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Row-column Addressed Arrays for Non-destructive Evaluation Applications

James P. Kirkpatrick, Paul D. Wilcox and Robert A. Smith

Abstract—Row-column Addressed (RCA) arrays are 2-d arrays formed by two orthogonal overlapping linear arrays made up of elongated elements. This substantially reduces the number of elements in the 2-d array. Modelled data are used to compare RCA arrays in pulse-echo mode to fully populated 2-d arrays for Non-destructive Evaluation (NDE) applications and an improved beamforming algorithm based on the total focusing method is tested. Improved beamforming has led to a less than half-wavelength diameter conical bottom hole being successfully detected experimentally using an RCA array, with a maximum signal-to-noise ratio of 17.0 dB (3.s.f). The average difference between the −6 dB drop width and the nominal drill bit diameter when sizing flat bottom holes experimentally using RCA arrays is also improved compared to plane B-scan algorithms from (1.29 ± 0.07) mm to (0.23 ± 0.04) mm. These developments demonstrate the advantages of using RCA arrays over conventional fully populated 2-d arrays and provides a basis for their use, and development, in the field of NDE.

I. INTRODUCTION

ROW-COLUMN addressed (RCA) arrays, also called crossed-electrode transducer arrays [1] or top orthogonal to bottom electrode arrays [2], are different from fully populated 2-d arrays in that they are effectively formed by two orthogonally orientated linear arrays with \( n_x \) transmit elements being orthogonal to \( n_x \) receive elements [Fig. 1]. RCA arrays therefore reduce the number of interconnections, compared to a fully populated 2-d array, by a factor of \( \frac{1}{2} n_x \). These reductions enable the production of 2-d arrays that would currently be impossible to manufacture while also reducing the amount of data per frame. Provided the imaging performance of RCA arrays can match that of fully populated 2-d arrays, their benefits are clear.

RCA arrays were originally invented for use in through-transmission Non-destructive Evaluation (NDE) in order to decrease the number of elements in 2-d arrays and increase the scanning speed [3]. In addition to continued use in through-transmission NDE [4], [5], these arrays have been used for different applications including particle manipulation [6], photoacoustic imaging [2], and medical imaging [1]. In the simplest case, image volumes can be formed by using the assumption that each A-scan originates from a fictional element located at the intersection between the physical transmit and receive elements [Fig. 1b] [7]. Recently, a number of developments have been made in the medical ultrasonics field because of increased interest in RCA arrays. Advances in the design of, and imaging using, RCA arrays have been well summarised in [8] and [9] which are currently the two most definitive papers on RCA arrays in the medical field. Improved beamforming techniques for medical imaging are shown and a new apodisation scheme was demonstrated to reduce the edge effects of elongated RCA array elements and significantly increase the imaging quality to a level comparable with fully populated 2-d arrays. Similarly, the field of NDE has made significant advances in the last 20 years with the growing use of arrays in industry as well as the increased adoption of the Total Focusing Method (TFM) [10]. Many of these advances have come about by the use of systematic approaches which combine modelling and experiment. Array models are often used to predict the performance of particular arrays and test potential improvements, as well as being used to assess imaging algorithms in both medical [11] and NDE fields [12], [13].

While advances have been made in both the medical and NDE fields, the use of RCA arrays for NDE has been predominantly limited to through-transmission. However, in 2011, a commercial device which used an RCA array and operated in pulse-echo mode, was produced specifically for NDE. While the device has been introduced into industry, there is a lack of literature covering the use of RCA arrays in pulse-echo mode for NDE applications. In 2014, Wong et al. [7] attempted to use a medical RCA array to demonstrate the use of RCA arrays for NDE. This approach, however, was not representative of the state-of-the-art in NDE because of, for example, the size of the array, the use of a medical array, surface scanning and reference subtraction. Also, results were shown using images with low dynamic range. Although the basic imaging algorithms and arrays for medical and NDE fields are similar, the requirements are often different. These differences include: the importance of signal-to-coherent and signal-to-random noise, the strength and quantity of scatterers and the use of multiple modes. Therefore, with the additional possibility of an NDE specific RCA array, there is a need to review pulse-echo RCA array use for NDE using current state-of-the-art processes and imaging algorithms on representative samples.

The organisation of this paper is as follows. First, the methodology used to obtain modelled and experimental data is introduced. Then, the data processing algorithms for imaging and evaluation, are proposed. Modelling results are evaluated using the Point Spread Function (PSF) in section III-A and
experimental results are compared using the detectability and sizing of holes in acrylic in section III-B. A discussion of the results follows in section IV before conclusions are drawn in section V.

II. METHOD

A. Modelling

A 3-d frequency-domain-linear-time-invariant model was used to calculate the frequency domain array response, \( H_{tr} \), as a function of frequency, \( f \), of an array to a perfect point reflector target located at \( d \) in a semi-infinite homogeneous medium when transmitting and receiving on elements \( t \) and \( r \) respectively. The equation for this approach is given by:

\[
H_{tr}(f) = \Psi_0(f) \frac{1}{|b_t| |b_r|} \Gamma_t(\theta_t, \varphi_t, f) \Gamma_r(\theta_r, \varphi_r, f) \exp \left( -i2\pi f \frac{|b_t| + |b_r|}{\nu_l} \right),
\]

where

\[
\theta_j = \arccos \left( \frac{b_j \cdot \hat{z}}{|b_j|} \right),
\]

\[
\varphi_j = \arctan \left( \frac{b_j \cdot \hat{y}}{b_j \cdot \hat{x}} \right),
\]

\[
b_j = e_j - d,
\]

\( \Psi_0 \) is the input pulse frequency spectrum, \( \Gamma \) is the element directivity function, \( \theta \) and \( \varphi \) are the polar and azimuthal angles between an element with its centre located at \( e \) and the target respectively, \( \nu_l \) is the longitudinal wave velocity in the medium, \( j \) represents either \( t \) or \( r \) for transmit or receive indices respectively and \( i = \sqrt{-1} \). The directivity functions used can be found in [14], [15]. Beam spread and wave propagation are shown explicitly. This model is only valid in the far field of the elements.

The length of the RCA array elements means that targets of interest are likely to be in the near field of the elements. To allow (1) to be used, the RCA array elements were modelled by dividing each element into a number of sub-elements for which the far field assumption was valid. The responses for these sub-elements were calculated, in transmit and receive, before being summed to give the total response for each elongated element combination as shown by the equation:

\[
H_{tr}(f) = \sum_{k,m} H_{k,r_m}(f),
\]

where \( k \) and \( m \) are the sub-element indices for transmit and receive elements respectively. The position of the point reflector was always kept beneath the centre of the array. A 3-cycle Hann-weighted sine wave was used as the input pulse. For consistency with the experimental set-up, acrylic was used as the medium for modelling using \( v_l = 2700 \text{ m s}^{-1} \) and a shear wave velocity of \( v_s = 1100 \text{ m s}^{-1} \). Details of the arrays used in modelling can be found in Table I.

B. Experiment

1) Experimental Array: Data were collected using an RCA array called DolphiCam (DolphiTech, Model CF08). This array first propagates ultrasound through a soft, conformable silicone based coupling pad which is used to make contact with the sample surface so that the array can be used with or without couplant. Details of this array can be found in Table II.

2) Sample: An acrylic test sample was manufactured in order to qualitatively validate the findings of the model. Acrylic was chosen because it closely matches the acoustic properties of Carbon Fibre Reinforced Polymer (CFRP) for which the experimental array was designed. However, unlike CFRP, acrylic is homogeneous which matches the modelling conditions. The test sample was 8 mm thick and contained a 0.3 mm diameter conical bottom hole as well as a series of flat bottom holes with diameters of 1 mm, 2 mm, 4 mm, 5 mm and 10 mm. These were all drilled to a depth of 3 mm, which meant that their tips were approximately 5 mm deep when measured from the front surface with the experimental array. The 0.3 mm conical bottom hole was chosen as the smallest target defect to try to resolve experimentally and is below the half-wavelength diffraction limit of \( \approx 0.4 \text{ mm (1.s.f)} \). The flat bottom holes were chosen because they have a response...
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3) Data Collection: 12 complete and independent datasets were taken for each defect using a single element aperture in transmit and receive. 8 software averages were taken for each A-scan using the manufacturer’s software and no focussing was used when collecting data. A matched filter is used by the device by default [16] to filter the data because of the double square-wave used for excitation. The raw dataset was then extracted from the device and stored along with the calibration stand-off values which were provided by the manufacturer. Couplant was used to improve coupling performance. 12 complete and independent datasets were also taken from areas of the acrylic sample where no holes were located.

C. Imaging

1) General Case: Two main imaging algorithms were used for both modelled and experimental data. These were the Plane B-scan and TFM algorithms, which are unfocussed and focussed algorithms respectively. The plane B-scan image, $I$, at a voxel located at $p$, is given by the equation:

$$I(p) = \sum_{t,r} \Lambda_j h_{tr} \left( \frac{|c_t|^2 + |c_r|^2}{v_j} \right),$$

(3)

where

$$c_j = e_j - p,$$

$\Lambda_j$ is the aperture function and $h_{tr}$ is the time domain array response. $\Lambda_j$ is given by the equation:

$$\Lambda_j = \begin{cases} 1 & \text{if } \sqrt{(c_j \cdot \hat{x})^2 + (c_j \cdot \hat{y})^2} \leq \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

(4)

where $a$ is the aperture width. Plane B-scans for RCA arrays were generated using the approximation that an A-scan from any orthogonal transmit-receive combination originated from a fictional element located at the intersection of the physical elements. For example, if the elongation direction of the elements are in the $\hat{y}$ and $\hat{x}$ directions for transmit and receive respectively, then the centre of the fictional element would be given by $e_j = (e_{tx}, e_{rx}, e_{jz})$. Two different Plane B-scan apertures were used. These were $a = \ell_1$ and $a = 4\ell_1$ which are denoted PB1 and PB4 respectively. PB4 was used in order to emulate the default data collection method set by the DolphiCam device which is that 4 elements are used in transmit in order to increase the amount of energy put into the system and, by widening the aperture, to increase the downwards component of the directivity function.

Similarly, the TFM image is given by the equation:

$$I_p = \sum_{t,r} Y_j Y_r h_{tr} \left( \frac{|c_t| + |c_r|}{v_j} \right),$$

(5)

where $Y_j$ is the angle limit function given by the equation

$$Y_j = \begin{cases} 1 & \arccos \left( \frac{c_j \cdot \hat{z}}{|c_j|} \right) \leq \theta_{limit} \\ 0 & \text{otherwise} \end{cases}$$

(6)

and $\theta_{limit}$ is the angle limit at the pixel. The RCA-adapted TFM algorithm uses the shortest time path between the centre line of an element and a voxel in both transmit and receive. This algorithm is preferable for NDE compared to the proposed medical algorithm suggested in [8], [9] because there is no defocussing in post-processing. However, this is at the cost of a lower input energy leading to a decrease in signal to random noise ratio.

One final step was added to all algorithms used, which was to divide the amplitude of every voxel by the number of transmit-receive combinations which contribute to the voxel. This is especially important when there is a combination of angle limiting and a large amount of refraction because the range of contributions per voxel is large. This division by the number of contributions helps to normalise the response from reflectors at different locations. Without it, responses from a reflector at each voxel would vary considerably in amplitude.

When imaging the experimental data, a broadband frequency domain Gaussian filter with $\Delta f = 150\%$ was applied to the raw A-scans before imaging. It was also assumed, for simplicity, that the array was parallel to the interface between the coupling pad and the sample in all directions. In line with this assumption a single value for the thickness of the coupling pad, averaged across the whole array, was calculated using the highest amplitude reflection within the first 50 time points for each A-scan. This coupling pad thickness was used in all the path calculations. Modifications were made to the RCA-adapted TFM algorithm because of refraction due to the interface between the coupling pad and the sample. The interface was discretized and the shortest time path was calculated by finding the shortest time path for each element to interface point to voxel combination in 2-d. Apart from modifications relating to the coupling pad, data from the DolphiCam device

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<td>Array width range</td>
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Similarly, the TFM image is given by the equation:

$$I_p = \sum_{t,r} Y_j Y_r h_{tr} \left( \frac{|c_t| + |c_r|}{v_j} \right),$$

(5)

where $Y_j$ is the angle limit function given by the equation

$$Y_j = \begin{cases} 1 & \arccos \left( \frac{c_j \cdot \hat{z}}{|c_j|} \right) \leq \theta_{limit} \\ 0 & \text{otherwise} \end{cases}$$

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were processed using the imaging algorithms described in (3) and (5). An angle limit of $\theta_{\text{limit}} = 30^\circ$ was used during all experimental implementations of the RCA-adapted TFM algorithm. The imaging algorithms were also applied to the datasets taken on areas of the sample which did not contain holes. These images were used to quantify the Root Mean Squared (RMS) noise.

2) Simplification and Computational Efficiency: On a regular Cartesian imaging grid, symmetry reduces (5) for RCA arrays to two 2-d path calculations, one in transmit and one in receive, which improves the speed of the algorithm. Both are calculated using (5) in 2-d as if there were two linear arrays perpendicular in orientation with one in transmit and one in receive. For example, if the elongation direction of the elements are in the $\hat{y}$ and $\hat{x}$ directions for transmit and receive respectively, the transmit path calculation would be calculated in the $\hat{y}$ plane (the plane for which $\hat{y}$ is the normal) and the receive path calculation in the $\hat{x}$ plane, as shown schematically in Fig. 1b. Where $n_{x_t} = n_{y_r}$ and $n_{y_t} = n_{x_r}$ symmetry simplifies the problem further and improves the speed of the algorithm. In this case, only $n_{x_t} \times n_{y_t} \times n_{y_r}$ paths need to be calculated. This is substantially less than the number of calculations required for full 3-d path calculations for every element for fully populated 2-d arrays using the TFM algorithm.

D. Evaluation

1) Point Spread Function Evaluation: The PSF was calculated for both types of array and all imaging algorithms considered by applying the imaging algorithms to the modelled data. The PSF was used to compare the imaging performance of both fully populated 2-d arrays and RCA arrays and their respective imaging algorithms. A dB scale, normalised to the maximum amplitude of the PSF, was used. The imaging performance was quantified using a measurement of the $-6\,\text{dB}$ drop volume of the PSF, $V_{-6\,\text{dB}}$, which was calculated by summing the number of voxels with an amplitude greater than $-6\,\text{dB}$ and multiplying it by the volume of one voxel, $V_p$. The uncertainty in this value, $\alpha_{V_{-6\,\text{dB}}}$, was estimated using the maximum uncertainty in the volume of a sphere placed in a regular 3-d Cartesian grid, which was calculated using the relationship between the surface area and volume of a sphere, yielding an uncertainty of $\alpha_{V_{-6\,\text{dB}}} = \left(\frac{2}{3}\pi V_p V_{-6\,\text{dB}}^2 \right)^{\frac{1}{3}}$.

![Fig. 2. The automated procedure used to size the flat bottom holes. (a) The maximum amplitude within the region of interest was located. (b) All amplitudes below the 15 dB detectability threshold were then disregarded by setting them to zero and all voxels that satisfied the condition $|p - p_{\text{max}}| < 0.05 \, \text{mm}$ were selected. (c) The amplitudes of these voxels at each $(p_x, p_y)$ location were condensed into a single amplitude by taking the maximum and all locations with an amplitude greater than 6 dB below the maximum amplitude were selected. (d) The $-6\,\text{dB}$ drop width was given by the maximum distance between any two of these locations.](image-url)

### TABLE II

<table>
<thead>
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<th>Parameter Name</th>
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<td>Bandwidth ($-6,\text{dB}$)</td>
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<td>-</td>
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$^1$ Approximate values estimated from the frequency spectrum provided.
2) **Detectability and Sizing**: The datasets that were taken on areas of the sample where no holes were located were imaged using the imaging algorithms described above. The RMS of the image voxels in the range ($-6 \text{ mm} \leq p_z \leq -4 \text{ mm}$) was calculated for each dataset and for each imaging algorithm. This range was selected in order to calculate the RMS noise value around the defect location ($p_z \approx -5 \text{ mm}$). The mean of the 12 datasets was taken to give a single RMS noise value for each imaging algorithm. An arbitrary conservative detectability threshold of 15 dB above the RMS noise level was set for successful detection of a defect. Following detection, the flat bottom holes were sized using the following automated procedure [Fig. 2]. First, the location of the maximum amplitude in the image, $p_{\text{max}}$, was located within the region of interest [Fig. 2a]. The region of interest was selected to be ($-13 \text{ mm} \leq p_x \leq 13 \text{ mm}$), ($-13 \text{ mm} \leq p_y \leq 13 \text{ mm}$) and ($-6 \text{ mm} \leq p_z \leq -4 \text{ mm}$). All amplitudes below 15 dB were then disregarded by setting them to zero because the confidence that they originate from a defect is low. All voxels within the region of interest that satisfied the condition $(|p - p_{\text{max}}| - 2) \leq 0.05 \text{ mm}$ were selected in order to take account of any small deviation in surface height or orientation of the flat bottom hole [Fig. 2b]. The amplitudes of these voxels for different $p_z$ at each $(p_x, p_y)$ location were condensed into a single amplitude by taking the maximum [Fig. 2c]. Following this, all locations with an amplitude greater than 6 dB below the maximum amplitude were stored [Fig. 2c]. Lastly, the $-6 \text{ dB}$ drop width was given by the maximum distance between any two of these locations [Fig. 2d]. This process was repeated for all 12 datasets for each defect. The mean was used to give a best estimate of the $-6 \text{ dB}$ drop width and the standard error was used to quantify the uncertainty.

**III. RESULTS**

**A. Modelling**

Example PSF B and C-scan images for a target at $d = (0, 0, -5) \text{ mm}$ for the $n_e = 41$ case for both types of array and all imaging algorithms considered are shown in Fig. 3 and Fig. 4 respectively. The images are shown using a dB scale which is normalised to the maximum amplitude of the PSF and has a 40 dB dynamic range. The PSF was calculated for a target at $d = (0, 0, -5) \text{ mm}$ as a function of $n_e$ for both types of array and all imaging algorithms considered. $V_{-6 \text{ dB}}$ as a function of $n_e$ is shown in Fig. 5.

**B. Experiment**

The responses of the DolphiCam array to holes located at a depth of approximately 5 mm in acrylic were measured and imaged for all imaging algorithms considered. Example B and C-scan images for every hole considered are shown in Fig. 6 and Fig. 7 respectively. The images are shown using a dB scale which is normalised to the amplitude of the RMS noise and has a 40 dB dynamic range. The maximum value on the scale is equal to the maximum amplitude within the range ($-6 \text{ mm} \leq p_z \leq -41 \text{ mm}$). Finally, the DolphiCam array responses, imaged using all imaging algorithms considered, were used to size the diameters of the flat bottom holes. The $-6 \text{ dB}$ drop widths of the flat bottom holes are compared to the drill bit diameter used to make them in Fig. 8. The average difference between the $-6 \text{ dB}$ drop width and the nominal drill bit diameter across the whole range of flat bottom holes was $(1.6 \pm 0.1) \text{ mm}$, $(1.29 \pm 0.07) \text{ mm}$ and $(0.23 \pm 0.04) \text{ mm}$ for PB1, PB4 and TFM imaging algorithms respectively.

**IV. DISCUSSION**

**A. Point Spread Function Evaluation**

It is shown from the results in Fig. 3 and Fig. 4 that fully populated 2-d arrays provide a tighter PSF than RCA arrays. For a given $n_e$, $V_{-6 \text{ dB}}$ is shown to be smaller for fully populated 2-d arrays in all cases [Fig. 5a]. It can be clearly seen that the TFM based imaging algorithms have a narrower PSF and $V_{-6 \text{ dB}}$ than the PB1 and PB4 algorithms in all cases. This is as expected because the PB1 and PB4 algorithms are both unfocussed whereas the TFM based imaging algorithms are focussed. For low values of $n_e$, where the PSF extends outside of the array aperture, the division of the amplitude by the number of transmit-receive combinations which contribute to a voxel causes $V_{-6 \text{ dB}}$ to be artificially enlarged. At larger values of $n_e$, where the PSF is beneath the array, the results become stable.

It is shown in Fig. 5b that the TFM algorithms for the fully populated 2-d array and the RCA array quickly become within error of each other after this point. From $n_e = 65$, the difference in $V_{-6 \text{ dB}}$ is only $(0.2 \pm 0.9) \text{ mm}^3$. In terms of $V_{-6 \text{ dB}}$, the TFM imaging performance of the fully populated 2-d array and the RCA array can be considered comparable after this point. However, it can be seen from Fig. 3 and Fig. 4, that outside of the $V_{-6 \text{ dB}}$, the PSF for the RCA array is extended in all directions compared to the fully populated 2-d array. This is expected to result in a decrease in Signal-to-noise Ratio (SNR) [17], and therefore detectability, when using RCA arrays as opposed to fully populated 2-d arrays experimentally. This is because of the defocusing in one direction in both transmit and receive.

It has also been shown that the PB4 algorithm produced a smaller $V_{-6 \text{ dB}}$ compared to the PB1 algorithm. This was in agreement with the prediction that, due to the increase in aperture size, the directivity function is narrower. The limit reached by the PB1 and PB4 algorithms in Fig. 5a is because, once the $V_{-6 \text{ dB}}$ is beneath the array, any elements added to the side of the array do not contribute to $V_{-6 \text{ dB}}$. For the TFM-style algorithms, the $V_{-6 \text{ dB}}$ continues to decrease as more elements are added, but tends towards a limit as additional elements contribute less and less to the $V_{-6 \text{ dB}}$ due to both directivity and beam spread.

**B. Detectability**

The SNR for the smallest hole considered, the 0.3 mm conical bottom hole, was improved using the RCA-adapted TFM algorithm. This improvement led to the response being above the 15 dB detectability threshold, while the next highest response, using the PB4 algorithm, was below the threshold.
The SNR benefits of the RCA-adapted TFM algorithm also decrease as the hole diameter increases. This is expected to be because the orientation of the flat bottom holes is parallel to the sample surface and so the PB1 and PB4 algorithms are aligned to the high amplitude specular reflection from the flat bottom hole. The RCA-adapted TFM algorithm uses a much larger set of A-scans which contain, in addition, many other signals which contribute very little apart from noise.

The responses from the holes in Fig. 6 and Fig. 7 do not show the side lobes seen in Fig. 3 and Fig. 4 for the PB1 and PB4 algorithms. This is because of the refraction at the interface between the coupling pad and the sample. At this interface, the critical angle \( \theta_{\text{crit.}} \approx 20.6^\circ \) (3.s.f) for longitudinal waves. Therefore the coupling pad not only stops the edge waves propagating into the sample below the centre of the array, but also increases the applicability of the intersection approximation used for the PB1 and PB4 algorithms by narrowing the directivity function in terms of the waves propagating into the sample. The coupling pad also reduces the benefits of a broad directivity function used by the RCA-adapted TFM algorithm. In the absence of a coupling pad, the literature shows that side lobes have been successfully reduced using element apodisation outside the main array area [8], [9]. This would allow the use of the broad directivity function of the elements to be maximised for the RCA-adapted TFM algorithm.

The extension of the response, in terms of depth, from the holes seen for all algorithms in Fig. 6 is caused by the long length in time of the transmission pulse, which is approximately 5-cycles. The extension of the response, in terms of width in the \( p_y \) direction, from the 4 mm, 5 mm and 10 mm diameter holes seen in all algorithms in Fig. 7 is caused by the transducer design asymmetry, that is that each set of electrodes above and below the piezoelectric material are orthogonal.

C. Sizing

Finally, the accuracy and precision of sizing flat bottom holes in acrylic has been improved by using the RCA-adapted TFM algorithm. The average difference across the whole range of hole diameters has been improved by a factor of 5.61 (3.s.f). Fig. 8 shows that the RCA-adapted TFM algorithm consistently performs better than the PB1 and PB4 algorithms across the full range of hole diameters.

The systematic error in the \(-6\,\text{dB}\) drop width for the PB1 and PB4 algorithms towards the higher diameters of holes is caused by the beam profile not being circular for the RCA array. The PB1 and PB4 algorithms behave as expected towards the lower diameters of holes as they reach a sizing limit roughly proportional to their beam size [18]. Accurate values of beam size are difficult to calculate because of the refraction at the coupling pad.

The sizing limit for the RCA-adapted TFM algorithm was not reached and so cannot be conclusively stated in this paper. However, it cannot be below the half-wavelength diffraction limit.

V. Conclusion

The imaging performance of fully populated 2-d and RCA arrays was compared using frequency domain modelling tech-
Fig. 4. C-scan images at \( p_z = -5 \text{ mm} \) of the PSF with a target at \( d = (0, 0, -5) \text{ mm} \) for \( n_e = 41 \) from modelled data. Fully populated 2-d and RCA arrays are shown from top to bottom respectively. PB1, PB4 and TFM imaging algorithms are shown from left to right respectively.

Fig. 5. The \(-6 \text{ dB}\) drop volume of the PSF at \( d = (0, 0, -5) \text{ mm} \) as a function of \( n_e \) from modelled data for (a) all imaging algorithms and arrays, (b) for TFM based imaging algorithms only. A reduced number of error bars are shown for clarity.

Three different imaging algorithms were applied to the modelled data in order to obtain the PSF. These consisted of two plane B-scan algorithms, with apertures of \( a = \ell_1 \) and \( a = 4\ell_1 \), and TFM algorithms. A RCA-adapted TFM algorithm was proposed in order to take account of the geometry of RCA arrays. The \(-6 \text{ dB}\) drop volume of the PSF was used as the comparison metric to compare the imaging performance. For a given \( n_e \), RCA arrays were found to perform worse than fully populated 2-d arrays for imaging in all cases. However, the imaging performance of RCA arrays was comparable to that of fully populated 2-d arrays when using the RCA-adapted TFM algorithm. The imaging performance of RCA arrays was compared experimentally for the same three imaging algorithms. The RCA-adapted TFM algorithm was shown experimentally to improve the imaging performance in respect to the detectability of a very small defect. The RCA-adapted TFM algorithm also improved the accuracy and precision of sizing flat bottom holes. The proposed RCA-adapted TFM algorithm for NDE applications shows that RCA array imaging performance can be comparable to that of fully populated 2-d arrays. This means that the improvements provided by RCA arrays in terms of data volumes, numbers of connections and image-construction times, can be fully utilised in the field of NDE. It is recommended that further work is done to develop...
Fig. 6. B-scan images of a range of holes in acrylic at a depth of approximately −5 mm from experimental data. Increasing defect diameters are shown from top to bottom. CB and FB denote conical bottom and flat bottom respectively. PB1, PB4 and TFM imaging algorithms are shown from left to right respectively.
Fig. 7. C-scan images of a range of holes in acrylic at a depth of approximately $-5\,\text{mm}$ from experimental data. Increasing defect diameters are shown from top to bottom. CB and FB denote conical bottom and flat bottom respectively. PB1, PB4 and TFM imaging algorithms are shown from left to right respectively.
an RCA array without a coupling pad which uses apodised elements in order to maximise the benefits of the RCA-adapted TFM algorithm and minimise energy losses caused by the coupling pad.

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All data used in this paper are openly available for download from the University of Bristol data repository at https://doi.org/10.5523/bris.375rfm9pip3rj2wnd22fncnxvk.

REFERENCES


[15] G. F. Miller and H. Pursey, “The field and radiation impedance of a line element imaging, especially Eskil Skoglund, Fredrik Lingvall and Yngve Raudberget, for providing crucial information about the DolphiCam device, Stephen Iles for his advice and skill in manufacturing the sample, and David Hallam for his help and support during the project. This work was funded by the Engineering and Physical Sciences Research Council (EPSRC) grant number EP/L015587/1 through the UK Research Centre in Non-Destructive Evaluation (RCNDE) with additional funding provided by the Defence Science and Technology Laboratory (DSTL).

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