Comparing the impact of increasing condom use or HIV pre-exposure prophylaxis (PrEP) use among female sex workers

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1. Supplementary Methods

1.1 Model equations

The assumptions result in the following model equations:

\[
\begin{align*}
\frac{dy_0}{dt} &= (\lambda_1 y_1 + \lambda_2 y_2)(1 - y_0) - (\eta + \alpha_0)y_0 : \text{FSW} \\
\frac{dy_1}{dt} &= \lambda_0 y_0 (1 - y_1) - (\eta + \alpha_1)y_1 : \text{Clients} \\
\frac{dy_2}{dt} &= \lambda_3 y_0 (1 - y_1) - (\eta + \alpha_2)y_2 : \text{Pimps}
\end{align*}
\]

(S1)

The basic mathematical properties of model system (S1) are straightforward.
1.2 Basic reproductive number and stability

The basic reproductive number for model system (S1) $R_0$, at the disease-free equilibrium $(y_0, y_1, y_2) \rightarrow (0,0,0)$ using the approach in [1] is given by

$$R_0 = \sqrt{R_1 R_2 + R_3 R_4} \quad (S2)$$

Here, $R_1 = \frac{\beta_0 c_0}{(\eta + \alpha_1)}$ is the expected number of male clients infected by FSWs, $R_2 = \frac{\beta_1 c_1}{(\eta + \alpha_0)}$ is the expected number of FSW infected by clients, $R_3 = \frac{\beta_1 c_2}{(\eta + \alpha_0)}$ is the expected number of FSW infected by pimps and $R_4 = \frac{\beta_0 c_3}{(\eta + \alpha_2)}$, the expected number of pimps infected by FSW.

Using an approach in [2], we transform model system (S1) to

$$\frac{dX}{dt} = F(X, Z), \quad \frac{dZ}{dt} = G(X, Z), G(X, 0) = 0.$$

Here $X \in \mathbb{R}^m$ denotes proportions of uninfected individuals and $Z \in \mathbb{R}^n$ denotes the numbers/proportions of infected individuals. This satisfies the condition the following condition for global stability.

$$\hat{G} = \begin{pmatrix} 
\lambda_0 y_0 y_1 \\
y_1(\lambda_1 y_0 + \lambda_2 y_2) \\
\lambda_3 y_1 y_2 
\end{pmatrix} \geq 0$$

This implies that the disease-free equilibrium of model system (S1) is globally asymptotically stable thus when $R_0 < 1$ the disease dies out independent of the initial sizes of the sub-populations. This means that, $R_0$ as an epidemic threshold completely governs the dynamics of model systems (S1).

1.3 Model without pimps

The sub-model of system (S1) with clients only and FSWs has the following endemic prevalence.
\[
\begin{align*}
\left\{ \begin{array}{l}
y^*_0 = \frac{\lambda_0 \lambda_1 - (\eta + \alpha_0)(\eta + \alpha_1)}{\lambda_0 \lambda_1 + (\eta + \alpha_0)\lambda_1} = \frac{(\eta + \alpha_0)(\eta + \alpha_1)}{\lambda_0 \lambda_1 + (\eta + \alpha_0)\lambda_1} [R^2_0 - 1] \quad (S3) \\
y^*_1 = \frac{\lambda_0 \lambda_1 - (\eta + \alpha_0)(\eta + \alpha_1)}{\lambda_0 \lambda_1 + (\eta + \alpha_1)\lambda_0} = \frac{(\eta + \alpha_0)(\eta + \alpha_1)}{\lambda_0 \lambda_1 + (\eta + \alpha_1)\lambda_0} [R^2_0 - 1] \quad (S4)
\end{array} \right.
\]

Here \( R_0 = \sqrt{R_1 R_2} \) is the basic reproductive number for the model system without pimps and the endemic equilibrium (equations (S3) and (S4)) for this sub-model only exists when \( R_0 > 1 \).

We determine condom and PrEP use coverage levels that give the same impact in reducing the basic reproductive number.

\[
y_c = \frac{1}{\theta}(1 - (1 - e_{fc})^2). \quad (S5)
\]

Here the subscript “c” denotes “threshold quantity”. We also determine condom and PrEP use coverage levels that give the same impact in reducing HIV prevalence in the FSW at endemic prevalence. Using the endemic prevalence expressions in equations (S3) and (S4), the required PrEP coverage to achieve the same reduction in prevalence in FSWs as when condoms are used with a certain consistency \( f_1 \), is given by

\[
y_c = \frac{e f_1 ((\eta + \alpha_0)((\eta + \alpha_1) + \lambda_1(2 - e f_1)) + \lambda_0 \lambda_1(1 - e f_1))}{\theta(\eta + \alpha_0 + (1 - e f_1)\lambda_0)\lambda_1}. \quad (S6)
\]

By setting \( y_c = 1 \) in equation (S5), we derive critical consistency of condom use \( (f_c) \) beyond which PrEP use on its own cannot achieve the same impact in reducing HIV prevalence as condoms in FSW and this is given by

\[
f_c = \frac{e}{2e^2\lambda_1(\eta + \alpha_0 + \lambda_0)} \left( A - \sqrt{A^2 - 4\theta(\eta + \alpha_0 + \lambda_0)^2\lambda_1^2} \right) \quad (S7)
\]

where \( A = (\eta + \alpha_0)(\eta + \alpha_1) + \lambda_1(2(\eta + \alpha_0) + \lambda_0(1 + \theta)) \). The subscript “c” denotes threshold quantity.

Equations (S5) and (S6) are used to compare PrEP and condom use when PrEP coverage is at optimal levels.

Similar numerical calculations were also used to make the same comparison between the impact of PrEP and condoms in terms of HIV infections averted over different time frames.
1.4 Model with pimps

The sub-model of system (S1) with clients, FSWs and pimps, has the following endemic prevalence.

\[
\begin{align*}
\gamma_0^* &= \frac{a_0 (\lambda_0 (a_1 a_3 - \lambda_2 \lambda_3) + \lambda_0 \lambda_1 (\lambda_3 - a_2) + \sqrt{B}) - (2a_2 \lambda_0^2 + a_0^2 \lambda_3 (a_1 + \lambda_2))}{2\lambda_1 (a_0 + \lambda_0)(a_0 \lambda_2 - a_2 \lambda_0)} \\
\gamma_1^* &= \frac{\lambda_0 \lambda_1 (\lambda_3 - a_2) + \lambda_2 \lambda_3 (\lambda_0 - a_0) - a_1 (a_2 \lambda_0 + a_0 \lambda_3) + \sqrt{B}}{2\lambda_0 \lambda_3 (a_1 + \lambda_1 + \lambda_2)} \\
\gamma_2^* &= \frac{K(\lambda_0 \lambda_1 (\lambda_3 - a_2) + \lambda_2 \lambda_3 (\lambda_0 - a_0) - a_1 (a_2 \lambda_0 + a_0 \lambda_3) + \sqrt{B})}{4\lambda_0 \lambda_2 (a_1 + \lambda_1 + \lambda_2)(a_2 + \lambda_3)(a_2 \lambda_0 - a_0 \lambda_3)} \\
\end{align*}
\]

(S8)  

(S9)  

(S10)

Here, \[
\begin{align*}
a_0 &= \eta + a_0, a_1 = \eta + a_1, a_3 = \eta + a_3 \\
B &= ((a_2 \lambda_0 \lambda_1 - (\lambda_0 (\lambda_1 + \lambda_2) - a_0 \lambda_2) \lambda_3 + a_1 (a_2 \lambda_0 + a_0 \lambda_3))^2 \\
&+ 4\lambda_0 \lambda_3 (a_1 + \lambda_1 + \lambda_1)(a_2 \lambda_0 \lambda_1 + a_0 (\lambda_2 \lambda_3 - a_1 a_2))) \\
K &= ((a_2 \lambda_0 (\lambda_1 + 2\lambda_2) + \lambda_0 \lambda_1 \lambda_3 - a_0 \lambda_2 \lambda_3 + \lambda_0 \lambda_2 \lambda_3 + a_1 (a_2 \lambda_0 - a_0 \lambda_3) - \sqrt{B})
\end{align*}
\]

Here, we derive coverage levels of condom or PrEP that achieve the same impact in reducing HIV prevalence in the FSW for the model with pimps at endemic levels (similar to the analysis for the model without pimps).

Using the endemic prevalence expression in equations (S7), (S8) and (S9), the required PrEP coverage \( \gamma_c \) to achieve the same reduction in HIV prevalence amongst FSWs as condoms is:

\[
\gamma_c = \frac{a_1 (C - 2a_2 \lambda_0) (C - 2a_0 \lambda_3) + (C + 2a_0 \lambda_3) (\lambda_1 (C - 2a_2 \lambda_0) + \lambda_2 (C - 2a_0 \lambda_3))}{\theta (C + 2a_0 \lambda_3) (\lambda_1 (C - 2a_2 \lambda_0) + \lambda_2 (C - 2a_0 \lambda_3))}.
\]

Here \( C = \frac{E + \sqrt{F}}{D} \) where

\[
\begin{align*}
D &= (e f_1 - 1) (e f_2 - 1) (a_1 + \lambda_1 (1 - e f_1) + \lambda_2 (1 - e f_2)) \\
E &= a_2 \lambda_0 \lambda_1 (e f_1 - 1)^2 + (e f_2 - 1) (a_0 \lambda_2 (e f_2 - 1) + \lambda_0 (e f_1 - 1) (\lambda_1 (e f_1 - 1) + \lambda_2 (e f_2 - 1))) \lambda_3 \\
&+ a_1 (a_2 \lambda_0 (e f_1 - 1) + a_0 \lambda_3 (e f_2 - 1)) \\
F &= E^2 - 4\lambda_1 \lambda_3 (e f_1 - 1) (e f_2 - 1) (a_1 + \lambda_1 (e f_1 - 1) + \lambda_2 (e f_2 - 1)) (a_0 (a_1 a_2 - (e f_2 - 1)^2 \lambda_2 \lambda_3) \\
&- (a_2 \lambda_0 \lambda_1 (e f_1 - 1)^2).
\end{align*}
\]

We could not derive an explicit expression for the critical level of condom use \( (f_c^*) \) due to the complicated nature of the algebraic expression of \( \gamma_c \), and so this was estimated numerically to compare PrEP and condom use.
We determine condom and PrEP use coverage levels that give the same impact in reducing the basic reproductive number for the model with pimps.

\[ y_c = \frac{1}{\theta} \left[ 1 - \left( \frac{(1 - e f_c)^2 R_1 R_2 + (1 - e f_c / 3)^2 R_3 R_4}{R_1 R_2 + R_3 R_4} \right) \right]. \quad (S11) \]

Both models (with/without pimps) were used to derive expressions for the consistency of condom use and PrEP coverage that give the same impact in reducing \( R_0 \) or the endemic FSW HIV prevalence compared to a baseline where PrEP and condoms are not used. These expressions were used to derive the ‘critical threshold’ level of condom use beyond which PrEP use on its own cannot achieve the same impact on endemic HIV prevalence amongst FSWs. This threshold was used to assess the relative impact of PrEP through estimating the average increase in the consistency of condom use that gives the same impact as a unit increase in PrEP coverage for different endemic FSW HIV prevalences.

1.5 Model parameterization

The model is parameterized using data from different sources (Table 1). The probabilities of HIV transmission were varied in the range of values reported in [3] \((\beta_0 \beta_1) = [0.0001-0.011] \) to give different HIV prevalences or basic reproductive numbers. The frequency of partner acquisition among clients, FSWs and pimps is estimated using data from different studies [4-9], with baseline estimates for clients with FSWs and vice versa to be 5 and 50 partners/month, respectively. Based on previous modelling of the effect of pimps on HIV transmission amongst FSWs, four different scenarios for partner acquisition by FSWs with pimps and vice versa were considered [7]: (S1) \( c_2^* = 2 \) pimps per month, \( c_3^* = 8 \) FSWs per month and \( m = 2 \) sex acts between each FSW and pimp per month; (S2): \( c_2^* = 2 \) pimps per month, \( c_3^* = 4 \) FSWs per month and \( m = 2 \) sex acts between each FSW and pimp per month; (S3) \( c_2^* = 1 \) pimp per month, \( c_3^* = 4 \) FSWs per month and \( m = 1 \) sex acts between each FSW and pimp per month; and (S4) \( c_2^* = 1 \) pimp per month, \( c_3^* = 1 \) FSW per month and \( m = 1 \) sex act between each FSW and pimp per month. The rates of leaving being a sex worker client, FSW or pimp were estimated from previous studies [4, 6-8, 10-12, with baseline \( \alpha_0 = 0.25 \) (FSWs), \( \alpha_1 = 0.16 \) (clients) and \( \alpha_2 = 0.1 \) (pimps) per year. Individuals are assumed to die of HIV on average 8 years after acquiring HIV [13-15]. Condom efficacy is
estimated to be 85% [16-18] and PrEP efficacy is estimated to be 70% based on recent clinical trials [19-21]. Consistency of condom use and proportion of FSWs on PrEP are varied from 0-100% to consider different scenarios and make comparisons. We assume consistency of condom use between pimps and FSWs to be a third of the consistency of use between clients and FSWs [22]. Thus, in our numerical analysis we vary $f$ from 1% to 100% in steps of 1% for the model without pimps and for the model with pimps, $f_1$ is varied from 1% to 100% in steps of 1% and $f_2$ is assumed to be $\frac{1}{3}f_1$ (i.e. $f_2 = \frac{1}{3}f_1$).

2. Supplementary Results

![Figure S1: Relationship between PrEP coverage ($\gamma$) and consistency of condom use ($f_1$) required to achieve the same impact in reducing the basic reproductive number ($R_0$) for HIV transmission between FSWs and their clients. The effect of pimps is not included in this model and no condom use or PrEP use is assumed at baseline. We vary $f_1$ in the expression given in equation (S5) and other parameter values used are given in Table 1.](image)
Figure S2: Time series plots of the impact on FSW HIV prevalence over time of (a) condoms being used by FSWs with their clients; and (b) PrEP being used by FSWs only. The interventions are introduced at time $t = 40$ years with no PrEP or condom use prior to that time. We use initial prevalence $y_0 = 0.001$, $y_1 = 0.001$ and $f_1$ and $\gamma$ going from 0% at baseline to 20% or 40% with the interventions, $c_0 = 5$ partners/month and $c_1 = 50$ partners/month with other parameter values as given in Table 1. Coverage of PrEP ($\gamma$) is just amongst HIV negative FSWs whereas all FSWs use condoms with an average consistency of use ($f_1$).
**Figure S3:** The relative impact of PrEP compared to condoms for averting HIV infections after 5, 10 and 20 years for the model without pimps at different baseline FSW HIV prevalences. We use parameter values given in Table 1.
Figure S4: Levels of impact of condom use and PrEP by baseline HIV prevalence for models with different levels of pimp involvement, with impact estimated in terms of (a) % reduction in HIV prevalence and (b) % of infections averted among FSWs. We assumed PrEP use to be 50% with no condom use for PrEP scenario, and for condom use scenario we assumed no PrEP use but condom use to be 50% between clients and FSW partnerships and 17% between pimps and FSW partnerships. The same four pimp scenarios S1, S2, S3 and S4 were assumed. Other parameter values are as given in Table 1.
Figure S5. Relative impact of PrEP (i.e. average increase in condom consistency required to have the same impact as a unit increase in coverage of PrEP amongst HIV negative FSWs) for different baseline levels of condom use and FSW HIV prevalence. We considered a scenario S1 in which pimps have sex with 8 FSWs per month and FSWs have 2 pimps per month with 2 sex acts with each per month. Impact is either assessed in terms of decreases in FSW HIV prevalence (Figure S5(a)) or number of HIV infections averted over 10 years (Figure S5(b)). These were estimated using parameter values in Table 1.
Figure S6. Relative impact of PrEP (i.e. average increase in condom consistency required to have the same impact as a unit increase in coverage of PrEP amongst HIV negative FSWs) for different baseline levels of condom use and FSW HIV prevalence. We considered a scenario S2 in which pimps have sex with 4 FSWs per month and FSWs have 2 pimps per month with 2 sex acts with each per month. Impact is either assessed in terms of decreases in FSW HIV prevalence (Figure S6(a)) or number of HIV infections averted over 10 years (Figure S6(b)). These were estimated using parameter values in Table 1.
Figure S7. Relative impact of PrEP (i.e. average increase in condom consistency required to have the same impact as a unit increase in coverage of PrEP amongst HIV negative FSWs) for different baseline levels of condom use and FSW HIV prevalence. We considered a scenario S3 in which pimps have sex with 4 FSWs per month and FSWs have 1 pimps per month with 1 sex acts with each per month. Impact is either assessed in terms of decreases in FSW HIV prevalence (Figure S7(a)) or number of HIV infections averted over 10 years (Figure S7(b)). These were estimated using parameter values in Table 1.
Figure S8. Relative impact of PrEP (i.e. average increase in condom consistency required to have the same impact as a unit increase in coverage of PrEP amongst HIV negative FSWs) for different baseline levels of condom use and FSW HIV prevalence. We considered a scenario S4 in which pimps have sex with 1 FSWs per month and FSWs have 1 pimps per month with 1 sex acts with each per month. Impact is either assessed in terms of decreases in FSW HIV prevalence (Figure S8(a)) or number of HIV infections averted over 10 years (Figure S8(b)). These were estimated using parameter values in Table 1.
**Figure S9:** Sensitivity analysis on how the relative impact of PrEP changes for specific changes in model parameters. The relative impact of PrEP is defined as the average increase in condom consistency that is required to have the same impact in either decreasing FSW HIV prevalence (Figure S9(a)) or averting HIV infections over 10 years (Figure S9(b)) as a unit increase in coverage of PrEP amongst HIV negative FSWs. All projections are compared against a baseline impact scenario (shown as black dashed line) that assumes 40% HIV prevalence amongst FSWs, 60% condom use between FSWs and clients for the model without pimps. The parameter values variations considered are shown in the figure with the baseline values being number of clients per FSW - $c_1 = 50$ per month, number of FSWs per client, $c_2 = 5$ per month, yearly leaving rates for clients ($\alpha_1$), FSWs ($\alpha_0$) being 0.25 and 0.16, condom efficacy $e = 85\%$, PrEP efficacy $\theta = 70\%$.

**References**


