
Peer reviewed version

Link to publication record in Explore Bristol Research
PDF-document

This is the author accepted manuscript (AAM). The final published version (version of record) will be available online via IEEE.

University of Bristol - Explore Bristol Research
General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available: http://www.bristol.ac.uk/pure/about/ebr-terms
The effect of bi-dimensional power structure on supply chain decisions and performance

Xu Chen¹*, Xiaojun Wang², Ke Gong¹

1. School of Management and Economics, University of Electronic Science and Technology of China, Chengdu, 611731, P. R. China
2. Department of Management, University of Bristol, Bristol, BS8 1TZ, U.K.

Abstract: Operational decisions made by manufacturers and retailers are often affected by the power relationships of both vertical and horizontal competitions in a supply chain. Although many studies have been carried to assess how the power structure impacts supply chains, most of them only focus on the power dynamics in the vertical competition or horizontal competition. This paper systematically examines the impacts of bi-dimensional power structure on a supply chain. Considering a triadic setting that is made of two rival retailers and a common manufacturer, we compare the equilibriums for the supply chain and individual firms with five different possible power structures. The analysis results show that the bi-dimensional power dynamics have a significant impact. Furthermore, the impacts of the bi-dimensional power structure are also dependent on the nature of customer demand. This work provides important management implications that are supportive for companies to make strategic and operational decisions.

Keywords: supply chain management, marketing-operations interface, vertical and horizontal competition, power structure, game theory

* Corresponding author: Xu Chen, E-mail: xchenxchen@263.net; Tel:+86-28-83206622
1. INTRODUCTION

Operations decisions on inventory, pricing and orders made by manufacturers and retailers are usually influenced by their relative power against parallel competitors and their customers or suppliers. In most cases, a dominant position over other counterparties enables companies to drive through their strategic agenda throughout the supply chain and, more often, positions the dominant players in a driving force in negotiating contracts [6], [10], [42]. As Fang et al. [20] revealed in their research that while addressing its own priorities, channel leader’s actions in price setting can lead to the profit gain that is significant for each channel member. Using the food retail supply chains as an example, supermarkets, often regarded as the supply chain leader [12], [42], are the powerful player. Their powerful position in the supply chain enables them to push down wholesale prices when negotiating contract with their suppliers. When consumers may enjoy the low retail prices of food products, small producers and farms feel the pains of the squeezed profit margin. This is highlighted by the recent dispute in milk prices between the major supermarket chains and small dairy producers in the UK and France. In contrast, the dynamics in these negotiations may change if the food producers increase their supply chain power. This is mirrored by the recent price battle in 2016 between TESCO, the largest supermarket in the UK in terms of market share, and Unilever, UK’s biggest grocery manufacturer. The dispute between the two players escalated and went public as both players refused to back down initially, which resulted in availability issues for many Unilever products on supermarket shelves. Although the price dispute ended eventually as both parties took compromise to reach a resolution, it clearly shows the significance of supply chain power relationship in negotiating wholesale prices as well as setting up the retail prices.

It is critical for business managers to understand properly the power relationships existing in the supply chains to manage supply chains effectively and efficiently [16]. Although the existing literature have looked at issues including market power [1], [5], [30], [32], channel power [27], [35], [36], and supply chain power [3], [9], [16], [17], [18], [42], often only one-dimensional power structure, either horizontally between rival firms or vertically between supply chain parties, is considered in these studies to examine its impact on firms’ operational decisions and their performances. Therefore, this research investigates the following questions:
- How does the power relationship in the vertical and horizontal competitions affect the supply chain decisions?
- To what extent, the bi-dimensional power structure makes impact on the performances of the supply chain and individual firms?

To address these questions, we take into consideration both vertical and horizontal competitions a triadic supply network that consists of a common manufacturer and two retailers [14]. Five different supply chain power structures are analytically modelled using non-cooperative game theory with a focus on the interaction of supply chain firms. The different power structures are characterized by the different orders of event sequence, in which, the equilibrium are derived for both the manufacturer and the two retailers [10], [24]. Through the comparison of optimal supply chain decisions (e.g. prices and quantities) and profits derived in each game model, our research systematically examines the impacts of bi-dimensional power structure on the supply chain. Our analysis leads to many interesting insights.

This paper intends to make several contributions. First, our research complements to the existing literature on supply chain power relationship by incorporating both horizontal and vertical power relationships in the examination of their impact on the supply chain. It is one of few studies [29], [44] that have considered the bi-dimensional power structure in such an investigation. Second, using a realistic setting of the models, our analysis also reveals some novel insights that are not captured by other studies as well as reinforcing some views of supply chain relationship in the existing literature. These insights give us a better understanding of how supply chain firms set up prices in relation to their relevant power positions in the supply chain. It also provides important managerial insights that are valuable for firms in making critical strategic and operational decisions in order to enhance their competitiveness.

The remaining of the article is organized as below. It is started by presenting the theoretical grounding to our study through the review of relevant literature in Section II. The model description and assumptions are then provided in Section III. The equilibriums under various supply chain power structures are presented in Section IV. After that, the effects of vertical power relationship, horizontal power relationship, and potential market size are
discussed in Section V. In Section VI, numerical examples are presented to obtain additional managerial insights. Finally, we draw our conclusions by discussing our main research findings and research contributions, and avenues for future research.

II. LITERATURE REVIEW

The influence of power relationships on firms’ strategic and operational decisions is an important research area. This section reviews several streams of literature that are particularly relevant.

Studies on the impact of market power dynamics on companies’ decisions and performances have been reported substantially in the economics, operations, marketing literature. Moorthy [31] examined two identical firms competing on product pricing and quality. The study obtains the equilibrium strategy for each firm and finds that the firm should be differentiating its product from its competitor. Banker et al. [2] investigated the effect of competitive intensity on the equilibrium levels of quality and their study found that the relationship between quality and competitive intensity depends on the increased competition and other parameters. Van Mieghem and Dada [43], in their study of the value of various forms of postponement, developed a single period model for retailers offering a homogeneous good. Their analysis result shows how horizontal competition, the timing of operational decisions, and uncertainty affect firm’s strategic investment decision and its value. Hall and Porteus [25] developed a dynamic model of firm behaviour, in which, customer service is a key competition factor, and studied firm’s capacity decisions in respond to customer service and competition pressure. So [37] analysed the individual firms’ optimization problem and then studied the equilibrium solution in a competition involving multiple firms with an assumption of price and delivery time sensitive customer demand. Tsay and Agrawal [41] studied two independent retailers who compete for consumers directly using retail price and service. Nevo [32] found in his study on the ready-to-eat cereal industry that powerful firms gain high price-cost margins by their ability in maintaining a portfolio of differentiated products and influencing the perceived product quality when competing with their rival firms. The above mentioned studies mainly focus on the horizontal competition between rival firms and investigate the influences of the power dynamics in the marketplace on the firms’ decisions and financial performances.
Nevertheless, business competition is no longer the competition between individual firms but the competition between supply chains. Therefore, the interactions between different supply chain members and their power relationships have to be considered.

One relevant stream of literature deals with the impact of supply chain power relationships on operational decisions and performances. Among them Choi [13] investigated the impact of power structure on the operational decisions of a common retailer channel. Ertek and Griffin [19] looked at its impact on price, profits and the market sensitivity towards the price through analyzing the cases where the buyer or the supplier has superior bargaining power over its counterparty. Using game theory approach, Cai et al. [7] incorporated various power structures in the analysis of the effect of various pricing schemes on the supply chain with dual-channel competition. Similarly, taking power structure into consideration, Zhang et al. [46] examined the impact of products’ substitutability and channel position on pricing decision in two dual-exclusive channels from the game theoretical perspective. They found that the supply chain performs best in a balanced power structure. Using game theoretical approach, Shi et al. [39] examined the impacts of three different power structures on supply chains with price-dependent random demand. Their study shows that power structure makes impact on the efficiency of the supply chain and the impact is dependent on both expected demand and demand shock. Grennan [23] empirically analyzed the relationship between power dynamics, more specifically, firms’ bargaining ability and competitive advantage in the context of hospitals and medical device supplier. His findings indicate that the variation in purchasing prices of the same device is mainly due to the difference in bargaining ability. In their study on managing imbalanced relationship for the supply chain sustainability, Touboulcic et al. [42] shows the impact of power relationship on how individual members manage their supply chain relationships and the consequential impact of organizational response on the sustainability implementation. Chen and Wang [10] studied the optimal strategic choice between free and bundled channels in the context of the mobile phone sector from the perspective of power relationship. Using a similar approach, Chen et al. [9] examined the effect of power structure on the decision and performance of an online-to-offline retail service supply chain. Furthermore, Chen et al. [11] explore the role of power relationship in coordinating the supply chain with a goal of improving both environmental and economics performances. However, the
above mentioned studies mainly concentrate on the vertical power relationships between manufacturers and their downstream customers or between manufacturers and their upstream suppliers. In fact, while the vertical power relationships affect firms’ strategic and operational decisions, their decisions and performances are also subject to the market power relationships with their rival competitors.

Another relevant research stream is the study on supply chain performance using different power relationships between involved supply chain members. Ingene and Parry [26] investigated a two-part tariff problem through a monopoly manufacturer channel, which includes multi-retailer model with one manufacturer. Their results show that the optimal tariff from the monopoly manufacturer channel model can be more profitable than that from dyadic models. Choi [14] used a duopoly common retailer channel to model price competition. He indicates that the most common channel structure that includes multiple common retailers and manufacturers. The research findings show that while the horizontal product differentiation is beneficial to manufacturers, it is harmful to retailers. In contrast, horizontal store differentiation is beneficial to retailers but manufacturers are worse off. Considering a two-echelon supply network with one single supplier providing service to a number of rival retailers, Bernstein and Federgruen [4] examined the equilibrium behaviour of decentralized supply chains under uncertain demand. Netessine and Shumsky [34] investigated how the airline seat inventory decisions are affected by vertical and horizontal competition respectively in the airline industry, and discussed revenue-sharing contracts that coordinate these decisions. Their study considers the power structure in the horizontal competition but not in the vertical competition. More recently, in a triadic supply network with an unbalanced power relation, Geylani et al. [21] developed a theoretical model to demonstrate how a manufacturer responds strategically to the dominant retailer. Cai [8] examined the impact of channel coordination on supply chain performance through a comparison of four different supply chain structures. In his study, the revenue sharing contract is utilized to show that negotiation power between the channel members differs over different supply chain structures. Taking in consideration of different channel power structures, Pan et al. [36] examined different contract strategies in a triadic supply chain network. Guo and Iyer [22] analyzed multilateral bargaining in a supply network that consists of two rival retailers and a common manufacturer. In their study, the
effects of the timing of negotiations on the bargaining externality across the retailers and the price competition are investigated. In examination of return polity for the fashion supply chain, Li et al. [28] looked at the similarities and differences in the structural properties between the supply chains that produce the conventional newsvendor type of products and the fast fashion type of products. Although the above mentioned studies considered supply chain power relationships in either the monopoly common retailer channel or the monopoly manufacturer channel, only one dimensional power relationship (either vertical or horizontal) is considered when examining the effects of different power relationships on firms’ decisions and their performances.

As far as our understanding, Wu et al. [44] is among the few studies that investigated a similar research problem taking in consideration both the horizontal and vertical power structures. However, one power structure scenario considered in the study, where there is a Stackelberg game in horizontal competition between two retailers but the vertical competition between the supplier and each retailer is Bertrand competition, (in which, the two firms move simultaneously), is not logical. Stackelberg and Bertrand or Nash games are often adopted in the supply chain research to characterize different power structures, which is differentiated by the sequence of actions of involved firms [10], [33], [39]. Considering the power relationship in a triad, it does not make sense while there is an asymmetric power relationship between the two players, and at the same time, they also have symmetric power with the 3rd player in the triad. In addition, when modelling the decision sequence in their game models, the supplier announces the advanced and normal wholesale prices to retailers respectively. Such a setting is conducive to arbitrage in the market and may be harmful to a fair competition. In contrast, in our models, the manufacturer announces the same wholesale price to retailers, which is a more common industrial practice. Since the manufacturer supplies the same product to the two retailers and quantity discount is not applied in the model, it is fairer and more reasonable to announce the same whole price to the two retailers. More importantly, although the comparison of equilibrium profits and quantities among the game model was made, their analysis does not reveal the effect of both horizontal and vertical competitions and the power dynamics have on the manufacturer and the retailers’ optimal decisions and their performances, which is this study’s main purpose. Luo et al. [29] is the other study that considered all possible supply
chain power structures in their investigation of optimal pricing policies for differentiated brands. However, they study a monopoly common retailer channel, in which, two manufacturers supply goods to the same retailer. Furthermore, they focus on the combination of brand preference and customer valuation in their pricing model, which is quite different to our model.

III. MODEL DESCRIPTION AND ASSUMPTIONS

This research considers a supply chain environment that is made of a manufacturer and two rival retailers. The manufacturer supplies the same products to the retailers and sells to consumers through them. The model is illustrated as Figure 1 and the main notations used are listed in Table 1.

Fig. 1 The model

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>Unit wholesale price</td>
</tr>
<tr>
<td>( p_1 )</td>
<td>The unit retail price of Retailer 1</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>The unit retail price of Retailer 2</td>
</tr>
<tr>
<td>( c )</td>
<td>Manufacturer’s unit manufacturing cost</td>
</tr>
<tr>
<td>( m_1 )</td>
<td>The marginal profit of Retailer 1</td>
</tr>
<tr>
<td>( m_2 )</td>
<td>The marginal profit of Retailer 2</td>
</tr>
<tr>
<td>( q_1 )</td>
<td>Customer demand faced by Retailer 1</td>
</tr>
<tr>
<td>( q_2 )</td>
<td>Customer demand faced by Retailer 2</td>
</tr>
<tr>
<td>( \pi_m(w) )</td>
<td>The profit of manufacturer</td>
</tr>
<tr>
<td>( \pi_{r1}(p_1) )</td>
<td>The profit of Retailer 1</td>
</tr>
<tr>
<td>( \pi_{r2}(p_2) )</td>
<td>The profit of Retailer 2</td>
</tr>
<tr>
<td>( \pi_{mn} )</td>
<td>The maximum overall profit in the MS-N model</td>
</tr>
</tbody>
</table>
The maximum overall profit in the VN model

The maximum overall profit in the RS-N model

We assume that \( p_i > w > c \), which ensures both the manufacturer and the retailers to make profits, \( i = 1,2 \). The customer demand of retailer \( i \) is \( q_i = \alpha - \beta p_i + \gamma p_j \), \( i,j = 1,2 \) and \( i \neq j \), where \( \alpha \) means the potential market size, \( \beta \) means the self-price sensitivity and \( \gamma \) means the cross-price sensitivity. \( \beta > \gamma > 0 \) represents that the self-price sensitivity is higher than the cross-price sensitivity. This linear deterministic demand function is widely adopted in the marketing and supply chain management literature as an acceptable approximation of demand [9], [38]. Although other demand functions can also yield similar results, the linear deterministic demand function is adopted because it is more analytically traceable and enables us to obtain closed-form insights.

Based on above assumptions, we get that the manufacturer’s profit, \( \pi_m(w) \), is

\[
\pi_m(w) = w(q_1 + q_2) - c(q_1 + q_2)
\]

The first part of profit function is the manufacturer’s total revenue from retailers, and the second part means the total manufacture costs. Then

\[
\pi_m(w) = (w - c)[(\alpha - \beta p_1 + \gamma p_2) + (\alpha - \beta p_2 + \gamma p_1)] \quad (3.1)
\]

Replace \( p_i = w + m_i \) to (3.1), we get

\[
\pi_m(w) = (w - c)[2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] \quad (3.2)
\]

Retailer 1’s profit, \( \pi_{r_1}(p_1) \), is

\[
\pi_{r_1}(p_1) = p_1q_1 - wq_1
\]

The first part means retailer 1’s revenue from sale, and the second part is the ordering cost of retailer 1. Then

\[
\pi_{r_1}(p_1) = (p_1 - w)(\alpha - \beta p_1 + \gamma p_2) \quad (3.3)
\]

Similarly, retailer 2’s profit, \( \pi_{r_2}(p_2) \), is

\[
\pi_{r_2}(p_2) = p_2q_2 - wq_2
\]

The first part means retailer 2’s revenue from sale, and the second part is retailer 2’s order cost. Then

\[
\pi_{r_2}(p_2) = (p_2 - w)(\alpha - \beta p_2 + \gamma p_1) \quad (3.4)
\]

According to the vertical market power relationships between the manufacturer and the retailers, the power structure can be divided into three scenarios: Manufacturer Stackelberg
(MS) model, Vertical Nash (VN) model, and Retailer Stackelberg (RS) model.

IV. MODELS

A. Manufacturer Stackelberg (MS) models
In a MS model, the manufacturer is the market leader and the two competing retailers are the market followers. According to the power relationship between the two competing retailers, the horizontal market competition has two sub-games: Stackelberg and Nash. Thus, according to the bi-dimensional power relationships, there are two game models: MS and Nash (MS-N), and MS and Stackelberg (MS-S). We will discuss the two game models separately.

MS-N model
In the MS-N model, the manufacturer and the retailers make relevant decisions in sequence, and the decisions of the two competing retailers are simultaneously made. More specifically, retailer 1 determines her retail price given the retail price of retailer 2 and the manufacturer’s wholesale price, and retailer 2 decides her retail price given retailer 1’s retail price and the manufacturer’s wholesale price. Second, the wholesale price is then decided by the manufacturer using the response functions of the two competing retailers for her maximum profit. Finally, the manufacturer and the two retailers obtain their revenue after sales are made.

MS-S model
In the MS-S model, without loss of generality, retailer 1 is assumed as the leader and retailer 2 as the follower. The decision sequence of involved firms is described as follows. First, retailer 2 decides her retail price given retailer 1’s retail price and the manufacturer’s wholesale price. Second, given the manufacturer’s wholesale price, retailer 1 decides her retail price according to the response function of retailer 2. Third, the manufacturer’s wholesale price is determined based on the response functions of the two competing retailers. Finally, when end consumers’ demand is realized, the manufacturer and the two competing retailers obtain their revenue according.

B. Vertical Nash (VN) model
In the VN model, vertically, the manufacturer and the retailers have same supply chain power, and, horizontally, the two rival retailers have the equal market power.
In the VN model, the supply chain members simultaneously make their decisions. First, retailer 1 determines her retail price given retailer 2’s price and the manufacturer’s wholesale price, and retailer 2 decides her retail price given retailer 1’s retail price and the manufacturer’s wholesale price, and the manufacturer decides the wholesale price given retailers’ retail prices. Then, all supply chain members obtain their revenue after the customer demand is realized.

**C. Retailer Stackelberg (RS) models**

In a RS model, the retailers are regarded as the leader and the manufacturer is considered as the follower. According to horizontal market power between the two competing retailers, there are two sub-games: Stackelberg and Nash. Thus, according to the bi-dimensional power relationships, there are two game models: RS and Nash (RS-N), and RS and Stackelberg (RS-S). We will discuss the two models separately.

**RS-N model**

In the RS-N model, the detail of the event sequence is explained as follows. First, the wholesale price is determined by the manufacturer given retailers’ retail prices. Then, the two retailers simultaneously decide their retail prices given their rival’s retail price and the response function of the manufacturer. Finally, the supply chain members obtain their revenue respectively after the sales are realized.

**RS-S model**

In the RS-S model, the detail of the event sequence is as follows. First, the wholesale price is decided by the manufacturer given retailers’ retail prices. Second, given retail 1’s retail price, retailer 2 decides her retail price using the response functions of the manufacturer’s wholesale price for the maximum profit. Third, retailer 1 decides her retail price using the response function of retailer 2 and the manufacturer for the maximum profit. Finally, the supply chain members gain their revenue after customer demand is realized.

Regarding the manufacturer’s optimal wholesale price ($w^i$), retailer 1’s optimal retail price ($p^1_1$) and optimal order quantity ($q^1_1$), and retailer 2’s optimal retail price ($p^2_1$) and optimal order quantity ($q^2_1$) in the MS-N model ($i = mn$), MS-S model ($i = ms$), VN model ($i = n$), RS-N model ($i = rn$) and RS-S model ($i = rs$), the following lemma is derived.

**Lemma 1** The optimal decisions of both the manufacturer and the retailers are summarized as Table 2.
Table 2. Optimal decisions

<table>
<thead>
<tr>
<th>$i$</th>
<th>$i = mn$</th>
<th>$i = ms$</th>
<th>$i = n$</th>
<th>$i = rn$</th>
<th>$i = rs$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w^i$</td>
<td>$\frac{\alpha}{2(\beta - \gamma)} + \frac{c}{Z}$</td>
<td>$\frac{\alpha}{2(\beta - \gamma)} + \frac{c}{Z}$</td>
<td>$\frac{\alpha\beta}{(3\beta - \gamma)(\beta - \gamma)}$</td>
<td>$\frac{2\alpha\beta}{(7\beta - 3\gamma)(\beta - \gamma)}$</td>
<td>$\frac{\alpha(5\beta^3 + 45\beta^2\gamma - 15\beta\gamma^2 - 9\gamma^3)}{2(\beta - \gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)}$</td>
</tr>
<tr>
<td>$p^i$</td>
<td>$\frac{\alpha(3\beta - 2\gamma)}{2(\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{2(2\beta - \gamma)}$</td>
<td>$\frac{\alpha(6\beta^2 - \beta\gamma - 3\gamma^2)}{4(\beta - \gamma)(2\beta^2 - \gamma^2)} + \frac{(2\beta^2 + \beta\gamma - \gamma^2)c}{4(2\beta^2 - \gamma^2)}$</td>
<td>$\frac{\alpha(2\beta - \gamma)}{(3\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{3\beta - \gamma}$</td>
<td>$\frac{\alpha(5\beta - 3\gamma)}{(7\beta - 3\gamma)(\beta - \gamma)} + \frac{2\beta c}{7\beta - 3\gamma}$</td>
<td>$\frac{\alpha(72\beta^3 + 13\beta^2\gamma - 36\beta\gamma^2 - 9\gamma^3)}{(\beta - \gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} + \frac{2\beta(15\beta^2 + 11\beta\gamma - 6\gamma^2)c}{(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)}$</td>
</tr>
<tr>
<td>$q^i$</td>
<td>$\frac{\beta(\alpha - (\beta - \gamma)c)}{2(\beta - \gamma)}$</td>
<td>$\frac{(2\beta + \gamma)(\alpha - (\beta - \gamma)c)}{8\beta}$</td>
<td>$\frac{\beta(\alpha - (\beta - \gamma)c)}{3\beta - \gamma}$</td>
<td>$\frac{2\beta(\alpha - (\beta - \gamma)c)}{7\beta - 3\gamma}$</td>
<td>$\frac{(5\beta + 3\gamma)(\alpha - (\beta - \gamma)c)}{17\beta + 3\gamma}$</td>
</tr>
<tr>
<td>$p^i_2$</td>
<td>$\frac{\alpha(3\beta - 2\gamma)}{2(\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{2(2\beta - \gamma)}$</td>
<td>$\frac{\alpha(12\beta^3 - 2\beta^2\gamma - 7\beta\gamma^2 + \gamma^3)}{8\beta(\beta - \gamma)(2\beta^2 - \gamma^2)} + \frac{(4\beta^3 + 2\beta\gamma - \gamma^2 - \gamma^3)c}{8\beta(2\beta^2 - \gamma^2)}$</td>
<td>$\frac{\alpha(2\beta - \gamma)}{(3\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{3\beta - \gamma}$</td>
<td>$\frac{\alpha(5\beta - 3\gamma)}{(7\beta - 3\gamma)(\beta - \gamma)} + \frac{2\beta c}{7\beta - 3\gamma}$</td>
<td>$\frac{\alpha(73\beta^2 + 12\beta\gamma - 45\gamma^2)}{(\beta - \gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} + \frac{(29\beta^3 + 23\beta^2\gamma - 3\beta\gamma^2 - 9\gamma^3)c}{(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)}$</td>
</tr>
<tr>
<td>$q^i_2$</td>
<td>$\frac{\beta(\alpha - (\beta - \gamma)c)}{2(\beta - \gamma)}$</td>
<td>$\frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\alpha - (\beta - \gamma)c)}{8(2\beta^2 - \gamma^2)}$</td>
<td>$\frac{\beta(\alpha - (\beta - \gamma)c)}{3\beta - \gamma}$</td>
<td>$\frac{2\beta(\alpha - (\beta - \gamma)c)}{7\beta - 3\gamma}$</td>
<td>$\frac{\beta(29\beta^2 + 22\beta\gamma - 3\gamma^2)(\alpha - (\beta - \gamma)c)}{(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)}$</td>
</tr>
</tbody>
</table>
V. DISCUSSIONS

In the section, the effects of vertical power structure, horizontal power structure, and potential market size ($\alpha$) on the supply chain are explored respectively.

A. The effect of vertical power structure

It is started with analyzing the effect of power relationship in the vertical competition. Regarding the effect of vertical power structure on the pricing decisions, we obtain:

**Proposition 1** $w^{mn} > w^n > w^r$, $p^{mn}_i > p^r_i > p^n_i$, and $q^r_i > q^n_i > q^{mn}_i$, $i = 1, 2$.

It means that when the two competing retailers is in the horizontal Nash power structure, both the manufacturer’s optimal wholesale price and the retailers’ optimal retail price in the Manufacturer Stackelberg power structure are the highest. The wholesale price in the Retailer Stackelberg power structure is the lowest and the vertical Nash power structure has the lowest retail prices. The result shows that it is more likely for the manufacturer to set a high wholesale price when the manufacturer is more powerful party comparing to other supply chain members. The higher wholesale price will have a knock on effect on the retail price. On the contrary, the retailers’ optimal ordering quantities in the vertical Nash power structure are largest in size, and that in the Manufacturer Stackelberg power structure is smallest in size.

This is easy to explain as the low retail prices will stimulate the customer demand and result in higher ordering quantities from the retailers. It is opposite when there are high retail prices.

Regarding the effect of vertical power structure on the maximum profits of individual firms and the entire supply chain, we derive:

**Proposition 2** $\pi_m(w^{mn}) > \pi_m(w^n) > \pi_m(w^r)$, $\pi_{ri}(p^{rn}_i) > \pi_{ri}(p^n_i) > \pi_{ri}(p^{mn}_i)$ and $\pi^n_i > \pi^r_i > \pi^{mn}_i$, $i = 1, 2$.

It indicates that when the two competing retailers are in the horizontal balanced power structure, the manufacturer’s profit in the Manufacturer Stackelberg power structure is the highest because of the highest wholesale price, and that in the Retailer Stackelberg power structure is the lowest because of the lowest wholesale price. In contrast, the retailers’ profits in the Manufacturer Stackelberg power structure are the lowest, which is contributed by a high product cost and a subdued customer demand. Those in the Retailer Stackelberg power structure are the highest, which is contributed by a low product cost and a steady customer demand. In a word, for the manufacturer or the retailers, a more powerful position over other
supply chain members enables them to gain more profit.

When the two competing retailers have balanced market power, the maximum profit of the structure with vertical competition being Nash has higher maximum supply chain profit than those in the structures with vertical competition being Stackelberg games. This is can be explained by the fact that a balanced power relationship will drive down the retailer prices and therefore stimulate more customer demand. At the same time, a fair competition will enable the gained profit to be more evenly distributed to individual supply chain members. These outcomes will further improve the competitiveness of the supply chain. Therefore, from the supply chain’s point of view, more profit can be achieved in a balanced power structure.

**B. The effect of horizontal power structure**

Then, we analyze the effect of power relationships in the horizontal competition on the supply chain. To avoid the interference of vertical power dynamics, the analysis is based on the same vertical power structure. Regarding the effect of horizontal power structure on the pricing decisions, we obtain:

**Proposition 3**

1. In Manufacturer Stackelberg models, \( w_{mn} = w_{ms} \) and \( p_{1ms} > p_{2ms} > p_{2mn} = p_{1mn} \).
2. In Retailer Stackelberg models, if \( \beta = 3\gamma \), then \( w_{rn} = w_{rs} \) and \( p_{1rs} = p_{1rn} = p_{2rn} = p_{2rs} \); if \( \beta > 3\gamma \), then \( w_{rn} < w_{rs} \) and \( p_{1rs} < p_{1rn} = p_{2rn} < p_{2rs} \); if \( \beta < 3\gamma \), then \( w_{rn} > w_{rs} \) and \( p_{1rn} = p_{2rn} < p_{2rs} < p_{1rs} \).

Proposition 3(1) means that in the Manufacturer Stackelberg models, the manufacturer’s optimal wholesale prices are equal whenever horizontal competition is Nash or Stackelberg games. But the two competing retailers’ optimal retail prices in the Nash game are both lower than those in the Stackelberg games, and the optimal retail price of retailer 1 is higher than that of retailer 2 in the horizontal Stackelberg games. It is interesting that in a power structure with vertical competition being Manufacturer Stackelberg, the power dynamics of the horizontal competition have no effect on the optimal wholesale price. However, there is more competition between the two retailers in the game with horizontal competition being Nash and such a competitive market environment leads to lower optimal retail prices. In the horizontal Stackelberg games, retailer 1 is the leader and as a result a higher retail price will be set by retailer 1 than that set by retailer 2.
Proposition 3(2) means that in the Retailer Stackelberg models, if the self-price sensitivity is significant to a certain degree as compared to the cross-price sensitivity, the optimal wholesale price and the optimal retail prices in the game with horizontal competition being Nash are equal to those in the game with horizontal competition being Stackelberg. If the ratio between self-price sensitivity and cross-price sensitivity is higher than a critical value, both the optimal wholesale price and the optimal retail price of retailer 2 in the game with horizontal competition being Nash are lower than those in the game with horizontal competition being Stackelberg. But retailer 1’s optimal retail price in the game with horizontal competition being Nash is higher than that in the game with horizontal competition being Stackelberg. In contrast, if the same sensitivity ratio is lower than a critical value, the optimal wholesale price in the game with horizontal competition being Nash is higher than that in the game with horizontal competition being Stackelberg, but the retailers’ optimal retail prices in the game with horizontal competition being Nash are lower than that in the game with horizontal competition being Stackelberg. It indicates that, in a Retailer Stackelberg power structure, the power dynamics between the two retailers affect the optimal wholesale price. There is more competition in the game with horizontal competition being Nash than the game with horizontal competition being Stackelberg. However, in a Retailer Stackelberg power structure, more competition leads to a lower optimal retail price for retailer 2, but does not always result in a lower optimal retail price for retailer 1. Therefore, it is important for the firms to take the bi-dimensional power relationship in consideration when making important operations decisions.

Regarding the effect of horizontal power structure on the maximum profit of the supply chain, we obtain:

**Proposition 4**

(1) In Manufacturer Stackelberg models, \( \pi_m(w_{mn}) > \pi_m(w_{ms}) \), \( \pi_r(p_{1mn}^m) = \pi_r(p_{2mn}^m) < \pi_r(p_{1ms}^m) < \pi_r(p_{2ms}^m) \) and \( \pi_s^{mn} > \pi_s^{ms} \).

(2) In Retailer Stackelberg models, if \( \beta = 3\gamma \), then \( \pi_m(w_{rn}) = \pi_m(w_{rs}) \), \( \pi_r(p_{1rn}^m) = \pi_r(p_{1rs}^m), \pi_r(p_{2rn}^m) = \pi_r(p_{2rs}^m) \) and \( \pi_s^{rn} = \pi_s^{rs} \); if \( \beta > 3\gamma \), then \( \pi_m(w_{rn}) < \pi_m(w_{rs}) \), \( \pi_r(p_{1rn}^m) < \pi_r(p_{1rs}^m), \pi_r(p_{2rn}^m) > \pi_r(p_{2rs}^m) \) and \( \pi_s^{rn} < \pi_s^{rs} \); if \( \beta < 3\gamma \), \( \pi_m(w_{rn}) > \pi_m(w_{rs}), \pi_r(p_{1rn}^m) < \pi_r(p_{1rs}^m), \pi_r(p_{2rn}^m) < \pi_r(p_{2rs}^m) \) and \( \pi_s^{rn} > \pi_s^{rs} \).
Proposition 4(1) indicates that in the Manufacturer Stackelberg models, the maximum profits for the manufacturer and the entire supply chain in the game with horizontal competition being Nash is more than those in the game with horizontal competition being Stackelberg. The two competing retailers’ maximum profits in the game with horizontal competition being Nash are less than those in the game with horizontal competition being Stackelberg. It indicates that, in a Manufacturer Stackelberg power structure, more competition between retailers will benefit the manufacturer and the whole supply chain but hurt the retailers’ profits. One interesting finding in Proposition 4(1) is that in the Manufacturer Stackelberg models, the maximum profit of retailer 1 (leader) in the game with horizontal competition being Stackelberg is lower than that of retailer 2 (follower). It means that the follower will gain more profit than the leader. Based on this conclusion, neither retailer wishes to be first to announce its retail price. This may be explained by the fact that the price information is regarded as trade secret for retailers. Once it leaks and is known by their competitor, it will have a genitive impact on its own financial benefit.

Proposition 4(2) also indicates that in the Retailer Stackelberg models, if the ratio between self-price sensitivity and cross-price sensitivity reaches a critical value, all the manufacturer’s, the retailers’ and the whole supply chain’s maximum profits in the game with horizontal competition being Nash equal to those in the game with horizontal competition being Stackelberg. That is, in such a condition, the power relationship between the two retailers has no effect on the manufacturer’s, the retailers’ and the whole supply chain’s maximum profits. If the same sensitivity ratio is higher than this critical value, the manufacturer’s, retailer 1’s and the whole supply chain’s maximum profits in the game with horizontal competition being Nash are less than those in the game with horizontal competition being Stackelberg. But the maximum profit of retailer 2 is more than that in the game with horizontal competition being Stackelberg. That is, in such a condition, a more balanced power relationship between the two retailers will hurt the manufacturer, the leader retailer (retailer 1) and the whole supply chain, but will benefit the follower retailer (retailer 2). If the sensitivity ratio is lower than this critical value, both the manufacturer’s and the whole supply chain’s maximum profits in the game with horizontal competition being Nash are more than those in the game with horizontal competition being Stackelberg, but the retailers’ maximum profits
are less than those in the horizontal Stackelberg game. That is, in such a condition, a more balanced power relationship between the two retailers will benefit the manufacturer and the whole supply chain, but will hurt the two competing retailers.

C. The effect of potential market size

Now we discuss the effect of potential market size ($\alpha$) on the supply chain’s decisions and profits. The following proposition is obtained.

Proposition 5  (1) $w^i$, $p^1_i$, $q^1_i$, $p^2_i$ and $q^2_i$ all increase in $\alpha$; (2) $\pi_m(w^i)$, $\pi_{r1}(p^1_i)$, $\pi_{r2}(p^2_i)$ and $\pi_s^i$ all increase in $\alpha$. Where $i = mn, ms, n, rn, rs$.

This proposition means that in every power structure, when the potential market size ($\alpha$) is large, the manufacturer will set a high wholesale price and the retailers will set high retail prices and sell more products. Then the manufacturer, the retailers and the supply chain will gain more profits. In contrast, when the potential market size ($\alpha$) is low, the manufacturer will set a low wholesale price and the retailers will set low retail prices and sell less products in every power structure. Then the manufacturer, the retailers and the supply chain will be less profitable. It is not surprising that individual firms and the supply chain will benefit from a large market size. The manufacturer and the retailers should certainly make effort in expanding the potential market.

VI. NUMERICAL EXAMPLE

Although the analytical results in the last section provide many insights of the effect of the bi-dimensional power structure on the supply chain decisions and performances, we mainly look at the effects of vertical or horizontal power structure separately. In the numerical analysis, we attempt to combine both the vertical and horizontal dimension of the power structure and evaluate the significance of their impact on the manufacturer and retailers individually and collectively. To do this, we will compare the financial performances of individual firms and the whole supply chain under the possible five alternative power structures. In addition, as shown in the previous section that both self-price and cross-price sensitivities play an important role in the effect of horizontal power structure, further analysis about the demand sensitivities is also presented. First, we set $\alpha = 10$, $\beta = 3$, $c = 0.1$. The
demand function parameters and cost information used in the numerical analysis are comparable to the similar studies [10, 11, 29] in the literature. To illustrate how the ratio of own-price sensitivity and cross-price sensitivity affect the impact of vertical and horizontal power structure on the firms’ performance individually and collectively, a range of cross-price sensitivities values were simulated. The maximum profits of the manufacturer, the two retailers and the whole supply chain under different power structures are shown in Fig. 2-5.

![Graph](image)

**Fig. 2** Manufacturer’s maximum profit under different power structures

![Graph](image)

**Fig. 3.** Retailer 1’s maximum profit under different power structures
It supports the findings in the discussion that more power over its vertical supply chain partners enables the manufacturer or the two retailers to gain more profit as shown in Fig.2 that the manufacturer makes most profit in the Manufacturer Stackelberg games and in Fig.3-4 that the two retailers will gain more profits when they hold equal or more power as compared to the manufacturer. Collectively, the whole supply chain will deliver the best performance in the balanced power structure as shown in Fig.5. Furthermore, the numerical analysis also supports the finding of Proposition 4(2) that the horizontal power structure have no effect on the supply chain financial performance collectively and individually when the
ratio of own-price sensitivity and cross-price sensitivity reach a certain value (e.g. \( \beta/\gamma=3 \)).

More interestingly, the effect of horizontal power structure on the supply chain performances is less significant as compared to the effect of the vertical power structure. It is consistent in the Manufacturer Stackelberg games, the Retailer Stackelberg games, and the Nash game as illustrated from Fig.2 to Fig.5. This finding is important as it indicates that it is more economical viable for the retailers to enhance its power over its supply chain counterparties than over its market rivals. Clearly, the increase in the vertical power will bring the retailers more financial benefits. This may be explained by the fact that the first move advantage is significant in the negotiation with the supply chain counterparties but not in setting the retail price to compete with your market rivals. Nevertheless, this finding applies to a common manufacturer supply chain setting and might be different when applying to other supply chain settings.

\section*{VII. CONCLUSIONS}

This study, examines how decisions and performances of a supply chain are affected by the power relationship in both vertical and horizontal competitions. We consider a triadic supply chain setting. We define five different possible supply chain power structures using five non-cooperative games, which are expressed by the different sequences in which the wholesale and retail pricing decisions are made by the manufacturer and retailers, respectively. The optimal supply chain decisions (e.g. prices and quantities) and profits are derived in each game. Through the comparison of equilibrium decisions and performances, our research systematically examines the impact of bi-dimensional power structure on the supply chain. We highlight some interesting observations from our research findings.

\textbf{Observation 1:} Our findings show that, in most cases, the power relationships in both the vertical and horizontal competitions have a profound influence on the manufacturer’s and the retailers’ supply chain decisions. Nevertheless, the horizontal power relationship between the two retailers has no effect on the manufacturer’s wholesale price when the vertical competition is Manufacturer Stackelberg. It also has no effect on supply chain decisions when the ratio between the self-price sensitivity and the cross-price sensitivity meets a certain condition when the vertical competition is Retailer Stackelberg. It indicates that the power
aspect is the main factor in determining the wholesale price when the manufacturer is the dominant force. In contrast, other factor such as price sensitivities will also influence the wholesale pricing decision if the manufacturer is no longer the dominant one.

**Observation 2:** The implications of power relationship in vertical competition or horizontal competition on the supply chain decisions are clear. Generally, more power over its supply chain partners or rival competitors enables the manufacturer or the leader retailer to set up higher wholesale price or retail price to gain more profit, which is in line with most of previous studies [10], [44], [46] in the literature. However, the impacts of the bi-dimensional power structure on firms’ price decisions are much more complicated. The power relationship between the two retailers and the price sensitivity ratio will affect the optimal retail prices.

**Observation 3:** Similarly, the bi-dimensional power dynamics have a significant impact on the supply chain financial performance both individually and collectively, except the case that, in the game with vertical competition being Retailer Stackelberg. The power relationship in the horizontal competition between the two retailers has no effect on individual firms’ and the supply chain’s profits when the ratio between the self-price sensitivity and the cross-price sensitivity meets a certain condition. This finding is interesting as it indicates that when the retailers are the dominant force, the profit for the two retailers and the supply chain is mainly determined by the market competition, which is mainly characterized by the demand sensitivity towards the retailer’s own price or her competitor’s price rather than the power relationship between the two rival retailers.

**Observation 4:** While the implications of power relationship in vertical competition or horizontal competition on the firms’ and supply chain performance have been clearly shown in our analysis, the impact of bi-dimensional power structure on individual firms’ and the supply chain’s performance are much more complicated. Surprisingly, the vertical supply chain power relationship has a more significant impact on the financial performances of the individual firms or the whole supply chain than the horizontal power relationship.

The key contributions of this research are summarized as follows. Theoretically, this paper systematically examines the effects of bi-dimensional power structure on a triadic supply chain. Our finding also reveals that the impacts of bi-dimensional power structure depend on the nature of customer demand, which supports the view of Shi et al. [39]. This
work will provide a solid platform to investigate other strategic and operational problems that are affected by both vertical and horizontal power structure. Managerially, this work provides important implications that are supportive for companies to make strategic and operational decisions. For instance, our findings show that, from individual firms’ perspective, more power over their rival competitors or supply chain counter parties will enable firms to gain more profit. In contrast, the supply chain will gain extra financial benefits if a more balanced power relationship exists both vertically and horizontally. Therefore, it is important for individual firms to seek ways in enhancing their market and supply chain power in order to acquire superior financial benefits. On the other hand, strategically, it is crucial for managers to develop a more power balanced supply chain environment that promotes fair and effective competition to improve its supply chain competitiveness.

Similar to any other models previous studies, there are also some research limitations. For instance, we only consider a simplified model setting. Although it provides some interesting insights, one future extension is to consider a more common model setting that consists of multiple manufacturers as well as retailers. Second, our model assumes a linear deterministic demand. As our research findings indicated that the impact of bi-dimensional power structure on the supply chain are dependent on the nature of customer demand, which is the ratio between the self-price sensitivity and the cross-price sensitivity in this work. Another important research extension is to model the stochastic demand instead of the deterministic one. Finally, another plausible research extension is to consider other important operational and strategic decisions such as multiple products [45], supply chain coordination [11], [15], and supply contracts [40] in the investigation of bi-dimensional power structure.

Acknowledgments

This research is partially supported by the National Natural Science Foundation of China (No. 71272128, 71432003, 91646109).

References


25


K. C. So, “Price and time competition for service delivery,” *Manufacturing & Service*


Appendix

Lemma 1 proof: From (3.3), we get \( \frac{d\pi_{r1}(p_1)}{dp_1} = \alpha - 2\beta p_1 + \gamma p_2 + \beta w \), \( \frac{d^2\pi_{r1}(p_1)}{dp_1^2} = -2\beta < 0 \), so \( \pi_{r1}(p_1) \) is concave in \( p_1 \). Similarly, from (3.4), we get \( \frac{d\pi_{r2}(p_2)}{dp_2} = \alpha - 2\beta p_2 + \gamma p_1 + \beta w \) and \( \frac{d^2\pi_{r2}(p_2)}{dp_2^2} = -2\beta < 0 \), so \( \pi_{r2}(p_2) \) is concave in \( p_2 \). From \( \frac{d\pi_{r1}(p_1)}{dp_1} = \frac{d\pi_{r2}(p_2)}{dp_2} = 0 \),
we get $p_1 = p_2 = \frac{\alpha + \beta w}{2\beta - \gamma}$. Replace $p_1 = p_2 = \frac{\alpha + \beta w}{2\beta - \gamma}$ to (3.1), we get $\frac{d\pi_m(w)}{dw} = \frac{2\beta[\alpha \beta (\beta - \gamma)(2w - c)]}{2\beta - \gamma}$ and $\frac{d^2\pi_m(w)}{dw^2} = -\frac{4\beta(\beta - \gamma)}{2\beta - \gamma} < 0$, so $\pi_m(w)$ is concave in $w$. Let $\frac{d\pi_m(w)}{dw} = 0$, we get $w^{mn} = \frac{\alpha}{2(\beta - \gamma)} + \frac{c}{2}$. Replace $w^{mn}$ to $p_1 = p_2 = \frac{\alpha + \beta w}{2\beta - \gamma}$, we get $p_1^{mn} = p_2^{mn} = \frac{\alpha(3\beta - 2\gamma)}{2(\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{2(\beta - \gamma)}$. Then, we get $q_1^{mn} = q_2^{mn} = \frac{\beta[\alpha - (\beta - \gamma)c]}{2(\beta - \gamma)}$.

From (3.4), we get $\frac{d\pi_r(p_2)}{dp_2} = \alpha - 2\beta p_2 + \gamma p_1 + \beta w$ and $\frac{d^2\pi_r(p_2)}{dp_2^2} = -2\beta < 0$. So, $\pi_r(p_2)$ is concave in $p_2$. Let $\frac{d\pi_r(p_2)}{dp_2} = 0$, we get $p_2 = \frac{\alpha + \beta w + \gamma p_1}{2\beta}$, Replace $p_2 = \frac{\alpha + \beta w + \gamma p_1}{2\beta}$ to (2.3), we get $\frac{d\pi_r(p_1)}{dp_1} = \frac{\alpha(2\beta - \gamma)(\beta + \gamma)w - 2(2\beta^2 - \gamma^2)p_1}{2 \beta}$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = -\frac{(2\beta^2 - \gamma^2)}{\beta} < 0$.

So, $\pi_r(p_1)$ is concave in $p_1$. Let $\frac{d\pi_r(p_1)}{dp_1} = 0$, we get $p_1 = \frac{\alpha(2\beta + \gamma)(\beta + \gamma)w}{4\beta^2 - 2\gamma^2}$. Replace $p_1 = \frac{\alpha(2\beta + \gamma)(\beta + \gamma)w}{4\beta^2 - 2\gamma^2}$ to (3.1), we get

$$d\pi_m(w) \frac{dw}{dw} = \frac{\alpha(6\beta^2 - \beta\gamma - 3\gamma^2)}{4(\beta - \gamma)(2\beta^2 - \gamma^2)} + \frac{(2\beta^2 + \beta\gamma - 2\gamma^2)c}{(4\beta^2 - \gamma^2)}$$

and

$$d^2\pi_m(w) \frac{dw^2}{dw^2} = -\frac{a(12\beta^2 - 2\beta\gamma - 7\beta\gamma^2 + \gamma^3)}{8\beta(\beta - \gamma)(2\beta^2 - \gamma^2)} + \frac{(4\beta^2 + 2\beta\gamma - \gamma^2 - 3\gamma^2)c}{8\beta(2\beta^2 - \gamma^2)}.$$ Then, we get $q_1^{ms} = \frac{(2\beta + \gamma)(\beta + \gamma)c}{8\beta}$ and $q_2^{ms} = \frac{(4\beta^2 + 2\beta\gamma - \gamma^2)(\beta - \gamma)c}{8(2\beta^2 - \gamma^2)}$.

From (3.2), we get $\frac{d\pi_m(w)}{dw} = \frac{[2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] - 2(\beta - \gamma)(w - c)}{2\beta - \gamma}$ and

$$d^2\pi_m(w) \frac{dw^2}{dw^2} = -\frac{4\beta - \gamma}{2\beta - \gamma} < 0.$$ So, $\pi_m(w)$ is concave in $w$. From (3.3), we get $\frac{d\pi_r(p_1)}{dp_1} = \alpha - 2\beta p_1 + \gamma p_2 + \beta w$ and $\frac{d^2\pi_r(p_1)}{dp_1^2} = -2\beta < 0$. So, $\pi_r(p_1)$ is concave in $p_1$. From (3.4), we get $\frac{d\pi_r(p_2)}{dp_2} = \alpha - 2\beta p_2 + \gamma p_1 + \beta w$ and $\frac{d^2\pi_r(p_2)}{dp_2^2} = -2\beta < 0$. So, $\pi_r(p_2)$ is concave in $p_2$. From $\frac{d\pi_m(w)}{dw} = \frac{d\pi_r(p_1)}{dp_1} = \frac{d\pi_r(p_2)}{dp_2} = 0$, we get $w^n = \frac{\alpha \beta}{(3\beta - \gamma)(\beta - \gamma)} + \frac{(2\beta - \gamma)c}{3\beta - \gamma}$, $p_1^n = p_2^n = \frac{\alpha(2\beta - \gamma)(\beta - \gamma)}{(3\beta - \gamma)(\beta - \gamma)} + \frac{\beta c}{3\beta - \gamma}$. Then, we get $q_1^n = q_2^n = \frac{\beta[\alpha - (\beta - \gamma)c]}{3\beta - \gamma}$.

From (3.2), we get $\frac{d\pi_m(w)}{dw} = \frac{[2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] - 2(\beta - \gamma)(w - c)}{2\beta - \gamma}$ and...
\[
d^2\pi_m(w) = -4(\beta - \gamma) < 0. \text{ So, } \pi_m(w) \text{ is concave in } w. \text{ Let } \frac{d\pi_m(w)}{dw} = 0, \text{ we get } [2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] - 2(\beta - \gamma)(w - c) = 0, \text{ that is } [2\alpha - (\beta - \gamma)(p_1 + p_2)] - 2(\beta - \gamma)(w - c) = 0. \text{ Then, we get } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma}. \text{ Replace } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma} \text{ to (3.3), we get }
\]
\[
\frac{d\pi_{r1}(p_1)}{dp_1} = \frac{a(5\beta-3\gamma)}{2(\beta-\gamma)} + \beta(c - 3p_1) - \frac{(\beta-3)\gamma p_1}{2} \text{ and } \frac{d^2\pi_{r1}(p_1)}{dp_1^2} = -3\beta < 0. \text{ So, } \pi_{r1}(p_1) \text{ is concave in } p_1. \text{ Similarly, replace } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma} \text{ to (3.4), we get }
\]
\[
\frac{d\pi_{r2}(p_2)}{dp_2} = \frac{a(5\beta-3\gamma)}{2(\beta-\gamma)} + \beta(c - 3p_2) - \frac{(\beta-3)\gamma p_1}{2} \text{ and } \frac{d^2\pi_{r2}(p_2)}{dp_2^2} = -3\beta < 0. \text{ So, } \pi_{r2}(p_2) \text{ is concave in } p_2. \text{ From } \frac{d\pi_{r1}(p_1)}{dp_1} = \frac{d\pi_{r2}(p_2)}{dp_2} = 0, \text{ we get } p_1^{rn} = p_2^{rn} = \frac{a(5\beta-3\gamma)}{(7\beta-3\gamma)(\beta-\gamma)} + \frac{2\beta c}{7\beta-3\gamma}. \text{ Replace } p_1^{rn} \text{ and } p_2^{rn} \text{ to } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma}, \text{ we get } w^{rn} = \frac{2\beta c}{(7\beta-3\gamma)(\beta-\gamma)} + \frac{(5\beta-3\gamma)c}{7\beta-3\gamma}.
\]

Then, we get \[ q_1^{rn} = q_2^{rn} = \frac{2\beta(\alpha - (\beta-\gamma)c)}{7\beta-3\gamma}. \]

From (3.2), we get \[ \frac{d\pi_m(w)}{dw} = [2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] - 2(\beta - \gamma)(w - c) \text{ and } \frac{d^2\pi_m(w)}{dw^2} = -4(\beta - \gamma) < 0. \text{ So, } \pi_m(w) \text{ is concave in } w. \text{ Let } \frac{d\pi_m(w)}{dw} = 0, \text{ we get } [2\alpha - (\beta - \gamma)(2w + m_1 + m_2)] - 2(\beta - \gamma)(w - c) = 0, \text{ that is } [2\alpha - (\beta - \gamma)(p_1 + p_2)] - 2(\beta - \gamma)(w - c) = 0. \text{ Then, we get } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma}. \text{ Replace } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma} \text{ to (3.4), we get }
\]
\[
\frac{d\pi_{r1}(p_1)}{dp_1} = \frac{d\pi_{r2}(p_2)}{dp_2} = 0, \text{ we get } p_1^{rn} = p_2^{rn} = \frac{a(5\beta-3\gamma)}{(7\beta-3\gamma)(\beta-\gamma)} + \frac{2\beta c}{7\beta-3\gamma}. \text{ Replace } p_1^{rn} \text{ and } p_2^{rn} \text{ to } w = c - \frac{p_1 + p_2}{2} + \frac{\alpha}{\beta - \gamma}, \text{ we get } w^{rn} = \frac{2\beta c}{(7\beta-3\gamma)(\beta-\gamma)} + \frac{(5\beta-3\gamma)c}{7\beta-3\gamma}.
\]

Then, we get \[ q_1^{rn} = q_2^{rn} = \frac{2\beta(\alpha - (\beta-\gamma)c)}{7\beta-3\gamma}. \]
Proposition 1 proof: From lemma 1, we get \( w^{mn} - w^n = \frac{a - (\beta - \gamma)c}{6\beta - 2\gamma} \). Since \( w > c \), then from lemma 1 we get \( \alpha - (\beta - \gamma)c > 0 \). So, \( w^{mn} > w^n \). Similarly, from lemma 1, we get \( w^n - w^r = \frac{\beta[\alpha - (\beta - \gamma)c]}{(7\beta - 3\gamma)(3\beta - \gamma)} > 0 \), that is, \( w^n > w^r \). So, \( w^{mn} > w^n > w^r \). From lemma 1, we get \( p^{r_{mn}}_i - p^{r^n}_i = \frac{\beta[\alpha - (\beta - \gamma)c]}{2(7\beta - 3\gamma)(2\beta - \gamma)} > 0 \), that is, \( p^{r_{mn}}_i > p^{r^n}_i \). Similarly, from lemma 1, we get \( p^r_i - p^n_i = \frac{\beta[\alpha - (\beta - \gamma)c]}{(7\beta - 3\gamma)(3\beta - \gamma)} < 0 \), that is, \( p^r_i > p^n_i \). So, \( p^{r_{mn}}_i > p^r_i > p^n_i \). From lemma 1, we get \( q^{r_{mn}}_i - q^n_i = -\beta[\alpha - (\beta - \gamma)c] \frac{\beta - \gamma}{2(2\beta - \gamma)(3\beta - \gamma)} < 0 \), that is, \( q^{r_{mn}}_i < q^n_i \). Similarly, from lemma 1, we get \( q^{r^n}_i - q^n_i = -\beta[\alpha - (\beta - \gamma)c] \frac{\beta - \gamma}{(2\beta - \gamma)(3\beta - \gamma)} < 0 \), that is, \( q^{r^n}_i < q^n_i \). So, \( q^n_i > q^{r^n}_i > q^{r_{mn}}_i \).

Proposition 2 proof: From lemma 1 and (3.1), we get \( \pi_m(w^{mn}) - \pi_m(w^n) = \frac{\beta(\beta - \gamma)|\alpha - (\beta - \gamma)c|^2}{2(2\beta - \gamma)(3\beta - \gamma)} > 0 \), that is, \( \pi_m(w^{mn}) > \pi_m(w^n) \). Similarly, from lemma 1 and (3.1), we get \( \pi_m(w^n) - \pi_m(w^r) = \frac{2\beta^2(13\beta - 5\gamma)|\alpha - (\beta - \gamma)c|^2}{(7\beta - 3\gamma)(2\beta - \gamma)^2} > 0 \), that is, \( \pi_m(w^n) > \pi_m(w^r) \). So, \( \pi_m(w^{mn}) > \pi_m(w^n) > \pi_m(w^r) \). From lemma 1, (3.3) and (3.4), we get \( \pi_{r_i}(p^{r_{mn}}_i) - \pi_{r_i}(p^{r^n}_i) = -\frac{\beta(7\beta - 3\gamma)(\beta - \gamma)|\alpha - (\beta - \gamma)c|^2}{4(3\beta - \gamma)(2\beta - \gamma)^2} < 0 \), that is, \( \pi_{r_i}(p^{r_{mn}}_i) > \pi_{r_i}(p^{r^n}_i) \). Similarly, from lemma 1, (3.3) and (3.4), we get \( \pi_{r_i}(p^{r^n}_i) - \pi_{r_i}(p^v_i) = -\frac{\beta(5\beta^2 + 6\beta\gamma - 3\gamma^2)|\alpha - (\beta - \gamma)c|^2}{(7\beta - 3\gamma)(2\beta - \gamma)^2} < 0 \), that is, \( \pi_{r_i}(p^{r^n}_i) > \pi_{r_i}(p^v_i) \). So, \( \pi_{r_i}(p^{r^n}_i) > \pi_{r_i}(p^v_i) > \pi_{r_i}(p^{mv}_i) \). From lemma 1, (3.1), (3.3) and (3.4), we get \( \pi^n_3 - \pi^r_3 = \frac{2\beta(\beta - 3\gamma)(\beta - \gamma)|\alpha - (\beta - \gamma)c|^2}{([7\beta - 3\gamma)(2\beta - \gamma)]^2} > 0 \), that is, \( \pi^n_3 > \pi^r_3 \). Similarly, from lemma 1, (3.1), (3.3) and (3.4), we get \( \pi^{mn}_s - \pi^{r^n}_s = \frac{2\beta^2(3\beta - 6\gamma)(\beta - \gamma)|\alpha - (\beta - \gamma)c|^2}{([7\beta - 3\gamma)(2\beta - \gamma)]^2} < 0 \), that is, \( \pi^{mn}_s > \pi^{r^n}_s \). So, \( \pi^n_3 > \pi^{r^n}_s > \pi^{mn}_s \).

Proposition 3 proof: (1) From lemma 1, we get \( w^{mn} - w^{ms} = \frac{\alpha}{2(\beta - \gamma)} + \frac{c}{2} - \frac{\alpha}{2(\beta - \gamma)} + \frac{c}{2} = 0 \), that is, \( w^{mn} = w^{ms} \). From lemma 1, we get \( p^{mn}_1 - p^{ms}_1 = -\frac{\gamma^2[\alpha - (\beta - \gamma)c]}{4(\beta - \gamma)(2\beta^2 - \gamma^2)} < 0 \), that is, \( p^{mn}_1 < p^{ms}_1 \). From lemma 1, we get \( p^{mn}_2 - p^{ms}_2 = -\frac{\gamma^3[\alpha - (\beta - \gamma)c]}{8\beta(\beta - \gamma)(2\beta^2 - \gamma^2)} < 0 \), that is, \( p^{mn}_2 < p^{ms}_2 \).
\( p_2^{ms} \). From lemma 1, we get \( p_1^{ms} - p_2^{ms} = \frac{\gamma^2[(\alpha - (\beta - \gamma)c)]}{8\beta(2\beta^2 - \gamma^2)} > 0 \), that is, \( p_1^{ms} > p_2^{ms} \). So \( p_1^{ms} > p_2^{ms} > p_2^{mn} = p_1^{mn} \).

(2) From lemma 1, we get \( w^{rn} - w^{rs} = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{2(7\beta - 3\gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \), \( p_1^{rn} - p_1^{rs} = \frac{6\beta(\beta - 3\gamma)(\alpha - (\beta - \gamma)c)]}{(7\beta - 3\gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \), \( p_2^{rn} - p_2^{rs} = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{(7\beta - 3\gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \), and \( p_1^{rs} - p_2^{rs} = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \). So, if \( \beta = 3\gamma \), then \( w^{rn} = w^{rs} \), \( p_1^{rn} = p_1^{rs} = p_2^{rn} = p_2^{rs} \); if \( \beta > 3\gamma \), then \( w^{rn} < w^{rs} \), \( p_1^{rn} < p_1^{rs} < p_2^{rn} < p_2^{rs} \); if \( \beta < 3\gamma \), then \( w^{rn} > w^{rs} \), \( p_1^{rn} = p_2^{rn} < p_2^{rs} < p_1^{rs} \).

**Proposition 4 proof:** (1) From lemma 1 and (3.1), we get \( \pi_m(w^{mn}) - \pi_m(w^{ms}) = \frac{\gamma^2(2\beta + \gamma)(\alpha - (\beta - \gamma)c)]^2}{16\beta(2\beta - \gamma)(2\beta^2 - \gamma^2)} > 0 \), that is, \( \pi_m(w^{mn}) > \pi_m(w^{ms}) \). From lemma 1 and (3.3), we get \( \pi_r(p_1^{mn}) - \pi_r(p_1^{ms}) = -\frac{\gamma^3[(\beta - \gamma)c)]^2}{32\beta(2\beta^2 - \gamma^2)(2\beta^2 - \gamma^2)} < 0 \), that is, \( \pi_r(p_1^{mn}) < \pi_r(p_1^{ms}) \). From lemma 1, and (3.4), we get \( \pi_r(p_2^{mn}) - \pi_r(p_2^{ms}) = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]^2}{64\beta(2\beta - \gamma)^2(2\beta^2 - \gamma^2)^2} < 0 \), that is, \( \pi_r(p_2^{mn}) < \pi_r(p_2^{ms}) \). Then \( \pi_r(p_1^{mn}) = \pi_r(p_2^{mn}) < \pi_r(p_1^{ms}) < \pi_r(p_2^{ms}) \). From lemma 1, (3.1), (3.3) and (3.4), we get \( \pi_s^{mn} - \pi_s^{ms} = \frac{\gamma^3[(12\beta^2 - \gamma^2)^4 + 4(\beta^2 - \gamma^2) + 16\beta^2(\beta + \gamma)(\beta - \gamma)^4(\alpha - (\beta - \gamma)c)]^2}{64\beta(2\beta - \gamma)^2(2\beta^2 - \gamma^2)^2} > 0 \), that is, \( \pi_s^{mn} > \pi_s^{ms} \).

(2) From lemma 1 and (3.1), we get \( \pi_m(w^{rn}) - \pi_m(w^{rs}) = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{2(7\beta - 3\gamma)(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \). From lemma 1 and (3.3), we get \( \pi_r(p_1^{rn}) - \pi_r(p_1^{rs}) = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{2(7\beta - 3\gamma)^2(17\beta + 3\gamma)(6\beta^2 + \beta\gamma - 3\gamma^2)} \). From lemma 1 and (3.4), we get \( \pi_r(p_2^{rn}) - \pi_r(p_2^{rs}) = -\frac{3\beta(\beta - 3\gamma)(\beta + 3\gamma)^2(407\beta^3 - 137\beta^2\gamma - 193\beta\gamma^2 - 9\gamma^3)(\alpha - (\beta - \gamma)c)]^2}{2(7\beta - 3\gamma)^2(17\beta + 3\gamma)^2(6\beta^2 + \beta\gamma - 3\gamma^2)^2} \). From lemma 1, (3.1), (3.3) and (3.4), we get \( \pi_s^{rn} - \pi_s^{rs} = -\frac{(\beta - 3\gamma)(\beta + 3\gamma)(\alpha - (\beta - \gamma)c)]}{2(7\beta - 3\gamma)^2(17\beta + 3\gamma)^2(6\beta^2 + \beta\gamma - 3\gamma^2)^2} \) \{88\beta^2(\beta^2 - \gamma^2) + 196\beta(\beta^3 - \gamma^3) + 5\beta^4 + 1400\beta^3\gamma + 1204\beta^4](\beta - \gamma) + 47\beta^4 + 81\gamma^5 \}. So, if \( \beta = 3\gamma \), then \( \pi_m(w^{rn}) = \pi_m(w^{rs}) \), \( \pi_r(p_1^{rn}) = \pi_r(p_1^{rs}) \), \( \pi_r(p_2^{rn}) = \pi_r(p_2^{rs}) \) and \( \pi_s^{rn} = \pi_s^{rs} \); if \( \beta > 3\gamma \), then \( \pi_m(w^{rn}) < \pi_m(w^{rs}) \), \( \pi_r(p_1^{rn}) < \pi_r(p_1^{rs}) \), \( \pi_r(p_2^{rn}) < \pi_r(p_2^{rs}) \),
and $\pi_{s}^{rn} < \pi_{s}^{rs}$; if $\beta < 3\gamma$, $\pi_{m}(w^{rn}) > \pi_{m}(w^{rs})$, $\pi_{r1}(p_{1}^{rn}) < \pi_{r1}(p_{1}^{rs})$, $\pi_{r2}(p_{2}^{rn}) < \pi_{r2}(p_{2}^{rs})$, and $\pi_{s}^{rn} > \pi_{s}^{rs}$.

**Proposition 5 proof:** (1) From lemma 1, we get $\frac{dw_{mn}}{da} = \frac{1}{2(\beta - \gamma)} > 0$, $\frac{dw_{ms}}{da} = \frac{1}{2(\beta - \gamma)} > 0$,

$$\frac{dp_{1}^{mn}}{da} = \frac{3\beta - 2\gamma}{2(2\beta - \gamma)(\beta - \gamma)} > 0, \quad \frac{dp_{1}^{ms}}{da} = \frac{6\beta^{2} - \beta\gamma - 3\gamma^{2}}{4(\beta - \gamma)(2\beta^{2} - \gamma^{2})} > 0, \quad \frac{dp_{1}^{rs}}{da} = \frac{2\beta - \gamma}{(3\beta - \gamma)(\beta - \gamma)} > 0, \quad \frac{dp_{1}^{rn}}{da} = \frac{5\beta - 3\gamma}{7(\beta - 3\gamma)(\beta - \gamma)} > 0.$$

$$2\beta + \gamma > 0, \quad \frac{dq_{1}^{n}}{da} = \frac{\beta}{3\beta - \gamma} > 0, \quad \frac{dq_{1}^{rn}}{da} = \frac{2\beta}{7\beta - 3\gamma} > 0, \quad \frac{dq_{1}^{rs}}{da} = \frac{5\beta + 3\gamma}{17\beta + 3\gamma} > 0, \quad \frac{dp_{2}^{mn}}{da} = \frac{3\beta - 2\gamma}{2(2\beta - \gamma)(\beta - \gamma)} > 0,$$

$$\frac{dp_{2}^{ms}}{da} = \frac{12\beta^{3} - 2\beta^{2}\gamma - 7\beta^{2}y^{2} + 3y^{3}}{8\beta(2\beta^{2} - y^{2})} > 0, \quad \frac{dp_{2}^{rs}}{da} = \frac{5\beta - 3\gamma}{7(\beta - 3\gamma)(\beta - \gamma)} > 0, \quad \frac{dp_{2}^{rn}}{da} = \frac{\beta(73\beta^{2} + 12\beta \gamma - 45\gamma^{2})}{(\beta - \gamma)(17\beta + 3\gamma)(6\beta^{2} + \beta \gamma - 3\gamma^{2})} > 0,$$

$$\frac{dq_{2}^{n}}{da} = \frac{2\beta}{7\beta - 3\gamma} > 0, \quad \frac{dq_{2}^{rn}}{da} = \frac{\beta(2\beta^{2} + 22\beta \gamma - 3\gamma^{2})}{(17\beta + 3\gamma)(6\beta^{2} + \beta \gamma - 3\gamma^{2})} > 0.$$ So, $w^{i}$, $p_{1}^{i}$, $q_{1}^{i}$, $p_{2}^{i}$ and $q_{2}^{i}$ are all increase in $\alpha$, where $i = mn, ms, n, rn, rs$.

(2) From lemma 1 and (3.1), we get $\frac{d\pi_{m}(w^{mn})}{da} = \frac{\beta[\alpha - (\beta - \gamma)c]}{(2\beta - \gamma)(\beta - \gamma)} > 0, \quad \frac{d\pi_{m}(w^{ms})}{da} = \frac{4\beta^{2}[\alpha - (\beta - \gamma)c]}{(2\beta - \gamma)(\beta - \gamma)} > 0, \quad \frac{d\pi_{m}(w^{rn})}{da} = \frac{16\beta^{2}[\alpha - (\beta - \gamma)c]}{(2\beta - \gamma)(\beta - \gamma)} > 0, \quad \frac{d\pi_{m}(w^{rs})}{da} = \frac{59\beta^{3} + 45\beta^{2}\gamma - 15\beta y^{2} - 9\gamma^{3}}{2(\beta - \gamma)(17\beta + 3\gamma)(6\beta^{2} + \beta \gamma - 3\gamma^{2})} > 0.$ From lemma 1 and (3.3), we get $\frac{d\pi_{r1}(p_{1}^{rn})}{da} = \frac{\beta[\alpha - (\beta - \gamma)c]}{2(2\beta - \gamma)} > 0, \quad \frac{d\pi_{r1}(p_{1}^{ms})}{da} = \frac{(2\beta + \gamma)[\alpha - (\beta - \gamma)c]}{16\beta(2\beta^{2} - \gamma^{2})} > 0, \quad \frac{d\pi_{r1}(p_{1}^{rs})}{da} = \frac{5\beta + 3\gamma}{17\beta + 3\gamma}[\alpha - (\beta - \gamma)c] > 0.$ From lemma 1 and (3.4), we get $\frac{d\pi_{r2}(p_{2}^{rn})}{da} = \frac{\beta[\alpha - (\beta - \gamma)c]}{2(2\beta - \gamma)} > 0, \quad \frac{d\pi_{r2}(p_{2}^{ms})}{da} = \frac{4\beta^{2} + 2\beta y^{2} - 8\gamma^{2}}{(2\beta - \gamma)(2\beta^{2} - \gamma^{2})} > 0, \quad \frac{d\pi_{r2}(p_{2}^{rs})}{da} = \frac{5\beta + 3\gamma}{17\beta + 3\gamma}(\beta - \gamma)(6\beta^{2} + \beta \gamma - 3\gamma^{2}) > 0.$ Then $\frac{d\pi_{s}^{rn}}{da} > 0, \quad \frac{d\pi_{s}^{rs}}{da} > 0, \quad \frac{d\pi_{s}^{rn}}{da} > 0, \quad \frac{d\pi_{s}^{rs}}{da} > 0$. So, $\pi_{m}(w^{i}), \pi_{r1}(p_{1}^{i}), \pi_{r2}(p_{2}^{i})$ and $\pi_{s}^{i}$ are all increase in $\alpha$, where $i = mn, ms, n, rn, rs$.  

31

Dr Xiaojun Wang is Reader in Operation Management at School of Economics, Finance and Management, University of Bristol. His current research predominantly focuses on supply chain risk and resilience, low carbon manufacturing, eco-design, sustainability, and social media research. His research outputs have been published in many international journals including *Production and Operations Management*, *European Journal of Operational Research*, *OMEGA-International Journal of Management Science*, *International Journal of Production Economics*, *International Journal of Production Research*, *IEEE Transactions on Engineering Management*. He is currently working on several research projects funded by NERC, ESRC, the Royal Society, the Newton Fund, and the National Natural Science Foundation of China.

Ke Gong, Master student in School of Management and Economics, University of Electronic Science and Technology of China. His research interests are supply chain management and operations management.