Modular Generic Programming with Extensible Superclasses

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Abstract

“Generics for the Masses” (GM) and “Scrap your Boilerplate” (SYB) are generic programming approaches based on some ingenious applications of Haskell type classes. To achieve modularity, the GM and SYB approach have been extended by using some experimental language extensions such as abstraction over type classes and recursive instances. Hence, the type class encodings behind the GM and SYB approach become less practical and harder to understand.

We show that none of these type class features are necessary if we use the single feature of extensible superclasses, the complement of subclass extension. We formalize type classes with extensible superclasses as the combination of a previously introduced type-passing translation scheme and a general type class framework. Our results shed some new light on the use of type classes to support generic programming.

1. Introduction

Generic programming is a style of programming where a single generic function definition is applicable to a wide range of data types. This is in contrast to an ad-hoc polymorphic function which provides a separate definition for each data type. There are a number of compelling approaches which exploit ad-hoc polymorphism to support generic programming.

Prominent examples are “Generics for the Masses” (GM) [9] by Hinze and “Scrap your Boilerplate” (SYB) [17] by Lammel and Peyton Jones. These works employ Haskell type classes [22, 28], which are an elegant formulation to support ad-hoc polymorphism as an extension of Hindley/Milner. Besides Haskell, type classes can also be found in a number of other languages such as Clean [23], HAL [2] and Mercury [8, 14].

Unfortunately, the GM approach requires to update the class declarations with new method definitions for each ad-hoc type case. The consequence is that the GM approach is not modular. In some recent work [21] this problem has been addressed, by either relying on specific dispatcher functions for each generic function or using some non-standard type class extensions such as undecidable instances [4]. We believe that the proposed solutions are not entirely satisfactory and may affect the understandability of the GM approach.

The SYB approach has similar problems when it comes to the modular extension of generic functions. In [18], non-standard type class features such as type class abstraction [11] and recursive instances (a.k.a., recursive dictionaries, co-inductive type classes) [26] are employed to support modular, extensible generic functions.

In this paper, we take a fresh look at generic programming with type classes. Our contributions are as follows:

• We show that the GM and SYB approach can be made modular by employing extensible superclasses, a type class feature which supports the incremental extension of superclasses (Sections 5 and 4). In our opinion, extensible superclasses provide for a much more natural solution to the “modularity” problem.

• We formalize extensible superclasses using a combination of a previously proposed type-passing translation scheme by Thatte and our own Constraint Handling Rules based type class framework (Section 6).

In Section 7 we discuss further related work. We conclude in Section 8.

The GM and SYB approach make heavy use of type classes. Hence, we first give an introduction to Haskell type classes in the next section. Unless otherwise stated, we assume Haskell 98 type classes [22]. We implement a library which supplies evaluation and printing functions for a simple arithmetic expression language using type classes in the most straightforward way. In case we extend the functionality of our library, we wish to maintain close relations among the set of instances provided. But this requires to update existing class declarations which in turn forces us to recompile the entire program. Hence, modularity is broken. For very similar reasons, the GM and SYB approach struggle to achieve modularity.

In Section 3, we investigate why changing class declarations breaks modularity by taking a closer look at the dictionary-passing translation scheme [5, 26] underlying Haskell implementations. Haskell implementations support the modular extension of subclasses but not superclasses. If we switch to a type-passing translation scheme, we can incrementally introduce further superclasses without having to recompile existing code. This is the essence of our method to achieve modularity for the GM and SYB approach.

2. Haskell Type Classes

We start off with the definition and specific implementations of the evaluation function for a simple expression language.

```haskell
-- expression language
data Lit = Lit Int
data Plus a b = Plus a b
```
class Eval a where
  eval :: a -> Int

instance Eval Lit where
  eval (Lit n) = n

instance (Eval a, Eval b) => Eval (Plus a b) where
  eval (Plus a b) = eval a + eval b

The class declaration states that class Eval has a function eval
(also known as method) to evaluate values of type t to integers.
A type class constraint such as Eval t expresses that type t is a
member of class Eval. The instance declarations provide specific
implementations of eval on data types Lit and Plus a b. Notice
that the last instance declaration states that we can build an instance
of eval on type Plus a b if we can provide definitions for the
calls eval a and eval b. We commonly refer to (Eval a, Eval b)
as the instance context and to Eval (Prod a b) as the instance
head.

It is straightforward to define new cases by providing new instances.

data Minus a b = Minus a b
instance (Eval a, Eval b) => Eval (Minus a b) where
  eval (Minus a b) = eval a - eval b

We can also easily introduce new functions. For convenience, we
omit the instance bodies in the program text below. They do not
matter here.

class Print a where
  print :: a -> String

instance Print Lit
instance Print (Plus a b)
instance Print (Minus a b)
instance Print a => Print [a] -- (L)

The addition of the new cases and functions does not require to re-
compile any existing code. Hence, type classes support the modular
extension of libraries.

The trouble is that eval and print, respectively their type classes
Eval and Print, are only loosely connected. We support printing
lists, see instance (L), but there is no such instance for Eval.
This may result in some unexpected behavior for the user. For
example, consider the following user program.

literals = [Lit 1, Lit 2]

evalAndPrint = (eval literals, print literals)

We attempt to use eval and print on type [Lit] In case of eval,
there is no definition that deals with this. In Haskell speak,
the above yields an unresolved instance error Eval [Lit]. We
would much prefer to catch this error at the definition rather than
use site. What we would like is to guarantee that eval and print
are defined for the exact same set of instances.

In Haskell, we can give such guarantees via subclassing. We replace
Print’s class declaration as follows.

class Eval a => Print a where
  print :: a -> String

This declaration defines Print to be a subclass of Eval. Or putting
it the other way around, Eval is declared to be a superclass of
Print. Then, any Haskell implementation such as GHC [4] or
HUGS [12] will complain that there is an instance for Print [a],
see (L) above, but such an instance is missing for Eval.

Subclassing effectively guarantees that the set of instances of the
superclass are a subset of the set of instances of the subclass. Hence,
it is only natural to replace Eval’s class declaration by

class Print a => Eval a where

Thus, we can guarantee that for every print instance there is a
eval instance and vice versa. We have achieved our goal to catch
the error at the definition site. But there is a serious problem.

Haskell currently prohibits recursive subclass relations. But this
restriction can be safely lifted as we will argue later in Section 6.2.
The real problem is that our use of subclassing breaks modularity.
For example, at some later stage we may want to introduce
class Size a where
  size :: a -> Int

to compute the size of expressions. We leave out the instances for
simplicity. We would like to ensure that size operates on the exact
same set of types as eval and print. To impose this condition, we
will need to alter eval’s and print’s class declarations.

class (Eval a, Size a) => Print a
  where print :: a -> String

class (Print a, Size a) => Eval a
  where eval :: a -> Int

But changing existing class declarations means we need to recom-
pile existing code. Hence, modularity is broken. To better under-
stand why this is the case and how to fix the problem, we take a
look at possible ways to translate type classes.

3. Translating Type Classes

3.1 Dictionary-Passing Translation Scheme

In Haskell, we translate type classes by representing them via
dictionaries. These dictionaries hold the actual method definitions.
Each superclass dictionary is part of its (direct) subclass dictionary.
For example, the declarations

class Eval a where
class Eval a => Print a where

instance Eval Lit
instance (Eval a, Print a) => Size a

imply the dictionary declarations

data EvalDict a = E (EvalDict a) (a -> String)
data PrintDict a = P (EvalDict a) (a -> String)

We can thus easily access the superclass dictionary via its subclass
dictionary. The actual construction of dictionaries is described by
instance declarations. For example, instance (E1) shows that a
dictionary for Eval Lit exists. Instance (E2) shows how to build
a dictionary for Eval (Plus a b) given dictionaries for Eval a
and Eval b.

More formally, class and instance declarations specify a type class
proof system $\Delta \vdash d : TC\text{Dict}\ t$ where the environment $\Delta$
holds the set of given dictionaries and dictionary value $d$ of type
TC\text{Dict}\ t$ can be concluded from $\Delta$ with respect to a given set
of class and instance declarations.

In Figure 1, we give the specific type class proof system result-
ing from the above declarations. Rule (Var) allows us to lookup
assumptions from the environment. Each of the next four rules cor-
respond to one of the four instance declarations. We assume the
following dictionary construction functions.
The next example makes use of dictionary constructors belonging to instances in the translation. The program:

\[
\begin{align*}
\text{f2} & : (\text{EvalDict a, EvalDict b}) \rightarrow a \rightarrow b \rightarrow \text{Int} \\
f2 & (d1, d2) \ x \ y = \text{case } (\text{instP2} (d1, d2)) \ of \\
& \quad \text{P } eval \rightarrow \text{eval } (\text{Plus } x \ y)
\end{align*}
\]

translates to

f2 (d1, d2) x y = case (instP2 (d1, d2)) of
\[
\begin{align*}
P & \rightarrow \text{eval } (\text{Plus } x \ y)
\end{align*}
\]

The source program demands a dictionary for Eval (Plus a b) which we can supply by applying the dictionary constructor instP2 to the two dictionaries supplied by the annotation.

Here comes the important observation. In a dictionary-passing translation scheme, we can introduce new instances and subclasses without having to recompile existing code. But if we include a new superclass, say Size, in Print’s class declaration, we need to change the definition of existing superclass extension rules such as (PE). Hence, programs such as function f1 need to be recompiled.

### 3.2 Type-Passing Translation Scheme

Let’s see how to translate some of the above programs based on Thatte’s type-passing translation scheme [27]. We use a System F style target language extended with type case as found in intentional type analysis [7]. The main idea is to pass around types instead of dictionaries. These types are made available anyway by the standard translation of Hindley/Milner to System F [6].

Then, the program from before

\[
\begin{align*}
f1 & :: \text{Print } a \Rightarrow a \rightarrow \text{Int} \\
f1 & x = \text{eval } x
\end{align*}
\]

translates to

f1 = \lambda x:a. \text{eval } a \ x

None of the type classes are turned into dictionaries. We simply erase them. We directly use the type a to lookup the specific method definition. In the translation, function f1 remains unchanged and does not need to be recompiled. Hence, a type-passing translation scheme naturally supports the modular extension of superclasses. This is the essential feature we propose to simplify the “modular” type class encodings of the GM and SYB approach.

We yet need to discuss how to translate instances. In a dictionary-passing translation scheme each instance declaration is compiled into a separate dictionary constructing function. In a type-passing translation, we need to lump together all these instances. That is, for each method we need one central method lookup function to access the type-specific method definitions. Here is the method lookup function belonging to the above instance declarations of the Eval class.

\[
\begin{align*}
\text{eval} & = \Lambda a. \text{typecase } a \ of \\
& \quad \text{Lit } \rightarrow \ldots \\
& \quad \text{Prod } b \ c \rightarrow \ldots
\end{align*}
\]

Somebody may argue that this will break separate compilation in case we introduce new instances. But clearly we can compile the individual instances separately. Our assumption is that the linker collects the instance definitions (which are in pre-compiled form) and glues them together to build the method lookup function eval.

Another implementation detail is that the actual method definitions are only built at run-time. Hence, there may be a potential inefficiency if we use a type-passing translation scheme. For example, consider the situation where we need to built a definition for Print [\ldots[\text{Lit}\ldots]]. In Haskell’s translation scheme, we may be able to build Print [\ldots[\text{Lit}\ldots]]’s dictionary based on given dictionaries more efficiently. Obviously, we can improve the translation by using dynamic programming techniques etc.
Besides supporting the modular extension of superclasses, the type-passing translation scheme has a further advantage. For example, the program

\[
g :: \text{Eval } (\text{Plus } a \ b) \Rightarrow a \rightarrow b \rightarrow \text{Int}
g x y = \text{eval } x + \text{eval } y
\]
can be easily translated as follows

\[
g = \Lambda a, b. \lambda x : a. \lambda y : b. \text{eval } a \ x + \text{eval } b \ y
\]

The program text of \(g\) demands \text{Eval } a and \text{Eval } b. We can argue that these constraints follow (logically) from the constraint \(\text{Eval } (\text{Plus } a \ b)\) given by the annotation. The dictionary-passing translation scheme fails here because we must (constructively) build demanded dictionaries from given dictionaries. Rule (E2) in Figure 1 only tells us how to build \(\text{Eval } (\text{Plus } a \ b)\)'s dictionary given the dictionaries for \(\text{Eval } a\) and \(\text{Eval } b\). But the other direction, necessary to translate function \(g\), does not hold in general. The fact that under a type-passing translation scheme we can interpret instance declarations as "if-and-only-if" relations between instance context and instance head is interesting but not essential in our recast of the GM and SYB approach. Though, the ability to extend the set of superclasses is essential.

### 3.3 The Story So Far and The Next Steps

We summarize the main points. Under a dictionary-passing translation scheme, we cannot naturally support the extension of superclasses. Updating class declarations affects existing proofs, that is dictionaries, derived from the type class proof system. Hence, the program needs to be recompiled. Under a type-passing translation scheme, however, none of the existing proofs is affected if we introduce new superclasses. Hence, no recompilation is necessary.

In the upcoming sections, we revisit the GM and SYB approach and show that extensible superclasses is the crucial feature to support modularity. Existing Haskell implementations lack this feature. Therefore, the extensible version of GM [21] and SYB [18] rely on features such as abstraction over type classes and recursive instances. In essence, these features allow to mimic extensible superclasses under a dictionary-passing translation scheme. We will discuss this point in more detail for the SYB approach in Section 4. We argue that we can achieve modularity more directly by employing extensible superclasses. The type class encodings become more transparent and easier to maintain for the user.

The details of extensible superclasses are given in Section 6. We formalize them as an instance of our Constraint Handling Rules (CHR) based type class framework married with Thatté's type-passing translation method. In such a system, we do not derive dictionaries from type class proofs. We only need to check that type class proofs are valid. Hence, we can give the user the flexibility to specify additional proof rules which are not necessarily connected to any class or instance declarations. We choose the CHR formalism to specify such proof rules. For example, the CHRs

\[
\begin{align*}
\text{rule } \text{Print} \ a & \Rightarrow \text{Size} \ a \\
\text{rule } \text{Eval} \ a & \Rightarrow \text{Size} \ a
\end{align*}
\]
declare \text{Size} to be a superclass of \text{Print} and \text{Eval}. Hence, the CHR notation \(\Rightarrow\) can be read as "subclass of" which is somewhat contrary to the Haskell notation

\[
\begin{align*}
\text{class} \ \text{Size} \ a & \Rightarrow \text{Eval} \ a \\
\text{class} \ \text{Size} \ a & \Rightarrow \text{Print} \ a
\end{align*}
\]

Though, subclassing corresponds logically to Boolean implication. Hence, \(\Rightarrow\) is the more appropriate notation. Next, we take a look at the SYB and GM approach and make use of CHR proof rules to specify extensible superclasses.

### 4. Scrap Your Boilerplate

#### 4.1 A Non-modular Encoding

The SYB approach allows to write generic functions via a combinator library. We take a look at an example taken from [18] to get a better understanding of the modular extension problem. We introduce a standard generic function to compute the size of a data structure.

\[
gsize :: \text{Data } a \Rightarrow a \rightarrow \text{Int} 
\]

\[
gsize \ t = 1 + \text{sum } (\text{gmapQ } \text{gsize } t)
\]

The combinator \(\text{gmapQ}\) applies function \(\text{gsize}\) to each of the immediate children of \(t\). The result is a list of these sizes which are then summed up and incremented by one to obtain the total size. In general, there are further combinators besides \(\text{gmapQ}\) to write generic functions other than queries. These combinators are methods of the \text{Data} class. For simplicity, we only consider the \(\text{gmapQ}\) combinator.

\[
\begin{align*}
\text{class } \text{Typable} \ a & \Rightarrow \text{Data } a \ \\
\text{gmapQ } f & \Rightarrow \text{Data } a \rightarrow \text{Data } [a] \\
\text{gmapQ } f & \Rightarrow \text{Data } a \rightarrow \text{Data } [a] \\
\end{align*}
\]

Each time we introduce a new generic function such as \(\text{gsize}\), we define a new class \text{Size} with method \(\text{gsize}\). Thus, we can easily specify type-specific behavior of \(\text{gsize}\) by providing a \text{Size} instance. Here is the straightforward representation in Haskell

\[
\begin{align*}
\text{class } \text{Size} \ a & \Rightarrow \text{Data } a \ \\
\text{gsize} & \Rightarrow a \rightarrow \text{Int} \\
\end{align*}
\]

\[
\text{gsize } t = 1 + \text{sum } (\text{gmapQ } \text{gsize } t)
\]

The last case is the default, generic case and defines the behavior on all types that do not match \text{Name}. This is an example of an overlapping instance, an extension supported by GHC. Rank-2 types are necessary, for example see the upcoming instance (D) where we apply \(f\) on values of different type. The instances of the \text{Data} class can be derived automatically. Here are two of them.

\[
\begin{align*}
\text{instance } \text{Data } \text{Char} & \Rightarrow \text{gmapQ } f \ c = [] \\
\text{instance } \text{Data } a & \Rightarrow \text{Data } [a] \ \\
\text{gmapQ } f (x:xs) & \Rightarrow [f x, f xs]
\end{align*}
\]

Notice that we give \(\text{gmapQ}\) a rank-2 type, an extension which is supported by GHC. Rank-2 types are necessary, for example see the upcoming instance (D) where we apply \(f\) on values of different type. The instances of the \text{Data} class can be derived automatically. In the instance (D) where we apply \(f\) on values of different type. The instances of the \text{Data} class can be derived automatically. In this specific context, the combinator \(\text{gmapQ}\) expects (as its first argument) a function of type \(\forall a.\text{Data } a \Rightarrow a \rightarrow \text{Int}\) but the actual argument \(\text{gsize}\) has type \(\forall a.\text{Size } a \Rightarrow a \rightarrow \text{Int}\). There is clearly a mismatch and therefore the program will not type check. As pointed out in [18], we can fix the problem by making \text{Size} a superclass of \text{Data}.

\[
\begin{align*}
\text{class } (\text{Typable } a, \text{Size } a) & \Rightarrow \text{Data } a \ \\
\text{gmapQ } f & \Rightarrow (\forall b. \text{Data } b \Rightarrow b \rightarrow \text{Int}) \\
\text{gmapQ } f & \Rightarrow (\forall b. \text{Data } b \Rightarrow b \rightarrow \text{Int}) \\
\end{align*}
\]

Then, type \(\forall a.\text{Size } a \Rightarrow \text{Int}\) can be specialized to the type \(\forall a.\text{Data } a \Rightarrow \text{Int}\) and therefore the above instance (S) type checks. The down-side is that we need to recompile all existing code that refers to type class \text{Data}. This will happen for each newly introduced generic function. The SYB authors have recognized this problem.
4.2 The SYB3 Solution

The key idea behind the solution proposed in [18] (also known as the SYB3 approach) is to employ abstraction over type classes, a feature that has been suggested by Hughes [11] in a different context. Here is the rewritten program using type class abstraction.

```haskell
class (Typeable a, ctxt a) => Data ctxt a where
    gmapQ :: (forall b. Data ctxt b => b -> r) -> a -> [r]
instance Data Size t => Size t where
    gsize t = 1 + sum (gmapQ gsize t)
```

Notice that variable `ctxt` ranges over type classes instead of types (hence Hughes coined the term type class abstraction to refer to this feature). Then, the superclass described by `ctxt` is not fixed when class `Data` is declared. Thus, we can instantiate `ctxt` with `Size` later. See the instance declaration.

There are a number of further adjustments necessary. For example, abstraction over type classes introduces “ambiguous” types. Hence, the translation of programs may become ambiguous. Therefore, explicit type applications must be introduced. We cannot repeat all the details here and refer to the reader to [18].

The problem is that neither type class abstraction nor explicit type applications are features supported by any Haskell implementation. Although, it is possible to encode them by adding a auxiliary class `Sat` and type `Proxy`, the details are quite tricky and involve quite a bit of programmer effort in case we introduce new generic functions. As mentioned in [18], each generic function requires a record type, a `Sat` instance and a type proxy that needs to be inserted at the proper place. The encoding even makes it necessary to deal with another type class feature known as recursive instances. We briefly elaborate on this feature in Section 7.

4.3 Our Solution

In our proposed system of extensible superclasses, we can leave the declaration of class `Data` unchanged. We can introduce the new generic function `gsize` as presented in Section 4.1. All that is required is to impose the additional condition

```haskell
rule Data a ==> Size a
```

which declares `Size` to be a superclass of `Data`.

The program is then accepted in our system. Here is the type-passing translation for the `gmapQ` and `gsize` instances.

```haskell
gmapQ = \a \x. \lambda f. (\forall b. Data ctxt b => b -> r) \rightarrow a -> [r]
typecase a of
    Char -> []
    [t] -> case x of (y:ys) -> [f t x, f [t] xs]
```

Each type case corresponds to one instance. The formal parameter `f` has a polymorphic type. Hence, in the translation we supply `f` with additional type arguments as in `[f t x, f [t] xs]`. The translation of `gsize` instances is similar.

```haskell
gsize = \a \x. typecase a of
    Name -> ...
    t -> 1 + sum ((gmapQ a Int) gsize x)
```

In the generic case, the `gmapQ`-call is supplied with the type arguments `a` and `Int`.

A type-passing translation scheme allows us to keep type class relations flexible. We use dynamic type information to select the appropriate method definitions. Thus, our solution to obtain modular, extensible, generic functions is more light-weight (in terms of user programmer effort).

5. Generics for the Masses

Let’s take a look at the GM approach where we will encounter very similar problems as in case of the SYB approach.

The main idea behind the GM approach is to provide a uniform representation of data types in terms of unit, sum and product types. Generic functions are defined in terms of this uniform rather than the concrete structural representation of a data type. The programmer only needs to maintain a type isomorphism between the uniform and concrete representation. Thus, there is no need to extend the (new generic) definition of functions in case we include new data types. The trouble is that we cannot override generic with specific (ad-hoc) behavior in a modular fashion. We will see shortly why.

Here is a (over-simplified) presentation of the GM approach applied to our example from Section 2. In the GM approach, each generic function is an instance of the class `Generic`.

```haskell
data Lit = Lit Int
data Plus a b = Plus a b
data Iso a b = Iso {from :: a -> b, to :: b -> a}
class Generic g where
    lit :: g Lit
    plus :: g a -> g b -> g (Plus a b)
    view :: Iso a b -> g a -> g b
```

For simplicity, we assume that the concrete representations “literal” and “plus” are already part of the `Generic` class. They are structurally equivalent to the uniform representations for “unit” and “products” which are commonly found in the `Generic` class. Via the “view” function we can use the generic function on many Haskell data types given a type isomorphism between the data type and its structural representation. By including literal and plus from the start, we avoid defining some straightforward type isomorphisms which would make the whole presentation more noisy. We will see later in Section 5.1 an example of a type isomorphism. Notice that `g` in `Generic g` ranges over type constructors. This is an example of a constructor class [16] which is supported in Haskell.

Here is the generic definition of the evaluation function.

```haskell
newtype Ev a = Ev {eval' :: a -> Int}
instance Generic Ev where
    lit = Ev (\x -> case x of Lit i -> i)
    plus a b = Ev (\p -> case p of (Plus x y) -> eval' a x + eval' b y)
    view iso a = Ev (\x -> eval' a (from iso x))
```

In order to use the evaluator on its familiar type, we need a “dispatcher” function to select the appropriate case of a generic function. The most straightforward approach is to use an ad-hoc polymorphic (therefore extensible) function.

```haskell
class Rep a where
    rep :: Generic g => g a
instance Rep Lit where
    rep = lit
instance (Rep a, Rep b) => Rep (Plus a b) where
    rep = plus rep rep
eval :: Rep t => t -> Int
eval = eval' rep
```

The dispatcher function `rep` will select the appropriate generic case depending on the concrete type context. We can straightforwardly introduce new generic functions.
subclass. We cannot specify ad-hoc cases without breaking modularity.

On the other hand, we can easily introduce new (ad-hoc) cases by providing experimental type class extensions. This problem has been addressed [21]. The solution is inspired by the modular extension of the SYB approach and requires some experimental type class extensions.

The benefits of the GM approach compared to the Haskell type class approach become clear. In the GM approach, functions eval and print are represented by types which are instances of class Generic. Thus, both functions work on the same set of types. In the Haskell type class approach, we introduce a new class for each function and therefore we cannot give the same guarantees. On the other hand, we can easily introduce new (ad-hoc) cases by providing additional instances. This is a problem for the GM approach. We cannot specify ad-hoc cases without breaking modularity.

For example, data types Plus a b and Minus a b have the same uniform representation (as a product type). We obviously do not want to use the same generic definition for both types. To extend the Generic class with an ad-hoc “minus” case, we introduce a subclass.

class Generic g => GMinus g where
  minus :: g a -> g b -> g (Minus a b)
instance GMinus Ev where
  minus a b = Ev (\p \rightarrow case p of
                   (Minus x y) \rightarrow eval' a x - eval' b y)

The problem is that we cannot access this new case, unless we update the type of the dispatcher function rep.

class Rep a where
  rep :: GMinus g \Rightarrow g a
-- original code: rep :: Generic g \Rightarrow g a
instance (Rep a,Rep b) \Rightarrow Rep (Minus a b) where
  rep = minus rep rep
  eval :: Rep t \Rightarrow t \rightarrow Int
  eval = eval' rep

The original code will not type check for similar reasons as encountered in the SYB approach.

Alternatively, we could leave the dispatcher untouched and make GMinus a superclass of Generic.

class GMinus g => Generic g where
  minus :: g a -> g b -> g (Minus a b)
instance GMinus Ev where
  minus a b = Ev (\p \rightarrow case p of
                   (Minus x y) \rightarrow eval' a x - eval' b y)

The rule declaration guarantees that for each GMinus instance there is a Generic instance (as expected). This condition is required by the default instance definition. In Haskell, we would usually express such conditions via subclassing. But we want to make a point that under a type-passing translation scheme the introduction of sub-/superclasses is not necessarily connected to class declarations.

Next, we extend the dispatcher function by adding a new case for “minus”:

rule Generic g \Rightarrow GMinus g
instance (Rep a,Rep b) \Rightarrow Rep (Minus a b) where
  rep = minus rep rep

Via the rule declaration we introduce GMinus as a superclass of Generic. This is essential, otherwise, the instance declaration for the “minus” case will not type check. Two issues arise here.

The rule declaration claims that for each Generic instance there is a GMinus instance. Above we have defined an instance of Generic on type Pr a but there is no such instance declaration for GMinus. Hence, there seems to be a problem. In terms of the terminology we develop later in Section 6.2, the type class proof system is “non-confluent”. But wait! We forgot the generic, default definition for GMinus. The argument is that unless otherwise stated, for each Generic instance there is always a default GMinus instance. Hence, the type class proof system is confluent.

The second issue is that cyclic dependencies

rule GMinus g \Rightarrow Generic g
rule Generic g \Rightarrow GMinus g

among the Generic and GMinus class may threaten decidable type inference. Again, there is no problem here. Type inference remains decidable for such cases. We elaborate in Section 6.2.

We conclude that we can introduce new ad-hoc cases and easily inherit generic definitions without having to change any existing code. What we do next is override the default evaluator case with a new ad-hoc definition.

instance GMinus Ev where
  minus a b = Ev (\p \rightarrow case p of
                   (Minus x y) \rightarrow eval' a x - eval' b y)

In summary, we have achieved a modular extension of the GM approach. To convince ourselves that our solution works correctly, we consider the type-passing translation of all declared instances.
The syntax of class and instance declarations is as follows:

\[
\text{rep} = \lambda a. \text{typecase} a \text{ of}
\]

\[
\text{Lit} \to \lambda g. \text{lit} g
\]

\[
\text{Plus } b \text{ c} \to \lambda g. \text{plus } ((\text{rep } b) g) ((\text{rep } c) g)
\]

\[
\text{Minus } b \text{ c} \to \lambda g. \text{minus } ((\text{rep } b) g) ((\text{rep } c) g)
\]

The syntax is slightly more involved than outlined in Section 3. In the class declaration of Rep, the type of the method rep is locally constrained by Generic g. In case of the dictionary-passing translation scheme, we may therefore assume that the dictionary for Generic g will be locally supplied at the use site. Hence, the translation of the instance definitions takes an additional dictionary argument. The same principle applies in case of a type-passing translation scheme. For each type case branch, we introduce the local type abstraction \(\Lambda g\). At a use site we supply the appropriate type argument, for example see (rep b) g. The presence of constructor classes also makes it necessary to switch to System F, as the target language (this applies to both translation schemes).

The translation of the remaining instances is straightforward.

\[
\text{lit} = \lambda a. \text{typecase} a \text{ of}
\]

\[
\text{Ev} \to \text{Ev} \left(\lambda x. \text{case } x \text{ of Lit } i \to i\right)
\]

\[
\text{Pr} \to \text{...}
\]

\[
\text{plus} = \lambda a. \text{typecase} a \text{ of}
\]

\[
\text{Ev} \to \text{Ev} \left(\lambda p. \text{case } p \text{ of}
\right)
\]

\[
(\text{Plus } x y) \to \text{eval' } b x + \text{eval' } c y
\]

\[
\text{Pr} \to \text{Ev} \left(\lambda p. \text{case } p \text{ of}
\right)
\]

\[
(\text{Plus } x y) \to \text{print' } b x ++ " + " ++ \text{print' } c y
\]

\[
\text{minus} = \lambda a. \text{typecase} a \text{ of}
\]

\[
\text{Pr} \to \text{Ev} \left(\lambda b c. \text{Ev} \left(\lambda p. \text{case } p \text{ of}
\right)
\right)
\]

\[
(\text{Plus } x y) \to \text{eval' } b x - \text{eval' } c y
\]

Notice that we (automatically) included the default instance for printing.

Function eval translates to

\[
\text{eval} = \lambda a. \text{eval' } ((\text{rep } a) \text{ Ev})
\]

and the call

\[
\text{eval} \left(\text{Minus } (\text{Lit } 2) \text{ (Lit } 1)\right)
\]

translates to

\[
\text{eval} \left(\text{Minus } \text{Lit Lit} \text{ Lit } (\text{Lit } 2) \text{ (Lit } 1)\right)
\]

It should be clear that this program text correctly evaluates to 1.

6. Extensible Superclasses

We formalize extensible superclasses using a combination of our own CHR-based type class framework [25] and Thatte’s type-passing translation scheme. CHR stand for Constraint Handling Rule [3] and we use them to specify the type class proof system for extensible superclasses.

6.1 CHR Type Class Proof System

The syntax of class and instance declarations is as follows:

\[
\text{Types} \quad t ::= a \mid t \to t \mid T \text{ i}
\]

\[
\text{Type Classes} \quad tc ::= TC t
\]

\[
\text{Context} \quad Ctx ::= tc, ... t_c
\]

\[
\text{Constraint} \quad C ::= tc \mid C \land C
\]

\[
\text{Classes} \quad cls ::= \text{class } Ctx \Rightarrow TC a
\]

\[
\text{Instances} \quad inst ::= \text{instance } Ctx \Rightarrow TC t
\]

Notice that constraints and contexts effectively describe the same object, i.e. collections of type classes. Depending on the situation, we will use set, tuple or \("\land" \) notation.

We make use of CHRs of the following two forms:

\[
\text{rule } TC t \implies TC t'
\]

\[
\text{rule } TC t \iff TC t_1, ..., TC t_n
\]

where we assume that TC refers to a type class and \(t\) refers to a type.

The first CHR is referred to as a propagation rule and the second is referred to as simplification rule. Logically, the symbol \(\implies\) denotes Boolean implication and \(\iff\) denotes Boolean equivalence. CHRs have also a simple operational reading which we will ignore for the moment. In essence, CHRs model proof rules, similar to type class proof rules from Section 3.1.

Each declaration

\[
\text{class}(TC; a_1, ..., TC_n, a) \implies TC a
\]

translates to

\[
\text{rule } TC a \implies TC a
\]

\[
\text{rule } TC a \implies TC_n a
\]

Subclassing states subset relations among instances which can logically be expressed in terms of Boolean implication. Notice that the Haskell subclass arrow \(\Rightarrow\) is (logically speaking) the wrong way around!

Each declaration

\[
\text{instance}(TC; t_1, ..., TC_n, t_n) \implies TC t
\]

translates to

\[
\text{rule } TC t \iff TC_1 t_1, ..., TC_n t_n
\]

This is exactly the meaning required to describe type classes under a type-passing translation scheme. Recall that instances specify if-and-only-if relations.

Extensible superclasses can then be modeled straightforwardly by providing additional CHR propagation rules. For example, the following CHRs

\[
\text{rule } Eval a \implies Print a \quad -- (1)
\]

\[
\text{rule } Print a \implies Eval a \quad -- (2)
\]

state that Print is a superclass of Eval and vice versa. The introduction of cyclic CHRs such as (1) and (2) raises the concern whether we can maintain decidable type inference. We will address such issues further below.

We can also introduce “short-hand” notation via CHR simplification rules.

\[
\text{rule } EvalAndPrint a \iff Eval a, Print a
\]

The above introduces an abstract type class EvalAndPrint. We assume that there are no methods connected to this type class. In programs, that is in type annotations, we can then use EvalAndPrint as a short-hand for Eval a, Print a.

Based on this understanding of type classes in terms of CHRs, we can express type class proofs directly in terms of some familiar first-order logic statements. Let \(P\) be the set of CHRs, derived from class and instance declarations and specified by the programmer. Let \(C\) be a conjunction of (given) type class constraints and \(TC t\) a (demanded) type class constraint. Then, we can write \(P \models C \supset TC t\) to express that \(TC t\) is derivable from \(C\) under \(P\). The symbol \(\models\) denotes model-theoretic entailment. The statement \(P \models C \supset TC t\) holds if \(TC t\) can be satisfied in any (first-order) model of \(P\) and \(C\).
6.2 CHR Proof Checking

We briefly show how to verify that statements $P \models C \supset TC t$ hold. First, we perform some logical equivalence transformations. We have that $P \models C \supset TC t$ iff $P \models C \supset C \land TC t$. CHRs have a straightforward operational reading in terms of rewritings among constraints, more formally written $\rightarrow^* P$. Thus, we can check $P \models C \leftarrow C \land TC t$ by executing $C \leftarrow p_1$, and $C \land TC t \leftarrow p_2$ testing whether $C_1$ and $C_2$ refer to the same canonical form. Decidability and completeness of this check depend on whether CHRs are terminating and confluent.

Let’s take a look at the formal definition of the operational reading of CHRs. We assume that we are given a set $C$ of constraints where $TC t' \in C$ and consider the (rewriting) effect the different forms of constraints. CHRs can have on $TC t'$.

**Propagation step:** We can apply

$$ rule\ TC t \rightarrow TC_1 t'' \in P $$

by adding (propagating) $TC_1 \phi(t'')$ to $C$, if we find a substitution $\phi$ such that $t'$ and $\phi(t)$ are equal. Notice that we only perform matching (but not Prolog style unification). That is, we rewrite $C$ to $C \cup \{TC_1 \phi(t'')\}$, written $C \rightarrow p_1.C TC_1 \phi(t'')$.

We assume that CHRs are renamed before rule application. Notice that we avoid infinite propagation by prohibiting to fire a rule on the same constraint twice. We refer to [1] for further details.

**Simplification step:** We can apply

$$ rule\ TC t \leftrightarrow TC_1 t_1 ..., TC_n t_n \in P $$

if we find a substitution $\phi$ such that $t'$ and $\phi(t)$ are equal. Then, we can rewrite $C$ into $C' \{TC t' \cup \{TC_1 \phi(t_1), ..., TC_n \phi(t_n)\}\}$, i.e. the constraint resulting from $C'$ by replacing (simplifying) $TC t'$ with $TC_1 \phi(t_1), ..., TC_n \phi(t_n)$.

Each of the rewriting steps preserve the equivalence among constraints. Hence, the above checking method for $P \models \Delta \supset TC t$ is clearly correct.

We write $C \rightarrow C'$ to denote the exhaustive application of rewriting steps yielding the *final* constraint $C'$. We say a set of CHRs is *terminating* if for each constraint $C$ we find a final constraint $C'$ such that $C \rightarrow C'$. We say that a set of CHRs is *confluent* if different rewriting derivations starting from the same point can always be brought together again.

Let’s see whether the “(extensible superclass) examples” we have seen so far satisfy termination and confluence of CHRs.

We consider the “cyclic” CHRs.

\begin{align*}
\text{rule} & \quad \text{Eval} a \equiv \text{Print} a \quad \text{(1)} \\
\text{rule} & \quad \text{Print} a \equiv \text{Eval} a \quad \text{(2)}
\end{align*}

We find that

$$ \text{Eval} a \rightarrow \text{Eval} a, \text{Print} a \rightarrow \text{Eval} a $$

In the first step, we apply CHR (1) on Eval a and add Print a. In the second step, apply CHR (2) on Print a which adds the constraint Eval a. But wait, this constraint is already present. We assume set semantics. Hence, no further constraint will be added. Recall that we prevent firing the same rule twice on the same constraint. We have already fired the propagation rule on Eval a. Hence, Eval a \rightarrow Eval a, Print a. Hence, the above CHRs are terminating.

Here is another “cyclic” set of CHRs which arises from the SYB example. The (simplified) declarations

\begin{align*}
\text{class} & \quad \text{Size} a \Rightarrow \text{Data} a \quad \text{(Super)} \\
\text{instance} & \quad \text{Data} a \Rightarrow \text{Size} a \quad \text{(Inst)}
\end{align*}

yield

\begin{align*}
\text{rule} & \quad \text{Data} a \Rightarrow \text{Size} a \\
\text{rule} & \quad \text{Size} a \equiv Data a
\end{align*}

We find that

$$ \text{Size} a \rightarrow Data a \rightarrow Data a, Size a \rightarrow Data a, Size a $$

In the first step, we apply the simplification rule. Then, the propagation rule adds Size a. In the final step, we apply again the simplification rule and simplify Size a by Data a. But this constraint is already present (recall set semantics). The propagation rule has already been fired on Data a. Hence, Size a \rightarrow Data a, Size a. Hence, the above CHRs are terminating.

The SYB authors claim that the above program is non-terminating. That is, the rewriting steps implied by class and instance declarations may not terminate. As we show above this is not the case. There is also no connection to recursive instances. The “recursion” goes through a superclass!

We conclude. Although extensible superclasses introduce “cyclic” relations, resulting CHRs are terminating for the examples we have seen so far. We leave it for future work to establish syntactic conditions (in style of the Haskell conditions [22] imposed on instances) that guarantee termination. Note that for terminating CHRs there is an easy check of confluence by testing whether all “critical pairs” are joinable.

For example, consider

\begin{align*}
\text{rule} & \quad \text{Print} a \equiv \text{Eval} a \\
\text{rule} & \quad \text{Print} \text{Lit} \equiv \text{True}
\end{align*}

We consider the critical pair Print Lit. We find two non-joinable derivations Print Lit \rightarrow True and Print Lit \rightarrow \text{Eval} Lit. Hence, the above CHRs are non-confluent and hence we must reject the program. And rightly so, the first CHR claims that for each Print t there is a Eval t which is not the case because of the second CHR.

6.3 Type-Passing Translation Scheme

We highlight the main aspects. For simplicity, we only consider the translation of expressions and focus on the most interesting rules. The development is pretty much similar to the standard dictionary-passing translation method. The crucial difference is that we strictly erase type classes and use types to access specific method definitions.

We work with a simple expression language representing the core of a functional language with type classes. As our target language, we use a simplified version of System F. Function parameters are annotated with Hindley/Milner types which is sufficient here. There is also no need for a type case because we only consider the translation of expressions:

$$ \begin{align*}
\text{Type Schemes} & \quad \sigma ::= t \mid \forall a. \sigma \mid TC t \Rightarrow \sigma \\
\text{Expressions} & \quad e ::= x \mid \lambda e.e \mid e \mid let\ g = e in\ e \\
\text{Target} & \quad E ::= x \mid \lambda e : \sigma. E \mid E \mid \Delta a. E \mid E t
\end{align*} $$

We follow the common path and employ a type-directed translation scheme formulated in terms of judgments $C, \Gamma \vdash e : \sigma \rightarrow E$ where $C$ is a constraint holding the set of type class constraints, $\Gamma$ is an environment assigning type schemes to free variables, $e$ is an expression with type $\sigma$ and $E$ is a target expression. The judgment
C, Γ ⊢ e : σ → E states that a well-typed expression e with type σ
is translated to E. λ

Here are the most interesting translation rules:

(∀ Intro)

C, Γ ⊢ e : σ → E  a ∉ fr(Γ, C)
C, Γ ⊢ e : ∀a.σ → E a.

(∀ Elim)

C, Γ ⊢ e : ∀a.σ → E
C, Γ ⊢ e : [f/a]σ → E t

(⇒ Intro)

C ∧ TC t, Γ ⊢ e : t' → E
C, Γ ⊢ e : TC t ⇒ t' → E

(⇒ Elim)

C, Γ ⊢ e : TC t ⇒ σ → E  P ⊢ C ⊃ TC t
C, Γ ⊢ e : σ → E

The first two rules are familiar from translating Hindley/Milner to
System F  [6]. In rule (∀ Intro), we assume that fr(Γ, C) computes
the set of free (type) variables of an environment and constraint.
If a variable is not in this set we can universally quantify over
this variable. We use a System F style target language, therefore,
we make type abstraction (and later application) explicit via Λ, a
pronounced "big" lambda. In case of elimination (i.e. instantiation)
of a universal quantifier we use type application in the target term.
Up to know the same translation steps apply to the dictionary-
translating scheme as well.

The crucial difference between the dictionary- and type-passing
is manifested in the last two rules. Pushing a type class into a
type scheme has no effect on the translated program under a type-
passing scheme. See rule (⇒ Intro). That is, type classes are simply
erased during the translation. This is in contrast to the dictionary-
translating scheme where we would introduce a lambda-abstraction
because type classes are turned into dictionaries. In rule (⇒Elim),
we eliminate a type class if we can prove that this type class follows
from the given assumptions (represented by C) under the given set
of proof rules (represented by P), i.e. P ⊢ C ⊃ TC t holds.
Again, elimination has no effect on the translated program.
In the dictionary-passing scheme, we would need to derive a dictionary
out of the type class proof.

We leave it to future work to establish properties such as type
soundness and coherence. We expect that these properties follow
by straightforward application of methods and techniques found in
[27, 25]. For example, Thatte has already proven type soundness
whereas we have verified coherence if CHRs are confluent.

7. Related Work

7.1 Translating Type Classes

Thatte  [27] is the first to propose a type-passing translation scheme
for type classes. His main motivation is to provide an alternative
semantics where type classes can be interpreted co-inductively. For
example, consider the following program.

class Foo a where foo :: a->Int
instance Foo a ⇒ Foo a where foo = foo

Under the standard inductive interpretation of type classes, Foo t
has no meaning for any type t. Simply because we cannot (in-
ductively) rewrite Foo t to some simpler form. Under a co-inductive
interpretation, however, we can give Foo t the “undefined” mean-
ing.

In case we employ the standard dictionary-translating, the
meaning of co-inductive type classes can be explained in terms of
recursive dictionaries.

data Foo a = F (a->Int)
inst :: Foo a → Foo a
inst (F foo) = F foo

d :: Foo t
d = inst d

Among others, such an extension is required in the SYB3 ap-
proach  [18]. In our own work  [26], we formalize recursive instances
in the presence of a dictionary-passing translation scheme.

Thatte’s work does not include superclasses. We show here that
we can naturally support extensible superclasses in a combination
of Thatte’s type-passing translation scheme and our CHR-
based type class framework. The translation scheme we employ
in the CHR-based type class framework (from here-on referred
to as ATO) is in fact a hybrid. Similar to the dictionary-passing
scheme, we assume dictionary constructing functions. Though, in-
stance declarations describe if-and-only-if relations in ATO. For
example, in ATO the statement Print [a] is a correct.
In Haskell, we cannot verify this statement because it is not ob-
vious how to construct a proof (i.e. dictionary) for Print a out of
the proof for Print [a]. In ATO this problem is solved by assum-
ing that dictionary constructing functions are defined for all ground
instances. Of course, to implement such a scheme Thatte’s type-
passing translation method is the natural choice.

Another alternative translation scheme for type classes, similar to
Thatte’s type-passing method, appears also in the work by Pottier
and Gauthier  [24]. They use GADTs (also known as guarded recur-
sive data types  [31]) instead of System F extended with type-case
for the translation. The idea is to represent dictionaries via GADTs.
Here is the dictionary representation of the Print and Eval class
using the GADT notation as available in GHC.

data EvalDict a where
  EInstLit :: EvalDict List
  EInstProd :: EvalDict a (Prod a b)
data PrintDict a where
  PInstLit :: PrintDict Lit
  PInstProd :: PrintDict a (Prod a b)

The above constructor definitions require GADTs because the (out-
put) type changes. They directly correspond to the instance decla-
rations.

As in case of the type-passing translation scheme, we need to lump
together the instance definitions. Here is the translation of the Eval
instances.

eval :: EvalDict a → a→ Int
eval d = case d of
  EInstLit → 
  EInstProd → 

Up to here this looks exactly like Thatte’s type-passing translation.
This is not surprising given that the idea of GADTs can be traced
back to work on intentional type analysis.

However, the GADT-based translation scheme has a slight disad-
vantange when it comes to translating programs with sub-/superclasses.
For example, the program we have seen earlier

f1 :: Print a ⇒ a→ Int
f1 x = eval x

translates to the GADT program

f1 :: PrintDict a ⇒ a→ Int
f1 d = eval (super d) x
The function super to extract superclass from subclass dictionaries is defined as follows.

```haskell
super :: PrintDict a -> EvalDict a
super PInstLit = EInstLit
super (PInstProd a b) = EInstProd (super a) (super b)
```

The disadvantage of the GADT translation scheme is that each time we introduce a new sub-superclass we will need to adapt the definition of super. Though, this is still a local change. Hence, we do not need to recompile function $f_1$. The point is that in a type-passing translation scheme there are literally no changes necessary. Hence, we can argue that for a practical implementation Thatte’s type-passing translation method is the preferred choice over the encoding in terms of GADTs.

### 7.2 Generic Programming

Language extensions such as Generic Haskell [19] and PolyP [13] have direct support for generic functions but do not provide support for open extension. The Clean [23] supports both generic programming with the opportunity to override generic instances via specialization.

Wang, Chen and Khoo [29] apply aspect-oriented programming techniques to support openly extensible generic functions. Their idea is to use type classes to define the generic cases and type-scoped (also known as type-guarded) aspects for extensions. We yet have to work out the precise connections to our work. In [30], Washburn and Weirich propose a similar solution by modeling type class style overloading with aspects.

In [20], Löh and Hinze proposed a simple yet powerful solution to the problem of extensibility on both dimensions of functions and data types. Their source language allows definitions of function clauses and data constructors to be scattered in different modules; and merge them into one by preprocessing. It is obvious that many of the examples in this paper can be encoded in their language. The primary difference between our proposal and their’s is the use of data constructors versus class instances. There are pros and cons for both approaches. For some applications such as extensible eval, a data type encoding of expression appears to be more straightforward. For the others such as gainze, ad-hoc polymorphism avoids clumsy embedding of types into data constructors. Another notable advantage of our approach is static safety. A function called with arguments which has no instances defined on will result in a static error; instead of a run-type pattern matching failure as in Löh’s and Hinze’s system.

Much of our work, concerns explaining how to make type class encodings of the GM and SYB approach “modular” in terms of the alternative concept of extensible superclasses. In this respect, our work can be seen orthogonal to work by Hinze, Löh and Oliveira [10] who explain the “spine-view” underlying the SYB approach. It is interesting to note that they use GADTs in their example programs. As argued above GADTs are nothing else than a convenient source-language notation to mimic a type-passing translation scheme.

### 8. Conclusion

We have given a new perspective how to achieve modularity for the GM and SYB generic programming approach via extensible superclasses. In our opinion, extensible superclasses provide for a much more natural solution compared to previous solutions which require type class abstraction and recursive instances. We have formalized the main aspects of extensible superclasses using a combination of Thatte’s type-passing translation scheme and our own CHR-based framework. There is lots of future work ahead.

The translation scheme behind extensible superclasses demands significant changes to the dictionary-passing translation scheme currently employed in Haskell. We yet have to study the impact Thatte’s translation scheme has on existing compiler optimizations. Realistically, we do not expect that any time soon systems such as GHC will be able to support extensible superclasses. However, the experimental Haskell compiler JHC [15] implements a type class translation scheme that is very close to Thatte’s type-passing method. We consider this as evidence that a specialized Haskell compiler to support modular generic programming via extensible superclasses is feasible in the near future.

Thatte’s type-passing translation scheme resembles method lookup as found in OO languages. Modularity, local modification and separate compilation have been studied in OO languages for years. We hope that we can take advantage of results in this area and plan to pursue this topic in future work.

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### References


