to a single-mode fiber (SMF) and then addressed to a Silicon Single-Photon Avalanche Detector (SPAD), heralding the presence of the correlated signal photon (\( \lambda_s = 702 \) nm) that, after being SMF-coupled, is sent to a launcher and then to the free-space optical path, where the experiment for weak values evaluation is performed.

We have estimated the quality of our single-photon emission obtaining a \( g^{(2)} \) value (or more properly a parameter \( \alpha \) value [30]) of \((0.13 \pm 0.01)\) without any background/dark-count subtraction.

After the launcher, the heralded single photon state is collimated by a telescopic system, and then prepared (pre-selected) in a linear polarization state \( |\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle \) (by means of a calcite polarizer followed by a half-wave plate). The first weak interaction is carried out by a 2 mm long birefringent crystal (BC\textsubscript{V}) whose extraordinary (\( e \)) optical axis lies in the \( Y-Z \) plane, with an angle of \( \pi/4 \) with respect to the \( Z \) direction. Due to the spatial walk-off effect experienced by the vertically-polarized photons (i.e. along \( Y \) direction), horizontal- and vertical-polarization paths get slightly separated along the \( Y \) direction, inducing in the initial state \( |\psi_i\rangle \) a small decoherence (with our experimental \( g/\sigma \sim 0.15 \), being the “decoherence” coefficient on the off-diagonal elements \( \kappa_D = \exp[-g^2/(2\sigma)^2] \), we expect \( \kappa_D \sim 0.982 \); this is confirmed by the experimental tomographic reconstruction of the state, giving a fidelity \( F = 0.997 \) with respect to the theoretical predictions) that leaves it substantially unaffected.

Together with the spatial walk-off, the birefringent crystal also induces on this single-photon state a temporal walk-off and eventually a polarization change, both to be eliminated in order to avoid unwanted additional decoherence effects. We were able to nullify this
unwanted effects by adding another birefringent crystal of properly chosen length (1.1 mm) with the optical axis lying on the $X$ direction, in order to compensate the temporal walk-off without introducing any additional spatial walk-off. This second crystal is mounted on a piezo-controlled rotator (having a nominal resolution 0.001°) allowing almost perfect temporal compensation, i.e. to avoid any unwanted circularity in the polarisation state coming from the previous interaction.

After the weak interaction and the phase compensation in BC$_V$, the photon goes to the second weak interaction module. It is constituted by a system (BC$_H$) of two birefringent crystals rotated by 90° with respect to the previous one, i.e. the first crystal has its optical axis in the $X$-$Z$ plane, while the second one has the optical axis in the $Y$ direction, inserted between two half-wave plates. By rotating both wave-plates of the same angle with respect to the $H$-axis, one obtains the weak interaction on the linear polarisation state $|\psi\rangle$ with the polarisations separation appearing along the $X$ direction. This can be thought of as a simple example of the unitary evolution between weak interactions affecting the sequential weak value as discussed in [4].

After both WMs are performed, the photon meets a half-wave plate and a calcite polarizer, used to project the state onto the post-selected state $|\psi_f\rangle$, and then it is detected by a spatial-resolving single-photon detector prototype. This device is a two-dimensional array made of 32 × 32 “smart pixels” -each pixel includes a SPAD detector and its front-end electronics for counting and timing single photons [31]. All the pixels operate in parallel with a global shutter readout. The SPAD array is gated with 6 ns integration windows, triggered by the SPAD detector on the heralding arm. Being the heralding detection rate in the order of 100 kHz, the effective dark count rate of the array is drastically reduced by the low duty cycle, improving the signal-to-noise ratio.

The main results of our work are summarised in Fig. 2 where we have carefully chosen the pre- and post-selected states in order to show peculiar paradoxical properties predicted for sequential weak values -namely $|\psi_i\rangle = 0.588|H\rangle + 0.809|V\rangle$ and $|\psi_f\rangle = |H\rangle$ for Fig. 2(a), and $|\psi_i\rangle = 0.509|H\rangle + 0.861|V\rangle$ and $|\psi_f\rangle = -0.397|H\rangle + 0.918|V\rangle$ for Fig. 2(b). Here we plot the two weak values and the sequential one as a function of the angle $\theta$ of the polarisation projector $\tilde{\Pi}_\psi$ of the second weak interaction, showing a remarkable agreement with the theoretical predictions also in the case of anomalous weak values. An example of a paradoxical situation is represented by the case where, even
FIG. 1: Experimental setup and detection apparatus. (a) A frequency doubled mode-locked Ti:Sa laser pumps a LiIO$_3$ Type-I PDC crystal. The idler photon ($\lambda_i = 920$ nm) is coupled to a single-mode fiber (SMF) and addressed to a SPAD heralding the correlated signal photon ($\lambda_s = 702$ nm) that is prepared in a linear polarization state (pre-selection block) and sent to the in-air weak interaction apparatus. The first weak interaction is operated by the BC$_V$ system (composed of two orthogonal birefringent crystals, the first one realizing the weak interaction, the second one compensating temporal walk-off and decoherence effects), followed by the BC$_H$ block (identical to BC$_V$ but with a $90^\circ$ rotation of the $Z$ axis), in which the second weak interaction takes place. Just before and after BC$_H$, two half-wave plates are put in order to arbitrarily change the basis of this second measurement. Finally, the photon is post-selected and detected (SHG: Second Harmonic Generator; QWP: Quarter Wave Plate; HWP: Half Wave Plate; PBS: Polarizing Beam Splitter; BC: Birefringent Crystal). (b) Typical single data acquisition obtained with our spatial resolving single-photon detector (32X32 SPAD camera), after noise subtraction. It represents the number of counts acquired in 300 s versus the different pixels of the SPAD array. (c) The corresponding predicted probability distribution calculated according to the theory. (d) our SPAD camera prototype.
FIG. 2: Measured weak values (data points) compared with the theoretical predictions (dashed lines) for different $\hat{\Pi}_\psi$ (i.e. for different values of $\theta$, since $|\psi\rangle = \cos \theta |H\rangle + \sin \theta |V\rangle$). Blue and red points and lines correspond to the evaluations of the single-weak-value $\langle \hat{\Pi}_\psi \rangle_w$ and $\langle \hat{\Pi}_V \rangle_w$, respectively, while purple points and line represent the evaluation of the sequential-weak-value $\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w$. Uncertainty bars are evaluated on the basis of sequences of repeated measurements. The uncertainty bars are naturally bigger in the case of the evaluation of sequential-weak-values with respect to the case single-weak-values, since in the former case the quantity measured is a covariance of positions, while in the latter cases they are position mean values. The pre-selected and post-selected states are respectively $|\psi_i\rangle = 0.588 |H\rangle + 0.809 |V\rangle$ and $|\psi_f\rangle = |H\rangle$ for plot (a), and $|\psi_i\rangle = 0.509 |H\rangle + 0.861 |V\rangle$ and $|\psi_f\rangle = -0.397 |H\rangle + 0.918 |V\rangle$ for plot (b).

if one of the two single weak values is zero (within the uncertainty), the sequential weak value of the two non-commuting observables is significantly different from zero, e.g. in Fig. 2(a) when $\theta = 0.2\pi$ we obtain $\langle \hat{\Pi}_V \rangle_w = 0.03 \pm 0.03$, $\langle \hat{\Pi}_\psi \rangle_w = 1.44 \pm 0.04$, while $\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w = 0.69 \pm 0.15$, or when $\theta = 0.9\pi$ we have $\langle \hat{\Pi}_V \rangle_w = 0.04 \pm 0.03$, $\langle \hat{\Pi}_\psi \rangle_w = 0.35 \pm 0.04$, while $\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w = -0.46 \pm 0.10$. In particular, in the last case, we have a positive and an almost null positive single weak value associated to the two non-commuting observables, while the corresponding sequential weak value is negative, and with a modulus two orders of magnitude greater than the product of the single weak values. We also observe the surprising situation of having both one single weak value and the sequential weak value positive, while the other single weak value is negative (e.g. in Fig.2(b) when $\theta = 0.9\pi$ we obtain $\langle \hat{\Pi}_V \rangle_w = 1.40 \pm 0.04$, $\langle \hat{\Pi}_\psi \hat{\Pi}_V \rangle_w = 0.28 \pm 0.10$, while $\langle \hat{\Pi}_\psi \rangle_w = -0.24 \pm 0.03$).
the lines of [11], these are clear demonstrations of the “product rule” breakdown when weak values are concerned.

More generally, looking at Fig. 2(a) we can note that, despite the fact that \( \langle \hat{\Pi}_V \rangle_w \sim 0 \) everywhere, we have that both the single weak value of the other non-commuting observable and the sequential weak are significantly non-zero. Furthermore, for both of them we have observed anomalous weak values, i.e. weak values not bounded by the spectrum of the observables (in our case between 0 and 1). In Fig. 2(a) we observe \( \langle \hat{\Pi}_V \rangle_w > 1 \) and \( \langle \hat{\Pi}_V \rangle_w < 0 \), as well as \( \langle \hat{\Pi}_V \hat{\Pi}_V \rangle_w < 0 \). Analogously, in Fig. 2(b) we find in one case that all the weak values \( \langle \hat{\Pi}_V \rangle_w, \langle \hat{\Pi}_V \rangle_w, \) and \( \langle \hat{\Pi}_V \hat{\Pi}_V \rangle_w \) are larger than 1.

As pointed out also in Ref. [4], weak values present an internal consistency, thus they should be considered as the actual value of the parameters measured albeit the curious appearance of anomalous values. This internal consistency is also reflected in our data. In Fig. 2(a) looking at the data corresponding to \( \theta = 0.2\pi \) (in the following \( \hat{\Pi}_{\psi_0} \)) and \( \theta = 0.7\pi \) (in the following \( \hat{\Pi}_{\psi_0} \)) we observe that \( \langle \hat{\Pi}_{\psi_0} \rangle_w + \langle \hat{\Pi}_{\psi_0} \rangle_w = 0.97 \pm 0.06 \) in agreement with the general rule \( \langle \hat{\Pi} \rangle_w + \langle \hat{\Pi} \rangle_w = 1 \). Analogously, as in general \( \langle \hat{\Pi}_V \hat{\Pi}_V \rangle_w + \langle \hat{\Pi}_V \hat{\Pi}_V \rangle_w = \langle \hat{\Pi}_V \rangle_w \), in our case we observe that \( \langle \hat{\Pi}_{\psi_0} \hat{\Pi}_V \rangle_w + \langle \hat{\Pi}_{\psi_0} \hat{\Pi}_V \rangle_w = -0.05 \pm 0.22 \), in agreement with the theoretical prediction \( \langle \hat{\Pi}_V \rangle_w = 0 \), and the experimentally measured average value \( \langle \hat{\Pi}_V \rangle_w = 0.02 \pm 0.06 \).

Our uncertainties on the weak values presented in the paper and shown in the plots of Fig. 2 are obtained with the uncertainty propagation standard rules (coverage factor \( k = 1 \)) starting from the images collected by our 32 × 32 SPAD array. The statistical fluctuations on our data are obtained collecting 9 different images for each experimental point. After analyzing every image by itself, for each of the quantities \( g_x, g_y, \langle \hat{X} \rangle_f, \langle \hat{Y} \rangle_f \) and \( \langle \hat{X} \hat{Y} \rangle_f \) we extract the mean value and the corresponding uncertainty, i.e. the standard deviation on the average.

Summarising, we demonstrate an unprecedented measurement capability, providing information on two non-commuting observables at the same time, as well as on the correlation between them, a feature forbidden in the conventional (i.e. POVM-based) measurement framework of quantum mechanics.

In our sequential weak value experiment we exploit two weak couplings plus a “strong” post-selection measurement to obtain the simultaneous estimation of two single-operator weak values in connection with the same un-collapsed initial state,
as well as the sequential weak value of two (in general non-commuting) observables. This is more significant (as discussed for instance in Ref. [4] and in the recent Ref. [35]) than what can be obtained from a single weak interaction plus a strong post-selection measurement, namely only a single-operator weak value estimation and nothing else. Indeed, another weak value means more (non-counterfactual) information and interesting temporal correlations between non-commuting operators including anomalous and paradoxical weak values.

Furthermore, we note that single-operator weak value estimation exploiting a single weak interaction plus a strong measurement allows obtaining partial information about the complementary observables. For instance, one can employ a weak interaction depending on the first observable and then perform a strong final measurement on the second, in general complementary, observable. This was essentially the idea behind, e.g. wavefunction direct characterization experiments [16–18]. Nevertheless, sequential weak values are much richer, allowing one to obtain the single weak values of two (in our case) or more observables, as well as the sequential weak value of, in general, non-commuting observable at the same time, i.e. as a sequence of weak couplings on one and the same photon. This is possible due to the presence of two independent and distinguishable weak interactions before the final strong measurement. Sequential weak values have been recently exploited in a proof-of-principle experiment of direct measurement of density matrix [36], and they can also be exploited in quantum process tomography [33], which makes use of this very technique of estimating an unknown dynamics. It is also worth mentioning that this experiment does not only shed light on counterfactual computation [32], but in fact enables for the first time its careful experimental test. As proposed in [4], the measurement outcome $|\psi_f\rangle$ is counterfactual if it determines the computer’s outcome and if the sequential weak value of projections onto all of the “on” instances is zero. However, the possible applications of the powerful paradigm of sequential weak values is far from being completely explored. The fact that we have proven experimentally their feasibility on a real single quantum system, will hopefully foster more theoretical and experimental research in the next few years.

Remark: At the time of our submission, Ref. [36] appeared on the arXiv performing an experiment exploiting sequential weak values in an optical setup
similar to ours. The authors implemented their sequential weak values experiment performing, as a proof of principle, the direct measurement of the polarisation density matrix (of a single photon) using also the imaginary part of the weak value, where, for simplicity, the single photon source was replaced with a laser beam.

Acknowledgements

This work has been supported by EMPIR-14IND05 “MIQC2” (the EMPIR initiative is co-funded by the EU H2020 and the EMPIR Participating States) and the EU FP7 under grant agreement No. 308803 (project “BRISQ2”). E.C. was supported by ERC AdG NLST and by the Israel Science Foundation Grant No. 1311/14. We wish to thank Yakir Aharonov and Avshalom C. Elitzur for helpful discussions.


[28] Simultaneous measurements of non-commuting observables have been already claimed in the past. For instance, in the paper [G. A. Howland et al., Phys. Rev. Lett. 112, 253602 (2014)] the authors present a technique able to perform the direct characterisation of the wavefunction as in Ref. [16–18]. Specifically, Ref. [17] exploits a single weak interaction, as well as a strong final measurement to reconstruct the wavefunction. The paper [G. A. Howland et al., Phys. Rev. Lett. 112, 253602 (2014)] exploits a partial filtering (that cannot be considered “weak”), as well as a strong momentum measurements. The analogy between the two approaches is evident. Here we would like to underline that none of the two approaches is able to provide the estimation of the sequential weak value (of two non-commuting operators), at most they are able to provide the sequential weak average [5].


[34] This enables to infer the particles properties via quantum correlations (second order in g), rather than via the single operator value (first order in g, where g is the coupling strength of the von Neumann interaction Hamiltonian). Single weak values usually emerge as first order in g, when linearizing the time evolution created by this weak coupling. Sequential weak values are unique in invoking the next order in g, which corresponds to (local) correlations between the two measured observables. This resembles joint weak measurements performed on two entangled particles, with the important difference that now the two measurements are performed on one and the same particle and thus measure temporal, local correlations where the order of operators is important.
