
Peer reviewed version

Link to published version (if available):
10.1109/ACSSC.2007.4487375

Link to publication record in Explore Bristol Research
PDF-document

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EWF codes - generalization of LT codes:

Let us assume that the numbers \( k_1, k_2, \ldots, k_r \) such that \( k_1 < k_2 < \cdots < k_r = k \) determine the partition of the message block of length \( k \) into the groups of input symbols named windows, such that the first \( k_1 \) input symbols belong to the \( 1 \)-th window. Using windows, we divide the block into the groups of input symbols of unequal importance. The most important class consists of the first \( k_1 = k_1 \) input symbols in the block, the class of secondary importance consists of the next \( k_2 - k_1 \) input symbols and, in general, the class of the \( i \)-th order of importance, \( 2 \leq i \leq r \), is made of \( s_i \) input symbols with indices \( k_1 + 1, \ldots, k_i \). We describe the division into importance classes using generating polynomial \( \Omega(x) = \sum_{i=1}^{r} k_i x^i \), where \( k_i = \frac{k_i}{k} \). We define EWF code \( F_{EWF}(U, F, \Omega_1(x), \ldots, \Omega_r(x)) \) as a fountain code which assigns each output symbol to the \( j \)-th window with probability \( \Gamma_j \) and encodes chosen window using the LT code with distribution \( \Omega_j(x) = \sum_{i=1}^{r} k_i x^i \).

By generalizing the and-or lemma [5], we obtain the erasure probability evolution for input nodes of EWF codes deduced iteratively, as stated in the following lemma.

**Lemma 1.** For EWF code \( F_{EWF}(U, F, \Omega_1(x), \ldots, \Omega_r(x)) \), the probability \( y_{i,j} \) that the input node of class \( j \) is not recovered after \( i \) iterations of belief propagation algorithm applied at the overhead \( i \) is

\[
y_{i,j} = \frac{1}{\Gamma_j} \left( 1 - \exp \left( -1 + e \sum_{i=1}^{r} \Gamma_i \Omega_i^{(1)}(1 - \frac{1}{2^{s_i-1} \Pi_{r-1}}) \right) \right).
\]

This result allows us to compare asymptotic BER of EWF codes with that of weighted LT codes for the case of two importance classes.

**Conclusion:**

- We have constructed EWF codes - rateless codes which provide UEP and URT erasure correction properties which use windowing of the data set instead of weighted approach as in previously studied UEP rateless codes.
- We derived lower and upper bounds on ML decoding of EWF codes which confirm their advantages hold under ML decoding.
- Simple precoding scenario allows for targeting the average overhead necessary for successful decoding of different importance classes - strong URT property can be achieved.

**Simulation results:**

- A message block of length \( k = 3000 \) with two importance classes, denoted as More Important Bits (MIB) and Less Important Bits (LIB), with \( U_1 = 0.1 \).
- Using Lemma 1, choose the optimal distribution \( \Pi \) such that at a fixed overhead MIB BER is minimized, and LIB BER is in the order of magnitude of BER of UEP fountain codes.
- The choice of distribution \( \Omega(z) \) used on the MIB window provides an additional design parameter and enables EWF codes to outperform the weighted UEP fountain codes in terms of MIB BER.
- Precoding of EWF codes can be done separately for each importance class and thus, via choice of \( \Pi \), we can target the overheads at which full successful decoding of MIB symbols and LIB symbols is expected.
- This way, one can achieve strong URT property necessary for delay-constrained applications - even users with severe channel conditions may be guaranteed the successful reception of the base layer (MIB) [7].

**References:**


**Aim:** We introduce and study novel class of fountain codes for erasure channels with unequal error protection (UEP) and unequal recovery time (URT) properties, using a windowing strategy.