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FOUNTAIN CODING WITH DECODER SIDE INFORMATION

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Outline

- Review of fountain codes
- Distributed joint source channel coding (DJSCC) multicast: a priori side information about source at each receiver
- Source-channel fountain code design
  - Analytic relationship of code parameters for good channel and good source-channel fountain codes
  - Optimization of code parameters
- Reduction of the problem with systematic Raptor design
- Simulation results
Fountain codes: TX

transmitter produces a potentially infinite number of fountain encoding symbols (random, equally important descriptions of the source) and sprays them across the channel

[Byers, Luby, Mitzenmacher, Rege 1998]
Fountain codes: RX

receiver collects encoding symbols and attempts decoding when "enough" symbols are received

$k$ : size of information sequence
$k^\prime$ : "enough"

*erasure channel:* $k^\prime$ should be only slightly larger than $k$

*noisy channel:* $k^\prime$ should be only slightly larger than $k / \text{Capacity}$
Sparse graph fountain codes

- LT codes [Luby, 1998-2002]
- Raptor codes [Shokrollahi, 2003-2006]
Assumptions

• Each receiver has access to some side information \( Y \) correlated with \( X \).
• We model side information as if \( Y \) was the output of some virtual communication channel when \( X \) (uncoded) is its input.
• A single fountain code for both source compression and error correction near the optimal source-channel code rate of \( \frac{H(X|Y)}{\text{Capacity}(C)} \).
System model
Case 1: Virtual channel as the BEC

- Each receiver has knowledge of some portion of the information sequence.
- The transmitter does not know which part of the data is available at the receiver, but is able to estimate how much of the data is already known.
Using side information in decoding

Removal of the known input nodes from the graph changes the degree distribution at the output nodes!

\[ \Omega(x) = \Phi(1 - p + px) \]
Using relationship between incoming and resulting distribution

- The design goal is to choose such incoming distribution $\Phi$ so that the resulting distribution $\Omega$ behaves like a good fountain output symbol degree distribution.
- Numerical methods - nonnegative least squares approximation.
- Linear programming optimization of $\Phi$ using density evolution of asymptotic BER.
Asymptotic performance of the optimized dd’s
Precode + source-channel LT code

$k = 40000, p = 0.3$, actual channel: BEC

34% of the length of information sequence on average suffices
Related work


Applications: data synchronization scenarios
Case 2: Virtual channel as the BIAWGNC

- $\mathbf{Y} = \mathbf{X} + \mathbf{N}$, $\mathbf{N} = \mathcal{N}(0, \sigma^2)$
- The transmitter does not have access to $\mathbf{Y}$, but is able to estimate the noise variance of the virtual channel, i.e., the correlation between $\mathbf{X}$ and $\mathbf{Y}$.
- The receiver embeds the apriori “soft information” into the factor graph prior to running sum-product algorithm.

$$
m_{v,f}^{(i)} = \begin{cases} 
L(y_v) & , i = 0, \\
L(y_v) + \sum_{g \neq f} \mu_{g,v}^{(i-1)} & , i \geq 1.
\end{cases}
$$

$$
tanh\left(\frac{\mu_{f,v}^{(i)}}{2}\right) = \tanh\left(\frac{L(z_f)}{2}\right) \prod_{u \neq v} \tanh\left(\frac{m_{u,f}^{(i)}}{2}\right), \ i \geq 0
$$
Optimization of dd for Case 2

- Modification of linear programming optimization of fountain degree distributions [Etesami, Shokrollahi 2006]
- Semi-Gaussian approximation of density evolution [Ardakani, Kschischang 2004]
another way to resolve fountain decoding with DSI:

**systematic raptor**

$G_R = G_{LT} G_C$ is overall Raptor encoding, $G_{LT}$ - truncated LT matrix matrix. Set $G_k$ equal first $k$ rows of $G_R$.

Idea: if $G_k$ is invertible let $\hat{x} = G_k^{-1} x$ be the new input to be processed by standard Raptor.

First $k$ symbols of the output stream $y$ will be the same as the input!

**Diagram:**

- **Encoder**:
  - $c'$ to $c$
  - $c'$ to $e$
  - Processor
  - $Ae = [0^T \ c']^T$
  - LT encoder
  - $c$ to $e$

- **Decoder**:
  - $c'$ to $c$
  - $c'$ to $c$
  - Processor
  - $A_i e = [0^T \ e_i]^T$
  - LT decoder
  - $c$ to $c'$

This kind of system is being adopted for application-layer FEC for 3GPP MBMS, IP-datacast for DVB-h…
Reduction to channel coding

- The transmitter multicasts only the non-systematic Raptor encoding symbols.
- The receiver views the available side information as the output corresponding to the systematic symbols.
- No code design modifications are necessary.
- The price of higher encoding/decoding cost, Gaussian elimination at the transmitter for each message block.
$k \sim 3000$, noiseless actual channel

$k \sim 3000$, BIAWGN actual channel with 3 dB SNR
Conclusion

- Study of LT and raptor code design when some side information about source is available at the receivers
- Optimization procedure for output symbol degree distributions for non-systematic LT and raptor codes for different models of side information
- Application of systematic raptor design to fountain coding with decoder side information
- Implicit optimal systematic LT design [Nguyen, Hanzo 2008]
Thank you!