
Peer reviewed version

Link to published version (if available):
10.1109/PIMRC.2007.4394765

Link to publication record in Explore Bristol Research
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
http://www.bristol.ac.uk/pure/about/ebr-terms
PERFORMANCE ANALYSIS OF LAYERED RANDOM BEAMFORMING OFDMA WITH FEEDBACK REDUCTION

Congzheng Han, Simon Armour, Angela Doufexi, Joe McGeehan
Centre for Communications Research, University of Bristol
Woodland Road, Bristol, BS8 1UB, Bristol, U.K.

Yong Sun
Toshiba Research Laboratory
Queens Square, Bristol, BS1 4ND, Bristol, U. K.

Abstract—This paper presents a downlink performance analysis of a Layered Random Beamforming (LRB) - MIMO-OFDMA Physical Layer (PHY) with feedback reduction as applicable to future generation wireless communication systems. OFDMA is a popular multiple access candidate for future generation cellular communication systems which facilitates multi-user diversity by enabling multiple access in the frequency domain. LRB enables the exploitation of spatial multi-user diversity gain, spatial multiplexing capacity gain and layer spatial multi-user diversity gain, which is achieved by enabling the multiplex of data transmitted simultaneously to different destinations. Unlike a conventional beamforming system, an LRB system only requires effective signal to interference and noise ratios (ESINR) as feedback from every spatial layer of the MIMO channels and thus has potentially lower feedback requirements than a system which requires feedback of more detailed channel information. By combining the LRB technique with OFDMA, LRB-OFDMA can achieve an additional spectral multi-user diversity gain compared to the single carrier LRB system. However, in this case ESINR feedback on a per-sub-carrier basis is required in principle and the feedback requirements may thus increase substantially. This feedback requirement can be reduced by generating the feedback information on a cluster (group of sub-carriers) basis rather than on an individual sub-carrier basis. In this way, the system can exploit any correlation in the frequency response of the channel. The design of an LRB-OFDMA system is presented in this paper and the performance of the system is evaluated for different degrees of feedback reduction using various statistical channel models. A complete list of feedback requirements for various MIMO schemes is also presented.

I. INTRODUCTION

Research on future generation cellular systems has focused on supporting multi-user transmission, providing higher data rates and spectral efficiency and enabling improved coverage and communication reliability. An OFDMA system is one of the most promising PHY and multiple access candidates for future communication systems [1] and Long Term Evolution (LTE) of the Third Generation Partnership Project (3GPP) has already assumed that the downlink of the air interface would be OFDM based [2]. Its performance can be further improved by employing multiple antennas at both transmitter and receiver (MIMO). Compared to MIMO schemes employing Space-Time Block Codes (STBCs) for diversity gain and Spatial Multiplexing (SM) for increased transmission rates, Eigenbeamforming [3,4] is a capacity achieving transmission scheme that utilises singular value decomposition (SVD) and requires full channel state information (CSI) at the transmitter. However, when the channel varies rapidly in time, frequency and space, full CSI requires a significant amount of feedback information and may thus be inefficient or impractical for real time applications.

In a system with many mobile stations, there is likely to be at least one mobile station (MS) whose channel is near its peak at one time and/or frequency, provided different mobile stations experience independent fading channels. Application of a randomly generated beamforming pattern at the transmitter to achieve Opportunistic Beamforming is proposed in [5]. Provided there are many independent mobile stations in the environment, it can effectively exploit multi-user diversity in combination with transmit beamforming to attain the coherent beamforming capacity and only requires the feedback of signal to noise ratio (SNR) (no spatial information is required). By combining the opportunistic beamforming concept and SVD technique, [6] and [7] extend this theory to a single carrier MIMO system and develop the Random Beamforming (RB) and Layered Random Beamforming (LRB) technique respectively. RB is capable of achieving multi-user diversity gain and spatial multiplexing gain and supports one MS transmission at any time/frequency. LRB has been developed for systems employing a linear receiver and it is capable of achieving further multiple access by allowing different spatial layers to be separated and allocated to different MSs simultaneously. Therefore, compared to RB, LRB can achieve an additional layer spatial multi-user diversity gain but at the expense of MIMO order times more feedback. An ESINR feedback metric is proposed in [7] for a system with an MMSE based receiver and it indicates not only the eigenvalues of the MIMO channels and the mismatch between the random precoding matrix and the unitary matrix of the actual MIMO channels, but also spatial information and self-interference caused by the mismatch. Both RB and LRB techniques are applied to OFDMA system in [8] and performance analysis shows that they can achieve an additional spectral multi-user diversity gain. However, feedback is significantly increased since it is required on a per-sub-carrier basis. [9] applies opportunistic beamforming to an SDMA OFDMA system and suggests the possibility of exploiting the correlation of the channel in frequency in the selection of precoding matrices. A similar approach will be considered here for an LRB-OFDMA system.

In this paper, the design of an LRB-OFDMA PHY employing an MMSE receiver is presented. LRB-OFDMA is capable of achieving not only spatial multi-user diversity gain, layer spatial multi-user diversity gain and spatial multiplexing capacity gain (as for single carrier RB systems) but also an additional spectral multi-user diversity gain in frequency selective channels [8]. As OFDM(A) is a multi-carrier technique, generating feedback on a per-sub-carrier, per-OFDM symbol basis is infeasible in reality due the resultant overheads. The effects of frequency response correlation on a cluster- (multiple sub-carriers grouped for
feedback purposes) based random beamforming OFDMA system are presented using both numerical and simulation results. The possibility of grouping the feedback of subcarriers to reduce overall system feedback by exploiting the frequency response correlation is also evaluated in detail.

This paper is organized as follows. Section II describes the PHY model of LRB-OFDMA system. In Section III, the possibility of reducing the feedback required by an OFDMA system is investigated. The numerical analysis of the effect of frequency response correlation on cluster size design is presented and is followed by simulation performance analysis in various statistical channel models. Feedback required by different MIMO systems is also listed and compared. Section IV concludes the paper.

II. PHY MODEL

A. OFDMA

The total bandwidth of an OFDM system is divided into a series of sub-carriers and (provided suitable selection of parameters) each sub-carrier is subject to a flat fading narrowband channel. With proper coding and interleaving across frequencies, OFDM exploits frequency selectivity to its advantage as a source of diversity. For the case of OFDMA, sub-sets of usable sub-carriers can be grouped into multiple sub-channels which are then allocated to different mobile stations for multiple access purposes. A 2x2 MIMO architecture is considered in this paper but the analysis is readily extendible to higher MIMO orders.

B. Random Beamforming OFDMA System

Random beamforming was originally proposed for a single carrier MIMO system in [6] to attain a near optimal channel capacity with reduced feedback in a multi-user environment. [7] extends the analysis to a single carrier MIMO system with an MMSE receiver and proposes an ESINR feedback matrix which includes information on self-interference and noise. It enables different spatial layers of the MIMO channels to be separated and allocated either to one MS for the RB scheme or to different MSs for the LRB scheme.

For RB-OFDMA, a unitary matrix \( V_r \) is generated from the random channel matrix \( H_r \) and it is applied to the subcarriers of the OFDMA signal on a cluster basis (a cluster is considered to consist of an integer number of sub-carriers adjacent in frequency). Different \( V_r \) is generated for different clusters of sub-carriers. The received signal after FFT and guard interval removal becomes:

\[
Y_r = H_{r}V_r X_r + N_r
\]

where a subscript \( k \) denotes a MS index, \( s \) denotes a subcarrier index and \( H_{r}^k \) is a matrix containing MS \( k \)'s frequency responses of the channels between \( N_r \) transmit and \( N_r \) receive antennas at subcarrier \( s \). \( D_{r}^k \) is a diagonal matrix including all the singular values of \( H_{r}^k \), and \( U_{r}^k \) and \( V_{r}^k \) are the unitary matrices obtained by applying an SVD to \( H_{r}^k \). \( X_r^s \) denotes an \( N_r \times 1 \) matrix containing the transmit signals at subcarrier \( s \) at the basestation (BS) and \( N \) represents the additive complex Gaussian noise.

The MMSE filter computed by MS \( k \) for subcarrier \( s \) is:

\[
G_{s}^{k} = \left( (H_{r}^{k} V_{r}^{k})^H \left( (H_{r}^{k} V_{r}^{k})^H + SNR^{-1} \right) \right)^{-1} (H_{r}^{k} V_{r}^{k})^H.
\]

The received signal is multiplied by \( G_{s}^{k} \), and it becomes:

\[
G_{s}^{k} Y_{s}^{k} = G_{s}^{k} H_{r}^{k} V_{r}^{k} X_{r}^{k} + G_{s}^{k} N.
\]

For a 2x2 MIMO system, the MIMO channels have two subspaces that can be considered as 2 data streams transmitting through 2 parallel sub-channels. For data stream \( q \) at every sub-carrier, the MS \( k \) computes the effective ESINR for every data stream for every sub-carrier (the subcarrier index is omitted):

\[
ESINR_{q} = \| (A_{k}^{q} q)^H E_s \| + \| (G_{s}^{k}) q_{q}^{k} N \|^2.
\]

where \( A_{k}^{q} q_{q}^{k} \) denotes the average symbol energy, and \( q_{q}^{k} \) indicates the element located in row \( q \) and column \( j \). In an OFDMA system, feedback from every sub-carrier in a cluster will be required by the BS. To reduce the feedback for the RB-OFDMA system, every MS calculates the average data rate across all subcarriers in each cluster and sends it to the BS through the feedback channel. For cluster \( c \), if the index of the starting subcarrier is \( n \) and finishing subcarrier is \( m \), then the feedback of cluster \( c \) from MS \( k \) in frequency domain is:

\[
r_{k,c} = \frac{1}{m-n} \sum_{m} \sum_{q} \log_{2}(1 + ESINR_{q}^{k,c}).
\]

The BS then uses this feedback information to allocate the common channel of this cluster to the MS with the best \( r_{k,c} \). Alternatively, a proportional fair algorithm could be employed where the resources are allocated to the MS when its channel is near its own peak. In this paper, a linear MMSE receiver is adopted for the RB-OFDMA.

C. Layered Random Beamforming OFDMA System

For the LRB scheme, the different spatial layers can be allocated to a different MS to achieve an additional spatial multi-user diversity gain although it increases the feedback according to the number of spatial layers (MIMO order) compared with RB scheme. Compared to RB-OFDMA, LRB-OFDMA requires feedback from every spatial layer of the MIMO channels on a cluster basis. The MS \( k \) calculates the data rate of each spatial layer on a cluster basis by averaging the data rates of the sub-carriers in the frequency domain, as:

\[
r_{k,c} = \frac{1}{m-n} \sum_{m} \log_{2}(1 + ESINR_{q}^{k,c}).
\]

For every cluster, the BS allocates each spatial layer of the common channel to the MS whose channel conditions of the corresponding layer are best.

III. LAYERED RANDOM BEAMFORMING OFDMA WITH FEEDBACK REDUCTION

A. System Setup and Channel Models

The key parameters which are used in the simulation of the
system in this paper are the same as those in [10] and are shown in Table 1.

<table>
<thead>
<tr>
<th>Operating Frequency</th>
<th>5 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>100 MHz</td>
</tr>
<tr>
<td>FFT Size</td>
<td>1024</td>
</tr>
<tr>
<td>Useful Sub-carriers</td>
<td>768</td>
</tr>
<tr>
<td>Guard Interval Length</td>
<td>176</td>
</tr>
<tr>
<td>Sub-carrier Spacing</td>
<td>97.656 kHz</td>
</tr>
<tr>
<td>Useful Symbol Duration</td>
<td>10.24 µs</td>
</tr>
<tr>
<td>Total Symbol Duration</td>
<td>12.00 µs</td>
</tr>
<tr>
<td>Channel Coding</td>
<td>Punctured 1/2 rate convolutional code, constraint length 7, {133,171}</td>
</tr>
</tbody>
</table>

LRB-OFDM and LRB-OFDMA are simulated using a MIMO implementation of the channel A and E of the ESTI BRAN channel models [11]. The ESTI BRAN channel models have a sampling period, \( T_s = 10 \text{ns} \) and the rms delay spread \( \tau_{\text{rms}} \) increases from 50\( \text{ns} \) for model A to 250\( \text{ns} \) for model E. In this paper, two transmission modes of a possible link adaptive scheme [10] are considered and they are mode 1 employing BPSK with 1/2 coding rate and mode 6 employing 64QAM with 3/4 coding rate.

B. Best User Selection Criterion (Greedy Algorithm)

For the proposed OFDMA system, \( N_c \) data subcarriers are grouped into a number of clusters depending on the cluster size. The base station allocates each spatial layer of each cluster to one of the mobile stations depending on the feedback. A greedy algorithm is employed for LRB-OFDMA in order to maximize the overall system throughput and the details are described below:

Step 1: For every spatial layer of the MIMO channel, the BS collects the numerical average data rate \( r^c_{K} \) of all subcarriers in every cluster from every MS, where \( q \) is the spatial layer index.

Step 2: For spatial layer \( q \) of cluster \( c \), the MS \( K \) with the highest \( r^c_{K} \) is scheduled for transmission to maximize overall the system throughput.

\[
r^c_{K} = \max \{ r^c_{1}, r^c_{2}, \ldots, r^c_{K}, \ldots \} \quad (7)
\]

C. Exploiting Frequency Correlation Among Sub-carriers

For the LRB-OFDM system, feedback is needed for every subcarrier at every spatial layer of the MIMO channel and hence the feedback information becomes significant. In order to reduce this requirement, feedback from subcarriers can be grouped. The frequency responses of the subcarriers in one cluster of an OFDM symbol are correlated and the degree of correlation depends on the coherence bandwidth of the channel in frequency domain, which is inversely proportionally to the rms delay spread of the channel in the time domain. The degree of correlation among adjacent subcarriers is also affected by the subcarrier spacing and cluster size. To utilise this frequency correlation property, feedback for an LRB-OFDM system can be on a cluster basis and the cluster size can be appropriately chosen so that the feedback can be reduced without much degradation in throughput performance. The correlation between the effective channel gain of sub-carrier \( m \) and \( n \) in cluster \( c \) can be derived from [12]:

\[
r_{mn} = \frac{1 - e^{-\frac{\tau_{\text{rms}}}{\tau_s}}}{1 - e^{-\frac{\tau_{\text{rms}}}{\tau_s}}} \left( 1 + j2\pi \frac{m-n}{T_N} \right) \quad (7)
\]

where \( L \) is the length of guard interval, \( T_s \) is the sampling period, \( \tau_{\text{rms}} \) is the expected rms delay spread and \( N_s \) is the number of data sub-carriers.

Fig. 1 shows the correlation coefficient between the effective channel gains of the first and last subcarrier in a cluster as the cluster size increases in various ESTI BRAN channel scenarios [11]. The cluster size increases as \( m - n \) becomes larger. For channel models experiencing different \( \tau_{\text{rms}} \), as \( \tau_{\text{rms}} \) increases, the frequency responses of the subcarriers in one cluster become less correlated. For all the channel models, the frequency response correlation of the subcarriers decreases as the cluster size increases. If the correlation of the frequency responses of the subcarriers in one cluster is high (correlation coefficient is higher than 0.5), a randomly generated precoding matrix matching one subcarrier well is most likely to match the other subcarriers in that cluster well, too. As a result, a cluster of suitable size can be used as a feedback unit without degrading the overall throughput performance much relative to the case of minimum cluster size.

D. Reduced Feedback Layered Random Beamforming OFDMA Exploiting Frequency Correlation

The performance analysis in [8] suggests that LRB-OFDMA outperforms RB-OFDMA due to an additional layer spatial multi-user diversity gain. For this reason, the performance analysis and results presented in this section consider only LRB-OFDMA. Similar simulation has also been performed for RB-OFDMA and the results show that the effect of frequency response correlation on the performance of LRB-OFDMA also applies for the RB-OFDMA case.

Adopting different cluster sizes, the average numerical data rates of LRB-OFDMA at SNR = 10dB are presented in Fig. 2 to show the impact of cluster size on system performance in channel scenarios A and E. The average numerical data rates for different cluster sizes as the number of MSs increases are
calculated using equation (6) and selected using the greedy algorithm. In channel A, the average numerical data rate of cluster size up to 16 is still nearly identical to cluster size 1 due to the high frequency correlation of the subcarriers in the clusters (as shown in Fig. 1). Compared to channel A, channel E is much more frequency selective because of the higher rms delay spread. As a result, the correlation coefficient decreases much faster as the number of subcarriers per cluster increases. Only small cluster sizes up to 8 have performance close to that of a cluster size of 1. In addition, in channel E, the data rate drops faster than the same system in channel A as the cluster size increases further. The data rate difference between cluster size 256 and OFDM is very low in channel E, and it shows that the frequency correlation of the subcarriers approaches to the minimum when cluster size reaches 256. On the other hand, there is some frequency diversity that can be exploited when the cluster size reaches 256 in channel A.

To verify the numerical analysis on the impact of cluster size on system performance, Fig. 3 shows the BER performance of LRB-OFDMA in channel A with various cluster sizes when employing transmission mode 1. Since the correlation of frequency responses for up to 16 subcarriers is still high (as shown in Fig. 5 and 6) the BER performances of cluster size 1, 4, 8 and 16 are very close. Therefore, a well matched random beamforming matrix to one subcarrier is also very likely to be good for its adjacent subcarriers. The correlation of subcarriers in one cluster decreases as the cluster size increases due to the frequency selectivity of the channel and the BER performance degradation is approximately proportional to the increase in the subcarrier number per cluster. When the cluster size becomes very large and correlation coefficient falls well below 0.5, the BER performance of the OFDM system approaches that of the OFDMA system.

By considering a target BER of $10^{-3}$, the relationship between cluster size and system performance can also be demonstrated by plotting the cluster size versus the required SNR for different transmission modes in different channels as in Fig 4. When transmission mode 1 is employed, for a small cluster size, LRB-OFDMA in channel E has a steeper slope than the same system in channel A. This is because the frequency responses of the subcarriers diverge faster in channel E. As the cluster size increases and finally reaches the maximum shown in Fig. 4, the slope of channel E increases very slowly and nearly becomes flat when the cluster size is larger than 64. In contrast, there is a gradual increase in the required SNR until the cluster size reaches the maximum in channel A. This result shows that there is still some level of frequency correlation that can be used for grouping feedback information for larger cluster sizes and there is still some performance gain compared to the LRB-OFDM system (a special case of LRB-OFDMA system which has all data subcarriers in one cluster only).

The second part of Fig. 4 shows the performance of the same system in different cluster sizes adopting mode 6 in both channel A and channel E. As the cluster size increases, the required SNR for the system employing mode 6 increases more significantly compared to mode 1. This trend can be observed from Fig. 4 that the slope of required SNR for increasing cluster size is slightly steeper for mode 6 than mode 1 in both channel A and E. This is because the higher modulation and weaker coding make mode 6 more sensitive to errors and change in channel conditions. For larger cluster sizes, the increasing SNR required by LRB-OFDMA employing mode 6 in channel A still shows a higher frequency response correlation than the same system in channel E, which loses most of the frequency diversity gain after the cluster size reaches 64.

Both numerical and simulation results analysis suggests that based on the channel conditions, subcarriers of LRB-OFDMA can be grouped into clusters to attain a close performance to the conventional LRB-OFDMA system but with reduced feedback. With little performance degradation,
the subcarriers can be grouped into clusters as feedback units if the correlation coefficient is as high as 0.5. In real system design, cluster size may be affected by other system parameters. Generally, lower modulation and stronger coding schemes are more robust to errors, and therefore can support larger cluster sizes.

E. Feedback Required by Different MIMO Schemes

Table 2 gives a summary of the amount of feedback required to perform different MIMO schemes where RF stands for reduced feedback. Eigenbeamforming utilizing SVD, which is the capacity achieving MIMO scheme, offers the best performance of all but requires feedback of the full channel matrix. Compared to random beamforming systems achieving spatial multi-user diversity gain, single user SM-MMSE OFDM requires no feedback but gives the worst performance [8]. Although RB-OFDM requires the same amount of feedback as single carrier RB system, its numerical data rate is much lower than RB-OFDMA which is capable of fully exploiting the spectral multi-user diversity gain and supporting instantaneous multi-user transmission. Compared to RB schemes, LRB is capable of supporting multi-user transmission at any time and frequency and achieves an additional layer spatial multi-user diversity gain resulting in better numerical data rate [8] at the expenses of MIMO order times more feedback. Since feedback is required from every subcarrier of an OFDM symbol, both RB-OFDMA and LRB-OFDMA face a significant trade off between feedback and performance, especially for LRB-OFDMA. This feedback can be reduced by generating the feedback information on a cluster (group of sub-carriers) basis rather than on an individual sub-carrier basis while still maintaining a good throughput performance.

IV. CONCLUSIONS

By employing the Layered Random Beamforming technique in combination with OFDMA, a next generation system would be capable of effectively exploiting spatial multiplexing capacity gain as well as spatial, layer and spectral multi-user diversity gain. This is achieved at the expense of a requirement, in principle, to have feedback of ESINR information for every sub-carrier. Instead of having feedback for every subcarrier, frequency response correlation among subcarriers can be exploited to group adjacent subcarriers into clusters to reduce the actual feedback required while still maintaining good throughput performance. The numerical and simulated analysis presented in this paper has shown that this feedback reduction can be achieved to a degree without undue penalty in terms of system performance provided that certain channel dependent constraints are respected.

The authors wish to acknowledge the financial support of Dorothy Hodgkin Postgraduate Awards (DHPA) and Toshiba Research Europe Limited (TREL) and to thank Dr. Magnus Sandell of TREL for his technical input.

REFERENCES


<table>
<thead>
<tr>
<th>MIMO Scheme</th>
<th>System</th>
<th>Subcarrier Allocation</th>
<th>Cluster Size</th>
<th>Feedback Order</th>
<th>No. of MSs in the environment</th>
<th>No. of MSs Supported Simultaneously</th>
<th>Feedback From Every MS k</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>OFDM</td>
<td>S</td>
<td>S</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SVD</td>
<td>Single Carrier</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>K</td>
<td>1</td>
<td>$N_x N_s$</td>
</tr>
<tr>
<td>SVD</td>
<td>OFDM</td>
<td>S</td>
<td>S</td>
<td>1</td>
<td>K</td>
<td>1</td>
<td>$N_x N_s$</td>
</tr>
<tr>
<td>SVD</td>
<td>OFDMA</td>
<td>S</td>
<td>C</td>
<td>1</td>
<td>K</td>
<td>K</td>
<td>$N_x N_s$</td>
</tr>
<tr>
<td>RB</td>
<td>Single Carrier</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>K</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>RB</td>
<td>OFDM</td>
<td>S</td>
<td>S</td>
<td>1</td>
<td>K</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>RB</td>
<td>OFDMA</td>
<td>S</td>
<td>C</td>
<td>1</td>
<td>K</td>
<td>K</td>
<td>S</td>
</tr>
<tr>
<td>RB (RF)</td>
<td>OFDMA</td>
<td>S</td>
<td>C</td>
<td>1</td>
<td>K</td>
<td>K</td>
<td>(S/C)</td>
</tr>
<tr>
<td>LRB</td>
<td>Single Carrier</td>
<td>1</td>
<td>1</td>
<td>N</td>
<td>K</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>LRB</td>
<td>OFDM</td>
<td>S</td>
<td>S</td>
<td>N</td>
<td>K</td>
<td>1</td>
<td>N</td>
</tr>
<tr>
<td>LRB</td>
<td>OFDMA</td>
<td>S</td>
<td>C</td>
<td>N</td>
<td>K</td>
<td>K</td>
<td>N(S/C)</td>
</tr>
</tbody>
</table>

Table 2 Feedback Comparisons among Various MIMO Schemes in a Multi-user Environment