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Linguistic Modelling Using a Semi-Naïve Bayes Framework

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Abstract
A random set semantics is presented as a knowledge representation framework for learning linguistic prototypes. Within this framework a number of algorithms for learning prototypes are presented, based on grouping certain sets of attributes and evaluating joint mass assignments on labels. Such prototypes are then combined with a semi-Naïve Bayes classifier in order to determine classification probabilities. The potential of such linguistic classifiers is then illustrated by their application to a number of toy and benchmark problems.

Keywords: Random Sets, Mass Assignment, Label Prototypes, Semi-Naïve Bayes.

1 Introduction
The concept of “computing with words” in fuzzy logic was introduced by Zadeh [15]. He stated that fuzzy logic offered an intuitive method for modelling natural language where the meaning of words such as “small, medium and large” could be represented by fuzzy sets. This is a particularly appealing concept when dealing with real word problems where there is often imprecision and ambiguity. Zadeh’s idea was centred on using linguistic variables to represent linguistic constraints. This however generates a number of problems relating to both the semantics and computational complexity (see [12] for a discussion of these issues).

Here we present an extension to an alternative method proposed by Lawry [11]. The method uses random sets as a way of choosing appropriate labels for a given variable. Further more it builds on the ability of fuzzy sets to partition a domain into linguistic descriptions coupled with a new label semantics to give a feasible and consistent linguistic inference mechanism (see [8] and [10]). The method uses mass assignments on labels to provide a measure of the appropriateness of a given label for a particular value. Classification is performed using prototypes, consisting of a vector of mass assignments on labels, giving the appropriateness of words as labels for the feature values for a certain class. These are then used in conjunction with a Naïve or Semi-Naïve Bayes classifier (see [6] and [7]). As such the prototypes provide an aggregated linguistic description of the examples of that class in the database.

This paper introduces techniques that can be applied to counter the “the curse of dimensionality”[4]. It also provides a method for solving non-decomposable problems such as XOR, by introducing Semi-Naïve Bayes to weaken the independence assumption of Naïve Bayes [7].

2 Label Semantics
Consider an attribute describing a universe of discourse $\Omega$, assumed in this context to be a closed interval of real numbers. A finite set of labels $LA$ is defined over $\Omega$ to form a linguistic covering. For example, in the classification of diabetes the diastolic blood pressure may be recorded. From reading this data a doctor may conjecture, “The measured blood pressure is very high”. This would mean that an appropriate label for describing blood pressure would be very high $\in LA$. More formally, for each label $l \in LA$ a fuzzy set $\mu_l$ is defined, representing it’s meaning. Any value $x$ then generates a mass assignment (see Baldwin [1]) on labels as follows:

Let $\{l_1, ..., l_n\} = \{l \in LA \mid \mu_l(x) > 0\}$ be ordered such that $\mu_l(x) \geq \mu_{l+1}(x)$ then the mass assignment generated by $x$ is:

$$\{l_1, ..., l_i\}: \mu_l(x) - \mu_{l+1}(x) \text{ For } i = 1, ..., n - 1$$

$$\{l_1, ..., l_n\}: \mu_l(x)$$

$$\phi : 1 - \mu_l(x)$$
It is assumed that the distribution is on a random set \( \mathcal{D}_h \) describing the set of labels deemed as appropriate for \( x \), as it varies across some population of voters and is denoted \( m_{\mathcal{D}_h} \). (See [11] and [12])

In practice, it is undesirable to have mass associated with the empty set; hence the further assumption is made that \( \forall x \in \Omega: \max_{i \in L_\mathcal{A}} \mu_i(x) = 1 \).

Here this is accomplished using trapezoidal fuzzy sets with a 50% overlap, as in figure 1:

**Figure 1:** Trapezoidal Fuzzy Sets With 50% Overlap

### 3 Label Prototypes for Modelling Classification Problems

Consider a classification problem where a prototype is generated from a set of attributes \( X_1, \ldots, X_n \), describing the classes \( C_1, \ldots, C_r \). In this case a finite set of labels \( LA_r \) is defined over each of the variables \( X_i \). The database is partitioned into subsets corresponding to each class as follows:

Consider a training set of examples \( DB = \{ (x_1(i), \ldots, x_v(i)) | i = 1, \ldots, N \} \) where each example \( i \) has associated class \( C(i) \). From this database a set of sub-databases can be obtained for each class, \( DB_j = \{ (x_1(i), \ldots, x_v(i)) | C(i) = C_j \} \).

The attributes \( X_1, \ldots, X_v \) are now partitioned into subsets \( S_1, \ldots, S_w \) where \( w \leq n \) and for each \( S_i \), a joint mass assignment \( m_{i,j} \) is determined as follows: Suppose, w.l.o.g. \( S_i = \{ x_1, \ldots, x_w \} \) then the joint mass assignment is:

\[
\forall T_l \times \cdots \times T_v \in 2^L_l \times \cdots \times 2^L_v, m_{i,j}(T_l, \ldots, T_v) = \frac{1}{|DB_j|} \sum_{k \in DB_j} \prod_{r=1}^v m_{\mathcal{D}_r}(x_{r}(k))
\]

Section five describes how to automatically learn groupings of attributes for each class.

### 4 Estimating Classification Probabilities From Prototypes

We now give details of how to estimate class probabilities using label prototypes, which then can be incorporated into a Bayesian framework.

In machine learning it is common to make the “Naïve Bayes assumption” (see [7]) that all variables are conditionally independent given a class. This assumption is weakened in our case as the prototype describing a class may contain joint mass assignments; hence the classifier described is based on Semi-Naïve Bayes [6].

Bayes theorem states that for a vector of attribute values \( \{ x_1, \ldots, x_v \} \), the class probability can be expressed as follows:

\[
Pr(C_j | x_1, \ldots, x_v) = \frac{p(x_1, \ldots, x_v | C_j) Pr(C_j)}{p(x_1, \ldots, x_v)}
\]

This can be simplified to the following estimate, for the purpose of classification:

\[
Pr(C_j | x_1, \ldots, x_v) \propto p(x_1, \ldots, x_v | C_j) Pr(C_j)
\]

There now remains the problem of how to estimate the density function \( p(x_1, \ldots, x_v | C_j) \). In the current context it is assumed that the prototype for each class can be used to estimate the density function, as follows:

Consider the joint mass assignment generated for the attribute grouping \( S_i \) given class \( C_j \). Then if we assume that there is a uniform prior distribution on \( \times \Omega_r \) then the prior mass assignment on \( \times 2^L_i \) is given by:

\[
m(T_1, \ldots, T_v) = \prod_{r=1}^v \frac{m_{\mathcal{D}_r}(T_r)}{\sum_{r=1}^v m_{\mathcal{D}_r}(T_r)} u(x_1, \ldots, x_v) dx_1, \ldots, dx_v
\]

Where \( u(x_1, \ldots, x_v) \) is the uniform distribution on \( \times \Omega_r \) and \( u(x_r) \) the uniform distribution on \( \Omega_r \).

From this the density function for \( S_i \) based on \( m_{i,j} \) is given by:

\[
p(S_i | m_{i,j}) = \frac{p(x_1, \ldots, x_v | m_{i,j})}{\sum_{T_1, \ldots, T_v} m_{i,j}(T_1, \ldots, T_v)} \prod_{r=1}^v m_{\mathcal{D}_r}(T_r)
\]

Hence taking \( p(x_1, \ldots, x_v | C_j) \equiv p(x_1, \ldots, x_v | m_{i,j}) \) an estimate of the class probability is:

\[
Pr(C_j | x_1, \ldots, x_v) \propto C_j \prod_{r=1}^w p(S_r | m_{r,j})
\]
5 Grouping Methods

We now consider methods for finding attribute groupings that increase discrimination in the model. For a given problem it is impractical to search the complete space of all attributes groupings and then partition to see if discrimination can be increased, as the search space would be exponential. For example, 20 attributes would generate a search of order 20! comparisons for a search limited to attribute groupings of two variables. Instead a heuristic search strategy is adopted. It is proposed that the search be guided by a measure of importance for each \( S_i \), defined as follows:

5.1 Definition (Importance Measure)

Let the joint mass assignment for \( S_i \) given \( C_j \) be denoted \( m_{ij} \). For any input vector \( S_i \) the probability of class \( C_j \) can be estimated using Bayes theorem where:

\[
\Pr(C_j \mid S_i) = \frac{p(S_i \mid m_{ij})p(C_j)}{p(S_i \mid m_{ij})p(C_j) + p(S_i \mid m_{i\neg j})p(C_j)}
\]

Hence \( m_{ij} \) denotes the mass assignment for \( S_i \) given \( C_j \). The importance measured of group \( S_i \) for class \( C_j \) is then defined by:

\[
IM_{ij}(S_i) = \sum_{k \in DB} \Pr(C_j \mid S_i(k)) \frac{\sum_{k \in DB} \Pr(C_j \mid S_i(k))}{\sum_{k \in DB} \Pr(C_j \mid S_i(k))}
\]

Careful limits must be set on the maximal size for groupings when running this algorithm, since as dimensionality increases the number of data points per focal element of the joint mass assignment decreases exponentially. The use of fuzzy sets in this context allows us to partially overcome this problem by trading off granularity against dimensionality and vice-versa. Two search strategies have been developed based on this measure.

5.2 Guided Breadth First Search

Consider a breadth first search where the most important current grouping \( S_i \) is combined with all the other current groupings to see if the combination significantly increases discrimination. Next the second most important unused grouping is tested with the remaining unused groupings and so on. At the next stage the new groupings produced are tested in a similar manner and this continues until a terminating condition is satisfied. This method provides a fairly extensive search of the space of the partitions, but does limit the structure of the groupings generated.

5.3 Guided Depth First Search

Alternatively, consider a depth first search where the most important grouping \( S_i \) is tested with all other groupings to see if the combination increases discrimination. Next any new grouping produced is tested with the unused groupings to see if discrimination is further increased. This continues until some termination condition is satisfied. Next the process is repeated with the next most important unused grouping and so on, until all unused groupings have been tested. This allows for a richer structure of groupings but has the disadvantage that some important groupings may be missed.

We now consider two ways of determining whether a pair of attribute groupings should be combined. The first is based on a direct measure of correlation and the second on a measure of the change in importance resulting from the grouping. Before we can define the above mentioned correlation measure we must first define what is meant by the focal sets for a mass assignment \( m_{ij} \).

5.4 Definition (Focal Sets)

Let the focal sets, \( F_i \), of be given by:

\[
F = \{ S \subseteq LA \mid \exists x \in \Omega, m_{ij} (S) > 0 \}
\]

5.5 Definition (Correlation Measure)

Let \( F_i \) be the focal sets for \( S_i \) and \( F_j \) the focal sets for \( S_j \). Now let \( m_{ij} \) be the joint mass of \( S_i \cap S_j \) given \( C_j \).

\[
CORR(S_i, S_j) = \frac{1}{|F_i||F_j|} \sum_{R \in F_i} \sum_{T \in F_j} (m_{ij}(R,T) - m_{ij}(R)m_{ij}(T))^2
\]

Here a threshold must be used to determine whether attributes should be grouped. The nearer the correlation measure gets to 1 the higher the correlation between attribute groups.
An alternative to measuring correlation is to use the importance measure, as a guide to whether attribute groups should be combined, by trying to maximise the importance of any new grouping formed.

5.6 Definition (Improvement Measure)

Suppose we have two subsets of attributes $S_1$ and $S_2$ then the improvement in importance obtained by combining them can be calculated as follows:

$$IPM_{j}(S_1,S_2) = \frac{\min(IM_j(S_1),IM_j(S_2))}{IM(S_1,S_2)}$$

Like the correlation measure a threshold is required, and in this instance the closer the improvement measure is to 0 the more likely that the attribute groups will be combined.

6 Performance On Model and Benchmark Problems

In this section we present a number of examples showing how the methods described in section 5 perform on model and real world problems.

6.1 Example (Non-Decomposable Model Problem)

In this example a figure of eight shape is generated according to the parametric equations $x = 2^{-0.5} (\sin 2t - \sin t)$ and $y = 2^{-0.5} (\sin 2t + \sin t)$ where $t \in [0,2\pi]$ as is illustrated in figure 2.

![Figure 2: Figure Eight Classification Problem](image)

A point on the $[-1.6,1.6]^2$ domain is classified as legal if it is contained within the figure and illegal if contained outside. The database contained 961 training examples of the X and Y co-ordinates and their associated class.

The prototypes obtained were generated from placing 5 labels over both attributes universes the meanings of which corresponded to uniformly distributed trapezoidal fuzzy sets. As there are only two attributes in this problem the choice of search method is arbitrary, as both will obtain the same results. For the correlation method a threshold of 0.005 was set, producing the following attribute groupings: $Legal = \{x, y\}$, $Illegal = \{x\} \{y\}$. For the improvement measure a threshold of 0.895 was set and the following groupings were generated: $Legal = \{x, y\}$, $Illegal = \{x, y\}$

Caution must be taken here as the thresholds used are not optimised for the problem and could suggest why the correlation method chooses slightly different grouping to the improvement measure. We can, however intuitively see why these groupings were chosen. If this problem is thought of as an XOR problem then clearly only a grouping of $\{x, y\}$ for the legal class is required for adequate classification.

From the groupings obtained it is possible to plot the posterior distributions learned from the data. Figure 3 shows these for the improvement measure approach. These suggest an inverse relationship between the legal and illegal distributions as would be expected.

![Figure 3: The Posterior Distributions Learned](image)

The classifiers were then tested on a test set of 2119 unseen examples using the distributions for both the grouping methods and the following classification results were obtained:

<table>
<thead>
<tr>
<th></th>
<th>Correlation Measure</th>
<th>Improvement Measure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted Accuracy</td>
<td>Predicted Accuracy</td>
</tr>
<tr>
<td>Test</td>
<td>94.61248</td>
<td>95.93573</td>
</tr>
<tr>
<td>Training</td>
<td>94.58897</td>
<td>96.46202</td>
</tr>
</tbody>
</table>

Here slightly better results are obtained by using the improvement measure against the correlation
method. It should be noted that if the Naïve Bayes independence assumption is used to classify then substantially poorer classification accuracy is obtained:

<table>
<thead>
<tr>
<th>Test</th>
<th>Predicted Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>85.0662</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Training</th>
<th>Predicted Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>84.5994</td>
<td></td>
</tr>
</tbody>
</table>

6.2 Example (Sonar Data)

This problem is taken from the UCI online repository [14] and contains 208 examples obtained by bouncing sonar signals off metal cylinders and rocks. Each of the patterns contains a set of 60 numbers in the range [0,1], which represent the energy within a particular frequency ban, over a certain time period. The integration aperture for high frequency occurs later in time, since these frequencies are transmitted later during the cipher.

It should be noted that the data used is for the “aspect-angle dependent experiment”. Here the data set is split into training and test sets of 104 examples, where this split takes into account the aspect angle (see [9]). The data set was also normalised so that all attributes shared the same mean and standard deviation. Each attribute in the database had 2 labels placed over their mean and standard deviation. Each attribute in the database had 2 labels placed over their mean and standard deviation.

The depth and bread first search methods were applied with the two combination techniques and the following result were obtained:

Breath First Search Results:

<table>
<thead>
<tr>
<th>Predicted Class/True Class</th>
<th>Rock</th>
<th>Metallic Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>95.2381</td>
<td>4.7619</td>
</tr>
<tr>
<td>Metallic Cylinder</td>
<td>12.9023</td>
<td>87.09677</td>
</tr>
</tbody>
</table>

Depth First Search Results

<table>
<thead>
<tr>
<th>Predicted Class/True Class</th>
<th>Rock</th>
<th>Metallic Cylinder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock</td>
<td>98.1818</td>
<td>1.81818</td>
</tr>
<tr>
<td>Metallic Cylinder</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

These results demonstrated that in both the breadth and depth first searches the improvement measure obtained the best classification, with the depth first method slightly out performing the breadth first method for both correlation and improvement measure. If these results are then compared with those obtained using Naïve Bayes, as presented in table 5, then it can be seen that apart form the correlation method, using bread first search gave an increase in classification accuracy of a maximum of 12.5%.

<table>
<thead>
<tr>
<th>Training</th>
<th>Predicted Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>82.69231</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test</th>
<th>Predicted Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>80.76923</td>
<td></td>
</tr>
</tbody>
</table>

These results also highlight the trade off between granularity and dimensionality as good classification results are observed using only two labels on the 60 attributes.
These results can be comparable with Gorman and Sejnowski [9] who experiment with a back propagation neural network with 60 inputs and up to 24 hidden nodes, which are illustrated in table 6 and those of Frieb, Cristianini and Campbell [5] who used a Kernel Adatron Algorithm obtaining a classification accuracy of 95.2%.

| Table 6: Gorman and Sejnowski |
|-------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Hidden Nodes                  | 0               | 2               | 3               | 6               | 12              | 24              |
| Accuracy on Test Set %        | 73.1            | 85.7            | 87.6            | 89.3            | 90.4            | 89.2            |

6.3 Glass Identification Database

This problem is taken from the UCI online repository [14] and is constructed by forensic scientists. The database contains 214 examples of 7 different types of glass fragments found at the scenes of crime, which if correctly identified can be used as evidence. The attributes supplied are as follows:

<table>
<thead>
<tr>
<th>Type of glass:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 RI: Refractive index</td>
</tr>
<tr>
<td>2 Na: Sodium (unit measurement: weight percent in corresponding oxide, as are attributes 4-10)</td>
</tr>
<tr>
<td>3 Mg: Magnesium</td>
</tr>
<tr>
<td>4 Al: Aluminium</td>
</tr>
<tr>
<td>5 Si: Silicon</td>
</tr>
<tr>
<td>6 K: Potassium</td>
</tr>
<tr>
<td>7 Ca: Calcium</td>
</tr>
<tr>
<td>8 Ba: Barium</td>
</tr>
<tr>
<td>9 Fe: Iron</td>
</tr>
</tbody>
</table>

Here, splitting each sub-class evenly in the problem produced a test set of 109 examples and training set of 105 examples. Both search and grouping methods were applied, with each attribute having 3 labels place over their domain in a non-uniform manner and with attributes 8 and 9 being discarded. The experiment was repeated 100 times with randomly generated data sets constructed in the same manner and the following average classification accuracy obtained:

Breath First Search Results:

| Table 7: Correlation Measure Results |
|--------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Average                              | Upper Bound     | Lower Bound     | Variance        | Uncertain       |
| Test                                 | 65.33945        | 76.14679        | 55.9633         | 4.84518         |
| Training                             | 87.66667        | 95.2381         | 80.95238        | 3.12223         |

Threshold = 0.005, Max Grouping = 4

Depth First Search Results:

| Table 8: Improvement Measure Results |
|--------------------------------------|-----------------|-----------------|-----------------|-----------------|
| Average                              | Upper Bound     | Lower Bound     | Variance        | Uncertain       |
| Test                                 | 67.11927        | 77.98165        | 59.63303        | 4.40887         |
| Training                             | 91.60952        | 98.09524        | 86.66667        | 2.39309         |

Threshold = 0.895, Max Grouping = 4

Using this database a direct comparison can be made with results from Baldwin, Lawry and Martin (see [2]), who split the database into two equal training and test sets of 107 examples. Here using a breadth search with the improvement measure and setting the threshold to 0.895 with a max grouping of 4 attributes a classification accuracy of 71.03% can be obtained on the test set and 92.52% on the training set. This compares well with the results of 71% on the test set using a mass assignment prototype method [2] and a test set accuracy of 68% using a mass assignment ID3 system [3].

7 Conclusion

This paper shows that it is possible to obtain good classification accuracy using the Semi-Naive Bayesian framework presented. These
results also highlight a theorem due to Wolpert and Macready entitled “No Free Lunch Theorems For Search” [13], who suggested that no search can in general obtain optimum classification result for all problems. This is apparent if a comparison is made between the sonar data results section 6.2, were the depth first search method given the best classification accuracy, against the glass database, section 6.3, were the best classification result are obtained by using a breadth first search.

It has also been demonstrated that correlation is not always a good discriminator between classes. A better approach is to directly measure the improvement in discrimination obtained by any particular grouping of variables, whilst balancing granularity and dimensionality in the problem.

The methods described here represent an ongoing development of the proposed framework and further work is needed to optimise classification performance. Furthermore, the potential power of using “natural language querying” in such a framework has still to be investigated.

References


