Achieve a low carbon supply chain through product mix

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ABSTRACT

Purpose

In the era of climate change, industrial organizations are under increasing pressure from consumers and regulators to reduce greenhouse gas emissions. The paper examines the effectiveness of product mix as a strategy to deliver the low carbon supply chain under the cap-and-trade policy.

Design/methodology/approach

We incorporate the cap-and-trade policy into the green product mix decision models by using game-theoretic approach and compare these decisions in a decentralized model and a centralized model respectively. The research explores potential behavioural changes under the cap-and-trade in the context of a two-echelon supply chain.

Findings

Our analysis results show that the channel structure has significant impact on both economic and environmental performances. An integrated supply chain generates more profits. In contrast, a decentralized supply chain has lower carbon emissions. The cap-and-trade policy makes a different impact on the economic and environmental performances of the supply chain. Balancing the trade-offs is critical to ensure the long term sustainability.

Originality/value

The research offers many interesting observations with respect to the effect of product mix strategy on operational decisions and the trade-offs between costs and carbon emissions under the cap-and-trade policy. The insights derived from our analysis do not only help firms to make important operational and strategic decisions to reduce carbon emissions while maintaining their economic competitiveness, but also make meaningful contribution to governments’ policy making for carbon emissions control.

Keywords: carbon management; low carbon supply chain; cap-and-trade; product mix
1 Introduction

Climate change has brought about many problems, such as rising sea level, heat waves, storm surges, and reduction of human life expectation (Cheryl et al., 2013). The increasing carbon emissions are regarded as one important human factor that contributes to global warming. To combat man-made climate change, more than 190 countries reached a final deal for the most comprehensive international agreement ever to reduce their greenhouse gas emissions in 2015. According to the United Nation, over 140 countries of 197 Parties including the top emitters e.g. China, the United States, and India, have ratified the Paris climate agreement. Now, it is their responsibility to take immediate actions to reduce global greenhouse gas emissions (e.g. carbon emission) and transparently report the results. Many carbon emissions control policies such as mandatory carbon emissions capacity, carbon emissions tax, cap-and-trade, and investment in the carbon offsets are available to governments to achieve the national targets set in the agreement (Song and Leng, 2012; Choi, 2013; Konur and Schaefer 2004; Chen and Hao, 2015). Among them, cap-and-trade is a policy approach that attracts much attention. The approach first sets an overall cap, and allows companies to trade the unused portion of their cap to other companies with high greenhouse gas emissions. Accompanied by complementary regulatory measures, cap-and-trade is a sufficient or necessary condition for carbon emission reduction (Hanemann, 2010). In practice, many countries or regions, like the United States, European Union and China have used the policy to reduce carbon emissions. Cap-and-trade policies, such as European Union Emissions Trading Scheme, have been proven to be an important tool to address climate change, and become a major choice for investors to decentralize their investment risks (Zhu and Wei, 2013).

Managing carbon emissions has become an important area of policy making and research against the background of increasing political and societal concerns about climate change. Although, carbon reporting has become a mandatory requirement for companies in many countries, it is often not enough for individual firms to report and reduce their own carbon emissions. Because, whatever progress is made “in house”, a large proportion of carbon emissions typically occur outside the direct control of reporting companies but within the operations of suppliers in their procurement networks (Bhattacharya et al. 2014; Kumar et
Therefore, to achieve the low carbon objective, it is essential to reduce carbon emissions across the whole supply chain. In the UK, major supermarket chains such as Tesco and Sainsbury’s set carbon emission reductions targets for both store and distribution operation as well as their supply chains. Individual firms have to respond with innovative and effective strategies in achieving the carbon emission reduction targets while maintain economic competitiveness (Luo et al. 2016; Uyarra et al. 2016). One popular strategic response from the firms is to produce a mixture of green and standard products. Green products deliver the environmental benefit as they generate less carbon emissions but with higher operational costs. In contrast, standard products provide the economic benefit as they produce higher carbon emissions with lower operational costs. It is important for firms to balance the tradeoff between the economic and environmental objectives for the sustainable development (Lee et al. 2012; Fahimnia et al. 2015; Tseng et al. 2015; Wang 2015).

This research focuses on the cap-and-trade policy, and investigates their effects on supply chain decisions, profits and carbon emissions reduction. In this paper, a two-echelon supply chain that consists of a supplier and a manufacturer is considered. The manufacturer purchases materials from the supplier and then produces mixed products to sell to end users in order to reduce carbon emissions. The manufacturer is imposed by the cap-and-trade policy. The research questions in this paper are as follows:

1. What are the supplier’s optimal wholesale price, the manufacturer’s optimal retail prices and production quantities for mixed products in the cases of without and with cap-and-trade?

2. What effect does channel structure (integrated channel or decentralized channel) have on the supply chain’s decisions, profits and carbon emissions reduction?

3. What effect does the cap-and-trade policy have on the supply chain’s decisions, profit and carbon emissions reduction?

In order to answer these questions, we develop carbon efficient supply chain game models with mixed products under the cap-and-trade policy. This paper contributes to the existing literature by providing model-based insights on the impact that cap-and-trade policy have on supply chain decisions and performances. We incorporate the cap-and-trade policy

into the green product mix decision models by using game-theoretic approach and compare these decisions in a decentralized model and a centralized model respectively. We offer interesting observations with respect to the effect of the cap-and-trade policy on supply chain decisions and the tradeoff between costs and carbon emissions. The results derived from the proposed models provide many interesting management insights and important policy implications that enable governments and regulators to examine current policies on carbon emission control.

The remainder of this paper is organized as follows. Section 2 surveys related literature on carbon efficient supply chain management in particular the effort of the cap-and-trade policy on supply chain decisions and performances. Section 3 presents the model formulation and assumptions. Section 4 investigates the integrated channel model under the cap-and-trade policy, and derives the central controller’s optimal retail prices and production quantities of mixed products. In Section 5 we discuss the decentralized channel model under the cap-and-trade policy, and derive the supplier’s optimal wholesale price, the manufacturer’s optimal retail prices and production quantities of mixed products. Section 6 examines the effect of channel structure and cap-and-trade policy on the supplier’s and manufacturer’s decisions, profits and the supply chain’s carbon emissions reduction. In Section 7 we present the concluding remarks and highlight possible future work.

2 Literature review

Low carbon supply chain management is an important issue in operations management. Relevant studies on the carbon emission reduction problem have been well reported in the literature. Comprehensive literature reviews on “green”, “environmental”, “sustainable” operation and supply chain management can be found in Kleindorfer et al. (2005), Corbett and Klassen (2006), Srivastava (2007), Seuring and Müller (2008). To highlight our contributions, we only review the literature that is representative and particularly relevant to our study.

Driven mainly by carbon emissions reduction, more and more scholars have taken the carbon emissions-related issues into consideration. For instance, Song and Leng (2012) adopted Newsboy model to analyze the single-period problem under carbon emissions
policies. Ramudhin et al. (2010) considered trade carbon emissions using a multi-objective mixed integer programming model. Some literature has discussed the carbon emissions tax policy. Penkuhn et al. (1997) presented an optimization production planning model by integrating emission taxes for the process industries. Lee and Cheong (2011) and Lee (2012) used the case study of Korean automotive manufacturers to explore the roles and usefulness of carbon footprint and carbon accounting for carbon management in the automotive supply chain. Through case studies of Walmart and ZETA Communities, Plambeck (2012) provided examples of how companies can profitably reduce greenhouse gas emission through operations and supply chain management. Meng et al. (2013) adopted a computable general equilibrium model to simulate the effects of the carbon tax on the environment and economy in Australia. Another stream of literature discusses the green supply chain design. Chan et al. (2013) developed a comprehensive framework for eco-design evaluation. In their research, environmental performance measures including carbon emissions throughout a product life cycle were considered in assessing alternative product designs. Chen and Hao (2015) employed game theory models to study two competing firms with carbon emissions tax. Jiang and Chen (2016) proposed a newsvendor model that incorporates strategic customer behavior and carbon emissions-sensitive random demand. However, most of the above studies do not examine the impact of the cap-and-trade policy on firms’ decisions and the associated carbon emissions reduction performance, which is the focus of this study.

Among the few studies that concentrate on the effect of the cap-and-trade policy on operation decisions, one stream of research tackles the carbon emission problem through operations decisions at firm level which is under its direct control. Hua et al. (2011) investigated firms that manage carbon footprints in inventory management under the carbon emissions trading mechanism. They derived the optimal order quantity, and examined the impacts of carbon trade factors on order decisions, carbon emissions, and total cost. He et al. (2012) compared the effectiveness and efficiency of cap-and-trade and carbon taxes through a case study of electricity generation expansion planning decisions in a competitive market. Their finding showed that both policies have their relative advantages and disadvantages with respect to different criteria. Zhang and Xu (2013) investigated the multi-item production
planning problem under carbon cap-and-trade mechanism. They built a profit-maximization model and provided an efficient solution method with linear computational complexity to solve the optimal production policy and carbon trading decisions. Using stochastic customer demand, Chen and Wang (2016) investigated the retailer’s optimal ordering and transportation mode selection problem under the cap-and-trade policy. Chen et al. (2016) examined the impact of the cap-and-trade policy on firm’s behavior change in making warehouse management decisions. The above literatures only discuss the cap-and-trade policy from the view of one single firm and do not take its supply chain into consideration.

Another stream of research related to this paper centres on the effect of the cap-and-trade policy on supply chain decisions and carbon emission reduction performance. Benjaafar et al. (2013) presented a series of models to analyze the effect of different carbon emission policies, including cap-and-trade, on supply chain management decisions. Their numerical studies showed that adjustments to the ordering policy can significantly reduce emissions without considerably increasing cost. Their research also indicated that the presence of carbon emission policy can significantly increase the value of supply chain collaboration. Jaber et al. (2013) investigated the European Union Emissions Trading System from the perspective of the user, and presented a two-level supply chain model accounting for GHG emissions from manufacturing processes. They argued that when emissions and penalty costs are considered, a combination policy of carbon tax and emissions penalty was the most effective one, and supply chain coordination can minimize the overall cost. Jin et al. (2014) developed optimization models for major retailers to design their supply chains under various carbon policies including carbon tax, inflexible cap, and cap-and-trade. Their research found that the supply chain network design is highly sensitive to the carbon price for the cap-and-trade policy. Similarly, Palak et al. (2014) examined different carbon policies on supplier and mode selection decisions in the context of the biofuel supply chain. Their findings showed that cap-and-trade is a more efficient mechanism compared to other policies such as carbon offset. Du et al. (2013) studied emission-dependent supply chain consisting of one emission-dependent manufacturer and one emission permit supplier in the cap-and-trade system. Based on game-theoretically analysis, they derived the optimal supply chain decisions
in a cap-and-trade system and showed that it was possible to coordinate the supply chain with a certain condition. In the above literature, there is only one product considered in their models.

Many studies have looked product mix as an effective strategy for firms to improve the environmental performance while maintaining economic competitiveness. Among them, Letmathe and Balakrishnan (2005) proposed a liner program model and a mixed integer liner program model that can be employed by firms to estimate their optimal decision for product mix and production quantities considering typical production constraints as well as various environmental constraints. Incorporating capacity expansion features, Tsai et al. (2012) presented green product mix decision model to evaluate the benefits of various options of capacity expansion. Tsai et al. (2013) developed a mathematical programming model to evaluate the financial performance of green manufacturing technology investments and product mix decision through a combination of the theory of constraints and activity based costing. Galal and Moneim (2015) presented a mixed integer non-linear programming model for a manufacturing facility to maximize a sustainability index, which is comprised of the economic, environmental and social dimensions of sustainability. However, above studies explored the role of product mix strategy in addressing the environmental challenge from single firm’s point of view. Chen et al. (2015) studied a production model consisted of one manufacturer which produces a standard product and the green product that can be used as a substitute of the standard product. They investigated the effect of substitution and government carbon emissions reduction policies on the sustainable optimal production policy and the expected profit of the manufacturer. In this paper, the manufacturer’s decision variable is production policy only and the pricing policy is not taken into consideration. Different to them, our paper investigate the product mix strategy from the supply chain point of view taking in consideration of the cap-and-trade policy.

3 Model formulation and assumptions

We consider a two-echelon supply chain that consists of a supplier and a manufacturer. Raw material is purchased from the supplier. The manufacturer produces two kinds of products and
sells to the end-users. Product 1 is green product and product 2 is standard product. Before the production period, the manufacturer receives an initial allocation of emission allowances from the government. They can also buy additional allowances from or sell them to outside market. At the end of a compliance period, the manufacturer should not discharge more emissions than the allowance they hold. We assume that the supplier is the Stackelberg leader and the manufacturer is the follower. The sequence of events is as follows. Firstly, the supplier decides her wholesale price for the material. Then, the manufacturer decides the product quantities and the retail prices of product 1 and product 2 and carbon emissions trading quantity according to the customers’ demands and supplier’s wholesale price.

We denote our parameters and variables for model development as the following notations in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Descriptions</th>
</tr>
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<tbody>
<tr>
<td>$q_1$ and $q_2$</td>
<td>The manufacturer’s production quantities or customer demands for product 1 and 2 respectively.</td>
</tr>
<tr>
<td>$c_1$ and $c_2$</td>
<td>Unit production cost of product 1 and 2 respectively.</td>
</tr>
<tr>
<td>$p_1$ and $p_2$</td>
<td>Unit retail selling price of product 1 and 2 respectively.</td>
</tr>
<tr>
<td>$K$</td>
<td>Initial carbon emission allowances from government.</td>
</tr>
<tr>
<td>$k_1$ and $k_2$</td>
<td>Unit carbon emission of product 1 and 2 respectively.</td>
</tr>
<tr>
<td>$c$</td>
<td>Unit price of carbon emissions trading with outside market.</td>
</tr>
<tr>
<td>$E$</td>
<td>Manufacturer’s carbon emissions trading quantities with outside market.</td>
</tr>
<tr>
<td>$w$</td>
<td>Wholesale price for unit material.</td>
</tr>
<tr>
<td>$c_0$</td>
<td>Production cost for unit material.</td>
</tr>
</tbody>
</table>

In addition, to make the model more practical, the parameters must satisfy certain conditions for the model to make sense, so we assume:

(1) $p_i = \alpha - \beta q_i - \gamma q_j$, $i, j = 1, 2$ and $i \neq j$, where $\alpha$ denotes the maximum unit retail selling price of product $i$, $\beta$ is the self-demand sensitivity, and $\gamma$ is the cross-demand sensitivity. This is a linear inverse demand function, which is commonly used in the marketing and operations research literature (Padmanabhan and Png 1997; Shin and Tunca,
2010; Shang et al. 2016). Through the inverse demand function, the objective functions for both products are concave and hence the optimal decisions can be found with basic algebra. If this demand function is converted into a conventional linear price-demand function, it will turn into a demand function which is sensitive to both the standard product price and the green product price with the different coefficient beta and gamma.

(2) \( p_i > w + c_i > 0 \) for \( i = 1,2 \) and \( w > c_0 \). The first condition states that there is a positive profit margin for each product if it is sold to consumer market, and the second condition means that there is a positive profit margin for unit material if it is sold to the manufacturer.

(3) \( p_i - w - c_i - c > 0 \). This assumption ensures that the manufacturer has incentive to produce products. Otherwise, the manufacturer will sell directly her initial carbon emission allowances from government to outside market.

(4) \( c_1 > c_2 \) and \( k_1 < k_2 \). Because product 1 is produced by green techniques and product 2 is produced by standard techniques, so the unit production cost of product 1 is higher than that of product 2, and the unit carbon emission of product 1 is lower than that of product 2.

(4) \( \beta > \gamma > 0 \). This assumption is reasonable because retail price is relatively more sensitive to demand for its product than that of for competing product.

4 The integrated channel model with cap-and-trade

In this section, we discuss the integrated channel model with cap-and-trade. This is a basic benchmark. In the integrated channel, the decision problem faced by central controller is to decide the production quantity and retail price of product 1 and product 2 so as to maximize the profit, denoted by \( \pi^I(q_1, q_2, p_1, p_2) \). With the consideration that \( \pi^I(q_1, q_2, p_1, p_2) \) equals to \( \pi^I(q_1, q_2) \), the central controller’s profit with cap-and-trade in an integrated channel is

\[
\pi^I(q_1, q_2) = (p_1 - c_0 - c_1)q_1 + (p_2 - c_0 - c_2)q_2 - cE
\]

The first term is the central controller’s sale revenue from product 1 and the second term is the central controller’s sale revenue from product 2. Then

\[
\pi^I(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2 - c_0 - c_1)q_1 + (\alpha - \beta q_2 - \gamma q_1 - c_0 - c_2)q_2 - cE
\]
\[ s.t \quad k_1q_1 + k_2q_2 - E = K \quad (1) \]

The equal equation constraint condition means that the central controller’s total mixed products carbon emission minus the carbon emissions trading quantities with outside market is equal to the government’s initial carbon emission allowances. \( E > 0 \) means that the central controller will buy carbon emission allowances from the outside market, \( E = 0 \) means that the central controller will not trade with outside market, and \( E < 0 \) means that the central controller will sell carbon emission allowances to the outside market.

From (1), we get \( E = k_1q_1 + k_2q_2 - K \). Replace \( E \) to \( \pi^I(q_1, q_2) \), we get

\[ \pi^I(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2 - c_0 - c_1)q_1 + (\alpha - \beta q_2 - \gamma q_1 - c_0 - c_2)q_2 - c(k_1q_1 + k_2q_2 - K) \quad (2) \]

We investigate the central controller’s optimal production quantities (denoted by \( q_1^I \) and \( q_2^I \)) and retail prices (denoted by \( p_1^I \) and \( p_2^I \)) of product 1 and 2, and the optimal carbon emissions trading quantities with outside market (denoted by \( E^I \)) with cap-and-trade in an integrated channel, and obtain the following proposition:

**Lemma 1**

\[ q_1^I = \frac{\alpha - c_0}{2(\beta + \gamma)} - \frac{c_1 + c_2}{4(\beta + \gamma)} - \frac{c_1 - c_2}{4(\beta - \gamma)} - \frac{(k_1 + k_2)c}{4(\beta + \gamma)} + \frac{(k_2 - k_1)c}{4(\beta - \gamma)} , \quad q_2^I = \frac{\alpha - c_0}{2(\beta + \gamma)} - \frac{c_1 + c_2}{4(\beta + \gamma)} - \frac{c_1 - c_2}{4(\beta - \gamma)} - \frac{(k_1 + k_2)c}{4(\beta + \gamma)} + \frac{(k_2 - k_1)c}{4(\beta - \gamma)} \]

\[ p_1^I = \frac{\alpha + c_0}{2} + \frac{c_1}{2} - \frac{k_1c}{2} , \quad p_2^I = \frac{\alpha + c_0}{2} + \frac{c_2}{2} - \frac{k_2c}{2} \] and \( E^I = k_1q_1^I + k_2q_2^I - K \).

This proposition indicates that in the case of an integrated channel, there exist unique central controller’s optimal production quantities and retail prices of product 1 and 2, and optimal carbon emissions trading quantities with outside market. Since the cap-and-trade policy just affects the two products’ integrated cost, that is the cost of production cost and the cost of consuming carbon emissions allowances that can be sold for revenue or bought at cost, so the initial carbon emission allowances from government have no impact on the central controller’s optimal production quantities and retail prices of product 1 and 2. But they have an impact on the central controller’s optimal carbon emissions trading quantities with outside market and profit, that is, the central controller’s optimal carbon emissions trading quantities with outside market and profit both are an increasing function of initial carbon emission allowances from government in the case of an integrated channel.

Because \( c_1 > c_2 \) and \( k_1 < k_2 \), so we get the following corollary from proposition 1:
Corollary 1 (1) If \( c \geq \frac{c_1 - c_2}{k_2 - k_1} \), then \( q_1^l \geq q_2^l \); if \( c < \frac{c_1 - c_2}{k_2 - k_1} \), then \( q_1^l < q_2^l \). (2) \( p_1^l > p_2^l \).

The first result of this corollary means that in the case of an integrated channel, if the unit price of carbon emissions trading with outside market is higher \( (c \geq \frac{c_1 - c_2}{k_2 - k_1}) \), then the central controller will produce more green products. This is reasonable from the practice point of view. More green products mean less carbon emissions. Then the central controller will spend less to buy carbon emission allowances from the outside market when the initial carbon emission allowances from government is used up, or gain more revenue from selling the carbon emission allowances to the outside market when there is a surplus of the initial carbon emission allowances from government.

The second result of this corollary means that in the case of an integrated channel, the central controller will set higher retail price for green product (product 1) than that for standard product (product 2). This intuition is clear. Because the unit production cost of green product (product 1) is higher than that of standard product (product 2).

5 The decentralized channel model with cap-and-trade

Now, we examine the decentralized channel model with cap-and-trade. The manufacturer’s problem is investigated first.

5.1 The manufacturer’s problem

The decision problem faced by the manufacturer is to decide the production quantities and retail prices of product 1 and product 2, and to decide the carbon emissions trading quantity with outside market so as to maximize the profit, denoted by \( \pi_m(q_1, q_2, p_1, p_2) \). With the consideration that \( \pi_m(q_1, q_2, p_1, p_2) \) equals to \( \pi_m(q_1, q_2) \), so the manufacturer’s profit with cap-and-trade in a decentralized channel is

\[
\pi_m(q_1, q_2) = (p_1 - w - c_1)q_1 + (p_2 - w - c_2)q_2 - cE
\]

\[
s.t \quad k_1 q_1 + k_2 q_2 - E = K
\]

The equal equation constraint condition means that the manufacturer’s total mixed products carbon emission minus the carbon emissions trading quantities with outside market is equal to the government’s initial carbon emission allowances. \( E > 0 \) means that the
manufacturer will buy carbon emission allowances from the outside market, \( E = 0 \) means that the manufacturer will not trade with outside market, and \( E < 0 \) means that the manufacturer will sell carbon emission allowances to the outside market.

From (3), we get \( E = k_1 q_1 + k_2 q_2 - K \). Replace \( E \) to \( \pi_m(q_1, q_2) \), we get

\[
\pi_m(q_1, q_2) = (p_1 - w - c_1)q_1 + (p_2 - w - c_2)q_2 - c(k_1 q_1 + k_2 q_2 - K) \quad (4)
\]

Regarding the manufacturer’s optimal production quantities (denoted by \( q_1^* \) and \( q_2^* \)) and retail prices (denoted by \( p_1^* \) and \( p_2^* \)) of product 1 and 2, and the optimal carbon emissions trading quantities with outside market (denoted by \( E^* \)) with cap-and-trade in a decentralized channel, we have the following proposition:

**Lemma 2** \( q_1^* = \frac{a-w}{2(\beta+\gamma)} - \frac{c_1 + c_2}{4(\beta+\gamma)} - \frac{c_1 - c_2}{4(\beta-\gamma)} - \frac{(k_1+k_2)c}{4(\beta+\gamma)} + \frac{(k_2-k_1)c}{4(\beta-\gamma)} \), \( q_2^* = \frac{a-w}{2(\beta+\gamma)} - \frac{c_1 + c_2}{4(\beta+\gamma)} + \frac{c_1 - c_2}{4(\beta-\gamma)} - \frac{(k_1+k_2)c}{4(\beta+\gamma)} + \frac{(k_2-k_1)c}{4(\beta-\gamma)} \), \( p_1^* = \frac{a+w}{2} + \frac{c_1}{2} - \frac{k_1c}{2}, \ p_2^* = \frac{a+w}{2} + \frac{c_2}{2} - \frac{k_2c}{2} \) and \( E^* = k_1 q_1^* + k_2 q_2^* - K \).

The proof of this proposition is similar to that of proposition 1. This proposition indicates that in the case of a decentralized channel, there exist unique manufacturer’s optimal production quantities and retail prices of product 1 and 2, and optimal carbon emissions trading quantities with outside market. Since the cap-and-trade policy just affects the two products’ integrated cost, that is the cost of production cost and the cost of consuming carbon emissions allowances that can be sold for revenue or bought at cost, so the initial carbon emission allowances from government have no impact on the manufacturer’s optimal production quantities and retail prices of product 1 and 2. But they have an impact on the manufacturer’s optimal carbon emissions trading quantities with outside market and profit, that is, the manufacturer’s optimal carbon emissions trading quantities with outside market and profit both are an increasing function of initial carbon emission allowances from government in the case of a decentralized channel.

Because \( c_1 > c_2 \) and \( k_1 < k_2 \), so we get the following corollary from proposition 2:

**Corollary 2 (1)** If \( c \geq \frac{c_1-c_2}{k_2-k_1} \) then \( q_1^* \geq q_2^* \); if \( c < \frac{c_1-c_2}{k_2-k_1} \) then \( q_1^* < q_2^* \). (2) \( p_1^* > p_2^* \).

The proof of this corollary is similar to that of corollary 1. The first result of this corollary means that in the case of a decentralized channel, if the unit price of carbon
emissions trading with outside market is higher \( (c \geq \frac{c_1-c_2}{k_2-k_1}) \), then the manufacturer will produce more green products. This is reasonable from the practice point of view. More green products mean less carbon emissions. Then the manufacturer will spend less to buy carbon emission allowances from the outside market when the initial carbon emission allowances from government is used up, or gain more revenue from selling the carbon emission allowances to the outside market when there is a surplus of the carbon emission allowances from government.

The second result of this corollary means that in the case of a decentralized channel, the manufacturer will set higher retail price for green product (product 1) than that for standard product (product 2). This intuition is clear. Because the unit production cost of green product (product 1) is higher than that of standard product (product 2).

5.2 The supplier’s problem

The decision problem faced by the supplier in the case of decentralized channel and cap-and-trade is to decide the wholesale price of material so as to maximize the profit. The supplier’s profit with cap-and-trade in a decentralized channel, denoted by \( \pi_s(w) \), is

\[
\pi_s(w) = (w - c_0)(q_1^* + q_2^*)
\] (5)

We explore the supplier’s optimal wholesale price with decentralized channel and cap-and-trade (denoted by \( w^* \)), and obtain the following proposition:

**Lemma 3** \( w^* = \frac{\alpha + c_0}{2} - \frac{c_1 + c_2}{4} - \frac{(k_1 + k_2)c}{4} \).

This proposition indicates that there exists a unique supplier’s optimal wholesale price in the case of cap-and-trade in a decentralized channel.

From lemma 2 and 3, we get the manufacturer’s optimal produce quantities and retail prices in the case of cap-and-trade in a decentralized channel as following:

\[
q_1^* = \frac{\alpha - c_0}{4(\beta + \gamma)} - \frac{c_1 + c_2}{8(\beta + \gamma)} - \frac{c_1 - c_2}{8(\beta - \gamma)} + \frac{(k_1 + k_2)c}{8(\beta + \gamma)} + \frac{(k_2 - k_1)c}{8(\beta - \gamma)}
\] (6)

\[
q_2^* = \frac{\alpha - c_0}{4(\beta + \gamma)} - \frac{c_1 + c_2}{8(\beta + \gamma)} + \frac{c_1 - c_2}{8(\beta - \gamma)} - \frac{(k_1 + k_2)c}{8(\beta + \gamma)} - \frac{(k_2 - k_1)c}{8(\beta - \gamma)}
\] (7)

\[
p_1^* = \frac{3\alpha + c_0}{8} + \frac{3c_1 - c_2}{8} - \frac{(5k_1 + k_2)c}{8}
\] (8)

\[
p_2^* = \frac{3\alpha + c_0}{8} + \frac{3c_2 - c_1}{8} - \frac{(k_1 + 5k_2)c}{8}
\] (9)
6 Discussion

In this section, we discuss the effects of channel structure and cap-and-trade on the supply chain’s decisions, carbon emissions and profits.

6.1 The effect of channel structure

Take the effect of cap-and-trade on the supply chain decisions into consideration, we get the following proposition:

**Proposition 1** \( q_1^* < q_1^I, \ q_2^* < q_2^I, \ p_1^* > p_1^I \) and \( p_2^* > p_2^I \).

This proposition means that under the cap-and-trade policy, the production quantities of both green product and standard product in the decentralized channel are lower than that in the integrated channel, and the retail prices of both green product and standard product in the decentralized channel are higher than that in the integrated channel. That is, double marginalization in the case of cap-and-trade is the same as that in the traditional case without cap-and-trade. The cap-and-trade policy just affects the two products’ *integrated cost*, that includes production cost and the cost of consuming carbon emissions allowances that can be sold for revenue or bought at cost, and has no effect on the double marginalization.

With cap-and-trade, we define \( \theta^I = \frac{q_1^I}{q_1^I + q_2^I} \), which is the proportion of green product in the integrated channel, and define \( \theta^* = \frac{q_1^*}{q_1^* + q_2^*} \), which is the proportion of green product in the decentralized channel. Then we gain the following proposition:

**Proposition 2** If \( c \geq \frac{c_1 - c_2}{k_2 - k_1} \), then \( \theta^* \geq \theta^I \); if \( c < \frac{c_1 - c_2}{k_2 - k_1} \), then \( \theta^* < \theta^I \).

This proposition means that with cap-and-trade, the relationship of the proportion of green product in the decentralized channel and that in the integrated channel is decided by the unit price of carbon emissions trading with outside market. If the unit price of carbon emissions trading with outside market is higher, then the proportion of green product in the decentralized channel is higher than that in the integrated channel. That is, higher unit price of carbon emissions trading with outside market will enable the supply chains with decentralized channel to generate less carbon emissions. If the unit price of carbon emissions trading with outside market is lower, then the proportion of green product in the decentralized channel is lower than that in the integrated channel. That is, lower unit price of carbon emissions trading...
with outside market will enable the supply chains with integrated channel to generate less carbon emissions as compared to the decentralized channel. These observations are interesting and can contribute to the policy maker’s decision makings. The policy maker can encourage the manufacturer to produce more green products and less standard product by adjusting the unit price of carbon emissions trade with outside world according to the industry’s supply chain structure, and as a result enable to achieve low carbon supply chains.

In the case of cap-and-trade, the supply chain’s carbon emissions with integrated channel, denoted by $K^I$, is $K^I = k_1q^I_1 + k_2q^I_2$, and the supply chain’s carbon emissions with decentralized channel, denoted by $K_1$, is $K_1 = k_1q^*_1 + k_2q^*_2$. As to the effect of channel structure on supply chain’s carbon emissions, the following proposition is obtained:

**Proposition 3** $K_1 < K^I$.

From this proposition, we know that with cap-and-trade, the supply chain’s carbon emissions in the decentralized channel are lower than that in the integrated channel. Recalling proposition 1, this intuition is clear. In the decentralized channel, compared to the integrated channel, the retail prices of products are higher, which leads to less production quantities. Therefore, the supply chain’s carbon emissions in the decentralized channel are lower. From the environment point of view, the decentralized supply chain is better than that in the integrated one.

Regarding the effect of the cap-and-trade policy on the supply chain’s profit, the following proposition is obtained:

**Proposition 4** $\pi^I(q^I_1, q^I_2) > \pi_m(q^*_1, q^*_2) + \pi_s(w^*)$.

This proposition indicates that with cap-and-trade, the supply chain’s profit in the integrated channel is always higher than that in the decentralized channel. The result is the same as the case of without cap-and-trade.

### 6.2 The effect of cap-and-trade

Now, we discuss the effect of cap-and-trade on the supply chain’s decisions, carbon emissions and profit. Firstly, we discuss the decentralized channel model without cap-and-trade. This is a useful benchmark. The decision problem faced by the manufacturer is to decide the production quantity and retail price of product 1 and product 2 to maximize the profit,
denoted by $\pi^0_m(q_1, q_2, p_1, p_2)$. Considering that $\pi^0_m(q_1, q_2, p_1, p_2)$ equals to $\pi^0_m(q_1, q_2)$, then the manufacturer’s profit without cap-and-trade is

$$\pi^0_m(q_1, q_2) = (p_1 - w - c_1)q_1 + (p_2 - w - c_2)q_2$$

The first term is the manufacturer’s sale revenue from product 1 and the second term is the manufacturer’s sale revenue from product 2. Then

$$\pi^0_m(q_1, q_2) = (\alpha - \beta q_1 - \gamma q_2 - w - c_1)q_1 + (\alpha - \beta q_2 - \gamma q_1 - w - c_2)q_2 \quad (10)$$

Similarly, the decision problem faced by the supplier in the case of without cap-and-trade is to decide the wholesale price of material to maximize the profit. The supplier’s profit with decentralized channel without cap-and-trade, denoted by $\pi^s_0(w)$, is

$$\pi^s_0(w) = (w - c)(q_1^0 + q_2^0) \quad (11)$$

We investigate the manufacturer’s optimal production quantities (denoted by $q_1^0$ and $q_2^0$) and retail prices (denoted by $p_1^0$ and $p_2^0$) of product 1 and 2, and the supplier’s optimal wholesale price (denoted by $w^0$) without cap-and-trade, and obtain the following lemma:

**Lemma 4** $q_1^0 = \frac{\alpha - c_0}{4(\beta + \gamma)} - \frac{c_1 + c_2}{8(\beta + \gamma)} - \frac{c_1 - c_2}{4(\beta + \gamma)}$, $q_2^0 = \frac{\alpha - c_0}{4(\beta + \gamma)} + \frac{c_1 + c_2}{8(\beta + \gamma)} + \frac{c_1 - c_2}{4(\beta + \gamma)}$, $p_1^0 = \frac{3\alpha + c_0}{4} + \frac{3c_1 - c_2}{8}$, $p_2^0 = \frac{3\alpha + c_0}{4} + \frac{3c_2 - c_1}{8}$ and $w^0 = \frac{\alpha + c_0}{2} - \frac{c_1 + c_2}{4}$.

The lemma means that in the decentralized channel, there exist unique manufacturer’s optimal production quantities and retail prices of product 1 and 2, and supplier’s optimal wholesale price in the case of without cap-and-trade.

Take into consideration the effect of cap-and-trade on the supply chain decisions, that is, on the supplier’s optimal wholesale price, the manufacturer’s optimal retail prices and production quantities of fixed products, we gain the following proposition:

**Proposition 5** (1) $w^* < w^0$. (2) If $k_2 \geq k_1 + \frac{2(\beta - \gamma)}{\beta + 3\gamma}$, then $q_1^* \geq q_1^0$; if $k_1 < k_2 < k_1 + \frac{2(\beta - \gamma)}{\beta + 3\gamma}$, then $q_1^* < q_1^0$. (3) $q_2^* < q_2^0$, $p_1^* < p_1^0$ and $p_2^* < p_2^0$.

This proposition means that the supplier will set lower wholesale price, and the manufacturer will set lower retail prices and produce less standard products in the case of cap-and-trade than without. The relationship of production quantity of green product is decided by their unit carbon emissions. If the unit standard product carbon emissions are higher, then the manufacturer will produce more green products in the case of cap-and-trade.
than without, and vice versa. The intuition is clear. Under the cap-and-trade policy, it gives more incentive for the firms, who have high unit standard product carbon emissions, to produce more low carbon product in order to be economic viable.

In the case of without cap-and-trade, the supply chain’s carbon emissions (denoted by $K_0$) is $K_0 = k_1 q_1^0 + k_2 q_2^0$. Regarding the effect of the cap-and-trade policy on the supply chain’s profit and carbon emissions, the following proposition is obtained:

**Proposition 6** (1) $\pi_s(w^*) < \pi_s(w^0)$. (2) If $K > K^*$, then $\pi_m(q_1^*, q_2^*) > \pi_m^0(q_1^0, q_2^0)$; if $K = K^*$, then $\pi_m(q_1^*, q_2^*) = \pi_m^0(q_1^0, q_2^0)$; if $K < K^*$, then $\pi_m(q_1^*, q_2^*) < \pi_m^0(q_1^0, q_2^0)$, where $K^* = K_1 + \frac{1}{c} [\pi_m^0(q_1^0, q_2^0) - \pi_m(q_1^0, q_2^0)]$.

From proposition 6, we know that the supplier’s profit in the case of cap-and-trade is always lower than without. The intuition is clear. Under the cap-and-trade policy, the manufacturer will produce less green product (product 1) and standard product (product 2), then the demand faced by the supplier is reduced. At the same time, the supplier will set lower wholesale price in the case of cap-and-trade than without. The reduction of both demand and wholesale price leads to lower profit for the supplier in the case of cap-and-trade than without.

The relationship of manufacturer’s profit between the cases of with and without cap-and-trade is decided by the initial carbon emission allowances from government. That is, if the initial carbon emission allowance from government is high, then the manufacturer’s profit in the case of cap-and-trade is higher than without; if the initial carbon emission allowance from government is medium, then the manufacturer’s profit in the case of cap-and-trade is equal to without; if the initial carbon emission allowance from government is low, then the manufacturer’s profit in the case of cap-and-trade is lower than without. Since the threshold ($K^*$) is higher than the supply chain’s actual carbon emissions ($K_1$), then the policy maker would set the initial carbon emission allowance from government lower than the supply chain’s actual carbon emissions ($K_1$) to let the cap-and-trade policy take effect. So, the manufacturer’s profit in the case of cap-and-trade is also lower than without.

The supply chain’s carbon emissions in the case of cap-and-trade are lower than the case without the policy. In a word, the cap-and-trade policy can reduce the supply chain’s carbon emissions, but also have a significant impact of the economic performance of both the
supplier and the manufacturer. It is not a surprise that the cap-and-trade policy has a positive impact on supply chain carbon emission reduction as it is designed to tackle the carbon emissions challenge. Nevertheless, the detail in the design of the cap-and-trade policy such as the unit price of carbon emissions trade with outside world and the initial carbon emission allowance have significant implications to the supply chain firms. From governments’ point of view, it is important to achieve the carbon emissions reduction targets. However, such an achievement should not comprise the development of their economies. Therefore, in order to achieve the long-term sustainability, it is critical for policy makers around the world to balance the tradeoff between the economic and environmental objectives, and set up appropriate cap-and-trade policies according to different economic development stages and environmental statuses of their regions or countries.

7 Conclusions

In this paper, we investigate the incentive for a carbon efficient supply chain’s wholesale pricing, retail pricing and production policies under the cap-and-trade policy. A two-echelon supply chain is considered that consists of a supplier and a manufacturer. Based on game theory models, this research analytically explores potential behavioural changes in the context of a two-echelon supply chain when the cap-and-trade policy is applied to the manufacturing industry.

The research makes two main contributions. First, theoretically, it is important to reduce carbon emissions of the whole supply chain in order to achieve the low carbon objective. This research complements the existing literature by studying one widely adopted carbon emissions reduction policy, cap-and-trade, from the supply chain perspective. More specifically, we examine the effectiveness of product mix as a strategy to deliver a low carbon supply chain under the cap-and-trade policy. We analyse the impact of the integrated and decentralized channel structures on the supply chain economic and environmental performances under the cap-and-trade policy. Second, our research findings generate many interesting insights which do not only help firms to make important operational and strategic decisions to reduce carbon emissions while maintaining their economic competitiveness, but
also make meaningful contribution to governments’ policy making for carbon emissions control. For instance, our analysis results show that the channel structures have significant impacts on both economic and environmental performances. An integrated supply chain generates more profits. In contrast, a decentralized supply chain has lower carbon emissions. Furthermore, we found that the cap-and-trade policy harm the economic performance of both the supplier and the manufacturer while improving the supply chain’s environmental performance. With understanding of the policy impacts on supply chain decisions and performances, our findings will support policy makers to develop effective carbon emissions control policies that enable the sustainable development of their economy and environment.

Indeed, we see this paper as an early attempt to understand the relationship between the cap-and-trade policy and supply chain decisions on mixed products. Similar to the models previously published in the literature, the present model also has its own limitations, which imply fruitful directions for future research. For example, our model assumed the two-echelon supply chain consisting of a supplier and a manufacturer with a deterministic demand. It would be interesting to consider multi suppliers and/or multi manufacturers with stochastic demand and analyze the effect of the cap-and-trade policy on the supply chain’s decisions, lateral competition, profits and carbon emissions reduction. Furthermore, consumers are becoming more sensitive towards low carbon products due to an increasing environmental awareness. One important research extension is to consider the carbon emission sensitive demand in the modelling (Chen et al. 2017). In addition, the emission allowance purchasing and selling prices are assumed to be the same in our model. In reality, these carbon emission trading prices are different due to the transaction costs (Gong and Zhou 2013). Therefore, one important research extension is to consider the different purchasing and selling costs for the carbon emissions trading in modelling the effect of the cap-and-trade policy. Another extension of our work is to analyze the supply chain management with mixed products under other carbon emissions policies, such as mandatory carbon emissions capacity, carbon tax, and investment in the carbon offsets, and discuss the effect of these different policies on the supply chain’s decisions, profits and carbon emissions reduction.
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Appendix A

Proof of Lemma 1

(2) shows that 
\[ \frac{\partial \pi^l(q_1, q_2)}{\partial q_1} = \alpha - \beta q_1 - \gamma q_2 - c_0 - c_1 - \beta q_1 - \gamma q_2 - c k_1 = \alpha - 2\beta q_1 - 2\gamma q_2 - c_0 - c_1 - c k_1 \]
\[ \frac{\partial \pi^l(q_1, q_2)}{\partial q_2} = \alpha - \beta q_2 - \gamma q_1 - c_0 - c_2 - \beta q_2 - \gamma q_1 - c k_2 = \alpha - 2\beta q_2 - 2\gamma q_1 - c_0 - c_2 - c k_2 \]
then 
\[ \frac{\partial^2 \pi^l(q_1, q_2) \partial q_1^2}{\partial q_1^2} = -2\beta < 0 \]
so we get 
\[ \alpha - 2\beta q_1 - 2\gamma q_2 - c_0 - c_1 - c k_1 = 0 \]
and 
\[ \alpha - 2\beta q_2 - 2\gamma q_1 - c_0 - c_2 - c k_2 = 0 \] Then we get 
\[ q_1^l = \frac{\alpha - c_0}{2(\beta + \gamma)} - \frac{c_1 + c_2}{4(\beta + \gamma)} - \frac{c_1 - c_2}{4(\beta - \gamma)} - \frac{(k_2 - k_1)c}{4(\beta - \gamma)} \]
and 
\[ q_2^l = \frac{\alpha - c_0}{2(\beta + \gamma)} + \frac{c_1 + c_2}{4(\beta + \gamma)} + \frac{c_1 - c_2}{4(\beta - \gamma)} - \frac{(k_1 + k_2)c}{4(\beta - \gamma)} \]

Recalling 
\[ p_1^l = \alpha - \beta q_1 - \gamma q_2 \]
and 
\[ p_2^l = \alpha - \beta q_1 - \gamma q_2 \]
that is, 
\[ p_1^l = \frac{\alpha + c_0}{2} + \frac{c_1}{2} \]
and 
\[ p_2^l = \frac{\alpha + c_0}{2} + \frac{c_2}{2} - \frac{c_2}{2} \]
Then we get 
\[ E^l = k_1 q_1^l + k_2 q_2^l - K \]

Proof of Corollary 1

From lemma 1, we get 
\[ q_1^* - q_2^* = \frac{1}{2(\beta - \gamma)} [(k_2 - k_1)c - (c_1 - c_2)] \]. Recalling 
\[ c_1 > c_2 \] and 
\[ k_1 < k_2 \] we get that if 
\[ c \geq \frac{c_1 - c_2}{k_2 - k_1} \], then 
\[ q_1^* \geq q_2^* \]; if 
\[ c < \frac{c_1 - c_2}{k_2 - k_1} \], then 
\[ q_1^* < q_2^* \].

Similarly, from lemma 1, we get 
\[ p_1^* - p_2^* = \frac{c_1}{2} - \frac{c_2}{2} + \frac{k_2 c}{2} - \frac{k_1 c}{2} \]. Recalling 
\[ c_1 > c_2 \] and 
\[ k_1 < k_2 \], we get that 
\[ p_1^* > p_2^* \].

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Proof of Lemma 3

From lemma 2, we get \( \frac{dq_1^*}{dw} = \frac{dq_2^*}{dw} = -\frac{1}{\alpha(\beta + \gamma)} \). Then, (5) shows \( \frac{d\pi_s(w)}{dw} = q_1^* + q_2^* + (w - c_0)\left(\frac{dq_1^*}{dw} + \frac{dq_2^*}{dw}\right) = q_1^* + q_2^* - \frac{w - c_0}{\beta + \gamma}, \)\( \frac{d^2\pi_s(w)}{dw^2} = -\frac{2}{\beta + \gamma} < 0, \) so \( \pi_s(w) \) is concave in \( w \). Let \( \frac{d\pi_s(w)}{dw} = 0, \) we get \( \pi^* = \frac{\alpha + c_0}{2} - \frac{c_1 + c_2}{4} - \frac{(k_1 + k_2)c}{4}. \)

Proof of Proposition 1

Because \( w > c_0 \), from lemma 1 and 2, we directly get \( q_1^* < q_1^l, q_2^* < q_2^l, p_1^* > p_1^l \) and \( p_2^* > p_2^l. \)

Proof of Proposition 2

From lemma 1, we get \( q_1^l + q_2^l = \frac{\alpha - c_0}{\beta + \gamma} - \frac{c_1 + c_2}{2(\beta + \gamma)} - \frac{(k_1 + k_2)c}{2(\beta + \gamma)} \), and \( \theta^l = \frac{q_1^l}{q_1^l + q_2^l} = \frac{1}{2} + \frac{c_1 - c_2}{4(\beta - \gamma)} + \frac{(k_2 - k_1)c}{4(\beta - \gamma)} \). From lemma 2, we get \( q_1^l + q_2^l = \frac{\alpha - w}{\beta + \gamma} - \frac{c_1 + c_2}{2(\beta + \gamma)} - \frac{(k_1 + k_2)c}{2(\beta + \gamma)} \), and \( \theta^* = \frac{q_1^*}{q_1^* + q_2^*} = \frac{1}{2} + \frac{c_1 - c_2}{4(\beta - \gamma)} + \frac{(k_2 - k_1)c}{4(\beta - \gamma)} \). So, \( \theta^* - \theta^l = \left[ -\frac{c_1 - c_2}{4(\beta + \gamma)} + \frac{(k_2 - k_1)c}{4(\beta - \gamma)} \right] \left( \frac{1}{q_1^* + q_2^*} - \frac{1}{q_1^l + q_2^l} \right) \). If \( \frac{c_1 - c_2}{k_2 - k_1} \geq 0, \) then \( \theta^* \geq \theta^l. \) If \( \frac{c_1 - c_2}{k_2 - k_1} < 0, \) then \( \theta^* < \theta^l. \)

Proof of Proposition 3

From lemma 1 and 2, we get \( K_1 - K_l^l = k_1q_1^* - k_1q_1^l + k_2q_2^* - k_2q_2^l < 0, \) that is \( K_1 < K_l^l. \)

Proof of Proposition 4

From (2), (4) and (5), we get \( \pi_m(q_1, q_2) + \pi_s(w) = \pi^l(q_1, q_2). \) Considering the maximum of \( \pi^l(q_1, q_2), \) we get \( \pi^l(q_1^l, q_2^l) > \pi^l(q_1^*, q_2^*), \) that is, \( \pi^l(q_1^l, q_2^l) > \pi_m(q_1^*, q_2^*) + \pi_s(w^*). \)

Proof of Lemma 4

(10) shows \( \frac{d\pi_m(q_1, q_2)}{dq_1} = \alpha - \beta q_1 - \gamma q_2 - w - c_1 - \beta q_1 - \gamma q_2 = \alpha - 2\beta q_1 - 2\gamma q_2 - w - \)
\[
c_1, \quad \frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_2^2} = \alpha - \beta q_2 - \gamma q_1 - w - c_2 - \beta q_2 - \gamma q_1 = \alpha - 2\beta q_2 - 2\gamma q_1 - w - c_2, \text{ then}
\]
\[
\frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_1^2} = -2\beta < 0, \quad \frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_1 \partial q_2} = -2\gamma, \quad \frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_2 \partial q_1} = -2\beta < 0, \text{ so we get}
\]
\[
\frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_1 \partial q_2} = \frac{\partial^2 \pi_m^0(q_1, q_2)}{\partial q_1^2} = 4\beta^2 - 4\gamma^2 > 0. \text{ Therefore, } \pi_m^0(q_1, q_2) \text{ is a concave function of } q_1 \text{ and } q_2.
\]

Let \( \frac{\partial \pi_m^0(q_1, q_2)}{\partial q_1} = \frac{\partial \pi_m^0(q_1, q_2)}{\partial q_2} = 0 \), we get \( \alpha - 2\beta q_1 - 2\gamma q_2 - w - c_1 = 0 \) and \( \alpha - 2\beta q_2 - 2\gamma q_1 - w - c_2 = 0 \), then we get \( q_1^0 = \frac{\alpha-w}{2(\beta+\gamma)} - \frac{c_1+c_2}{4(\beta+\gamma)} - \frac{c_1-c_2}{4(\beta-\gamma)} \) and \( q_2^0 = \frac{\alpha-w}{2(\beta+\gamma)} - \frac{c_1+c_2}{4(\beta+\gamma)} + \frac{c_1-c_2}{4(\beta-\gamma)} \). Then \( p_1^0 = \alpha - \beta q_1^0 - \gamma q_2^0 = \frac{\alpha+w}{2} + \frac{c_1}{2} \) and \( p_2^0 = \alpha - \beta q_2^0 - \gamma q_1^0 = \frac{\alpha+w}{2} + \frac{c_2}{2} \), that is,
\[
p_1^0 = \frac{\alpha+w}{2} + \frac{c_1}{2} \quad \text{and} \quad p_2^0 = \frac{\alpha+w}{2} + \frac{c_2}{2} \quad \text{so that}
\]
\[
d\pi_m^0(w) = q_1^0 + q_2^0 + (w - c_0) \frac{d q_1^0}{dw} + \frac{d q_2^0}{dw} = q_1^0 + q_2^0 - \frac{w-c_0}{\beta+\gamma}, \quad d^2\pi_m^0(w) = -2 \frac{q_1^0}{\beta+\gamma}. \text{ Then, (11) shows } \frac{\partial \pi_m^0(w)}{\partial w} = q_1^0 + q_2^0 + (w - c_0) \frac{d q_1^0}{dw} + \frac{d q_2^0}{dw} = q_1^0 + q_2^0 - \frac{w-c_0}{\beta+\gamma}, \quad d^2\pi_m^0(w) = -2 \frac{q_1^0}{\beta+\gamma} < 0, \text{ so } \pi_m^0(w) \text{ is concave in } w. \text{ Let } \frac{\partial \pi_m^0(w)}{\partial w} = 0, \text{ we get } w^0 = \frac{\alpha+c_0}{2} - \frac{c_1+c_2}{4}. \text{ Replace } w^0 \text{ to } q_1^0, q_2^0, p_1^0 \text{ and } p_2^0, \text{ we get}
\]
\[
q_1^0 = \frac{\alpha-c_0}{4(\beta+\gamma)} - \frac{c_1+c_2}{8(\beta+\gamma)} - \frac{c_1-c_2}{4(\beta-\gamma)}, \quad q_2^0 = \frac{\alpha-c_0}{4(\beta+\gamma)} - \frac{c_1+c_2}{8(\beta+\gamma)} + \frac{c_1-c_2}{4(\beta-\gamma)}, \quad p_1^0 = \frac{3\alpha+c_0}{4} + \frac{3c_1-c_2}{8}, \quad \text{and}
\]
\[
p_2^0 = \frac{3\alpha+c_0}{4} + \frac{3c_2-c_1}{8}.
\]

**Proof of Proposition 5**

(1) From lemma 3 and 4, we get directly that \( w^* < w^0 \).

(2) From lemma 4 and (6), we get \( q_1^* - q_1^0 = \frac{c}{\beta(\beta+\gamma)(\beta+3\gamma)}[(\beta + 3\gamma)(k_2 - k_1) - 2(\beta + \gamma)k_1] \).

so, if \( k_2 \geq k_1 + \frac{2(\beta-\gamma)}{\beta+3\gamma} \), then \( q_1^* \geq q_1^0 \); if \( k_1 < k_2 < k_1 + \frac{2(\beta-\gamma)}{\beta+3\gamma} \), then \( q_1^* < q_1^0 \).

(3) From lemma 4 and (7), we get \( q_2^* < q_2^0 \). From lemma 1 and (8), we get \( p_1^* < p_1^0 \). From lemma 4 and (9), we get \( p_2^* < p_2^0 \).

**Proof of Proposition 6**

(1) From lemma 4, (6) and (7), we get \( q_1^* + q_2^* = q_1^0 + q_2^0 - \frac{(k_1+k_2)c}{4(\beta+\gamma)} < q_1^0 + q_2^0 \). From proposition 5, we get \( w^* < w^0 \). Then from (11), (5), lemma 3 and 4, we get \( \pi_s(w^*) - \)
\( \pi_s(w^0) = (w^* - c_0)(q_1^* + q_2^*) - (w^0 - c_0)(q_1^0 + q_2^0) < 0 \), that is, \( \pi_s(w^*) < \pi_s(w^0) \).

(2) From lemma 4, (6) and (7), we get \( K_1 - K_0 = k_1q_1^* + k_2q_2^* - k_1q_1^0 - k_2q_2^0 = -\frac{(k_1+k_2)^2c}{8(\beta+\gamma)} = -\frac{(k_2-k_1)^2c}{4(\beta-\gamma)} < 0 \), then \( K_1 < K_0 \).

(3) If \( K \leq k_1q_1^* + k_2q_2^* \), from (4) and maximum of \( \pi_m(q_1,q_2) \), we get \( \pi_m(q_1^0,q_2^0) = \pi_m(q_1^0,q_2^0) - c(k_1q_1^0 + k_2q_2^0 - K) \leq \pi_m(q_1^0,q_2^0) < \pi_m(q_1^0,q_2^0) \), that is, \( \pi_m(q_1,q_2) < \pi_m(q_1,q_2) \). If \( K \geq k_1q_1^* + k_2q_2^* \), from (4) and maximum of \( \pi_m(q_1,q_2) \), we get \( \pi_m(q_1^0,q_2^0) = \pi_m(q_1^0,q_2^0) - c(k_1q_1^0 + k_2q_2^0 - K) < \pi_m(q_1^0,q_2^0) \), that is, \( \pi_m(q_1,q_2) > \pi_m(q_1^0,q_2^0) \). Since \( K_1 < K_0 \), then there always exists a \( K^* \) satisfies \( \pi_m(q_1^0,q_2^0) = \pi_m(q_1^0,q_2^0) \), that is, \( \pi_m(q_1^0,q_2^0) - c(k_1q_1^0 + k_2q_2^0 - K^*) = \pi_m(q_1^0,q_2^0) \), so \( K^* = K_1 + \frac{1}{c}[\pi_m(q_1^0,q_2^0) - \pi_m(q_1,q_2^0)] \). \( \pi_m(q_1,q_2^0) \) increases in \( K \), so, if \( K > K^* \), then \( \pi_m(q_1,q_2^0) > \pi_m(q_1^0,q_2^0) \); if \( K = K^* \), then \( \pi_m(q_1,q_2^0) = \pi_m(q_1^0,q_2^0) \); if \( K < K^* \), then \( \pi_m(q_1,q_2^0) < \pi_m(q_1^0,q_2^0) \).

References


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