
Peer reviewed version

Link to published version (if available): 10.1109/PIMRC.2008.4699870

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A Game Theoretic Approach to Distributed Resource Allocation for OFDMA-Based Relaying Networks

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Abstract—In this paper, algorithms on distributed resource (spectrum and power) sharing for relay stations are investigated for downlink transmissions in an OFDMA-based relay-aided cell. Both system capacity and user fairness are considered. By grouping the relay stations into coalitions according to the set of users they are relaying, the optimal resource allocation can be solved by considering resource allocation within and among the coalitions. The algorithm for intra-coalition resource allocation is proposed by utilizing the key observations: for each data symbol transmitted from the base station to a user (in a subcarrier), only one among all the available relay stations is required to relay the symbol. The inter-coalition resource allocation is modeled by both a non-cooperative and a cooperative game, where the cooperative game is solved by a nonsymmetric Nash bargaining solution. Simulation results show that the non-cooperative algorithm outperforms random allocation by approximately 50% in system capacity with 3 relay stations in each coalition. The cooperative algorithm has approximately 5% loss in system capacity comparing with the non-cooperative algorithm, but achieves a significant gain in terms of fairness performance.

I. INTRODUCTION

Both orthogonal frequency division multiple access (OFDMA) and relaying are regarded as leading candidates for future generation cellular networks [1], [2]. OFDMA serves as a promising multiple-access technique for high-data-rate transmissions while relaying helps to increase system capacity, transmission reliability, as well as coverage. In an OFDMA relaying network, the base station (BS) first sends out data symbols which are carried by subcarriers, and relay stations (RS) retransmit these data symbols carried by their own subcarriers; most likely in different frequency bands and in a different order to explore multiuser diversity and utilize frequency selective fading channels. In this paper, based on the estimated channel conditions of the multiuser downlinks, we investigate a distributed optimal spectrum sharing and power allocation strategy for the RSs in a decode-and-forward OFDMA-based relaying system in terms of both total system capacity and user fairness.

OFDMA resource allocation without relaying is well reported in the literature [3], [4], [5], [6], while relaying cases have only been considered in a number of limited scenarios due to their high complexity [7], [8], [9]. All the previous works in OFDMA relaying either do not consider the resource allocation problem or make unrealistic assumptions to simplify the problem. In [10], [11], [12], the authors take a different approach and make use of the concept of bargaining in game theory to model subcarrier sharing among the user equipment (UE) for the OFDMA downlink/uplink transmissions (without relaying). The concept of a Nash bargaining solution is used in these studies, which provides a fair operation point in a distributed implementation.

Our approach differs from previous work since we consider practical assumptions on relaying scenarios. RSs are assumed to have individual power constraints and they use the same or different frequency spectrum for relaying (i.e., sensed by cognitive radio). Some RSs may be designated to the same set of UEs and they have shared objectives, while some RSs may be designated to a completely different set of UEs and they may compete among each other for the shared resource. Such a system can be naturally modeled as a game. Greedy subcarrier and power allocation schemes for each RS-UE link are smartly chosen to explore the global resource utilization efficiency in improving capacity and fairness.

We group the set of RSs designated to the same set of UEs into a coalition. The optimal resource allocation of the whole system is then divided into two subproblems: (1) how to share the spectrum and allocate power among the RSs in the same coalition, and (2) how to share the spectrum among different coalitions. For the first subproblem, we prove that the optimal resource allocation theorem for the relaying network proposed in our previous work [9] still holds under the new assumptions. A greedy algorithm is proposed. For the second subproblem, we first model it as a non-cooperative game and show that under good channel conditions, it converges fast. We then model the game as a cooperative bargaining problem and nonsymmetric Nash bargaining solutions are utilized for the weighted fairness among the coalitions. The proposed algorithms have low complexity and simulation results show that they effectively enhance the total capacity and maintain the user fairness.

The paper is organized as follows. Section II describes our relaying network model. In Section III, the preliminaries in game theory are introduced and the problem is formulated.
using game theory. In Sections IV, V and VI, we investigate the resource allocation problems both intra-coalition and inter-coalition. Section VII evaluates the performance of these algorithms. Finally, conclusions are drawn in Section VIII.

II. SYSTEM MODEL

This paper investigates the problem of resource allocation in an OFDMA-based relaying wireless network. OFDMA downlink transmissions (with $N$ data subcarriers) in a single cell are studied. The system consists of one base station (BS), $K$ mobile users or user equipment (UE) and $L$ relay stations (RS), and each is equipped with a single antenna. Inter-cell interference is not considered in the additive Gaussian noise. Let $\mathcal{U} = \{u_1, \ldots, u_K\}$ denote the set of UEs, and $\mathcal{R} = \{r_1, \ldots, r_L\}$ denote the set of RSs. Unlike most previous work, we consider a practical and flexible relaying scenario based on fixed relays. As shown in Figure 1(a), each RS $r_i$ relays for some UEs $\mathcal{U}_i \subseteq \mathcal{U}$. The set of UEs $\mathcal{U}_i$ can be determined in many ways, for example location proximity or the UEs who pay for the relaying service. In the case that some RSs use the same spectrum to relay for different UEs, the interference to each other is considered as noise. The set of RSs who relay for UE $u_i$ is denoted as $\mathcal{R}_i \subseteq \mathcal{R}$, i.e., a UE may have more than one RS that is dedicated to relay for it.

The relaying scenario works as follows. The BS firstly transmits information symbols to UEs, and RSs also decode the symbols and retransmit them (decode-and-forward relaying) in different time slots or frequency bands (free spectrum chunks detected by cognitive radio [13]). We assume that channel gains between the BS and RSs are high enough (e.g., selective decode-and-forwarding is performed) so that the probability of a decoding error at each RS is small regardless of the power and modulation schemes adopted at the BS. RSs select and allocate different spectrum (subcarriers) and power for each UE to utilize the multiuser diversity, as shown in Figure 1(b).

Perfect channel state information at the receivers is assumed, and it is fed back to the senders (BS or RSs). The RSs negotiate with the UEs after the subcarrier allocation so that the UEs are able to correctly combine the relayed symbols with the symbol sent by the BS for better decoding. We also assume that UEs send back to the BS the information of combine channels (e.g. combined signal-to-noise ratio). This allows the BS to exploit adaptive modulation and coding schemes to improve system capacity.

III. GAME THEORY CONCEPTS AND PROBLEM FORMULATION

Game theory aims to study the interactions among a set of decision makers, called players. The resource allocation problem is similar to a bargaining problem in game theory, where a set of decision makers bargain among themselves for some shared resource. In this section, we will first introduce the basic concepts of game theory and then formulate the problem.

A. Nash Bargaining Solution

Let $N = \{1, \ldots, n\}$ denote the set of players and let $\mathcal{F}$ denote a closed and convex subset of $R^n$, representing the set of feasible payoff allocations that the players can get if they all work together. Let $d = (d_1, \ldots, d_n)$ denote the disagreement payoff allocation that the players would expect if they did not cooperate, and suppose that $\{y \in \mathcal{F} | y_i \geq d_i, \forall i \in N\}$ is a nonempty bounded set. The pair $(\mathcal{F}, d)$ is called an $n$-person bargaining problem [14].

Definition 1: A solution to the bargaining problem $(\mathcal{F}, d)$, $\phi(\mathcal{F}, d)$, is called a Nash bargaining solution (NBS), if the following axioms are satisfied [14].

1) Weak Pareto Efficiency: there is no other vector $y \in \mathcal{F}$ such that $y_i > \phi_i(\mathcal{F}, d)$ for every $i \in N$.
2) Individual Rationality: $\phi(\mathcal{F}, d) \geq d$.
3) Scale Covariance: For any linear transformation $\psi$ of $\mathcal{F}$, $\phi(\psi(\mathcal{F}), \psi(d)) = \psi(\phi(\mathcal{F}, d))$.
4) Independence of Irrelevant Alternatives: For any closed convex $\mathcal{G} \subseteq \mathcal{F}$, if $\phi(\mathcal{G}, d) \in \mathcal{G}$, then $\phi(\mathcal{G}, d) = \phi(\mathcal{F}, d)$.
5) Symmetry: if $\mathcal{F}$ is invariant under all exchanges of agents, then $\phi_i(\mathcal{F}, d) = \phi_j(\mathcal{F}, d), \forall i, j \in N$.

There is exactly one bargaining solution that satisfies the above axioms, which is the NBS, stated in the following theorem [14].

Theorem 1: There is a unique solution function $\phi(\mathcal{F}, d)$ that satisfies all five axioms in Definition 1, and the solution satisfies,

$$\phi(\mathcal{F}, d) \in \arg \max_{x \in \mathcal{F}, x \geq d} \prod_{i=1}^{n} (x_i - d_i)$$

B. Nonsymmetric Nash Bargaining Solution

Similar to the NBS, the nonsymmetric Nash bargaining solution [14], [15] satisfies,

$$\phi(\mathcal{F}, d) \in \arg \max_{x \in \mathcal{F}, x \geq d} \prod_{i=1}^{n} (x_i - d_i)^{w_i}$$

where $w_i$ represents the weight of player $i$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 2: A dictatorial solution [15] of player $i$, $\phi_i^P(\mathcal{F}, d)$, represents the optimal utility of player $i$ by assuming all other players adopting the strategies to achieve the utilities at the disagreement point $d$.

Definition 3: A solution to the bargaining problem $(\mathcal{F}, d)$, $\phi(\mathcal{F}, d)$, is called a nonsymmetric Nash bargaining solution (NNBS), if the following axioms are satisfied [15].

1) Weak Pareto Efficiency.
2) Individual Rationality.
3) Independence of Irrelevant Alternatives.
4) Disagreement Point Convexity (DPC): let $\tilde{x} = \phi(F, d)$ be a solution outcome, then the disagreement point $d' = (1 - \lambda)d + \lambda\tilde{x}$ leads to the same outcome.
5) Domination of Weighted Dictatorial Solution (DWD($w$)): with weights $\sum_{i=1}^{n}w_i = 1$, a solution $\phi(F, d)$ is domination of weighted dictatorial solution if,

$$\phi(F, d) \geq \sum_{i=1}^{n} w_i \phi^D_i(F, d),$$

where $\phi^D_i(F, d)$ are dictatorial solutions.

It is proven in [15] that, when $d = 0$, a NNBS corresponds to the solution for weighted proportional fairness by solving $\max_{x \in F} \sum_{i=1}^{n} w_i \log x_i$.

### C. Problem Formulation

Given the definitions and properties of various solutions to a bargaining problem, we formally formulate our problem in this section.

We define the set of players as the set of RSs $\mathcal{R}^2$ with size $L$. The utility $v(r_i)$ of the RS $i$ is defined as,

$$v(r_i) = \sum_{u \in U_i} R(u),$$

which is the summation of the information rates of the UEs that it relays for. The information rate $R(u_j)$ for UE $u_j$ is defined by the summation of the Shannon capacity of each subcarrier by considering the BS-UE signal-to-noise ratio (SNR), as well as the RS-UE SNR, where the signals from other RSs that do not relay for this subcarrier are treated as interference.

We define a coalition $S$ to be the group of RSs that relay for the same subset of UEs, i.e., if $U'$ is any subset of $U$, then,

$$S(U') = \{r_i | r_i \subseteq U_i\}$$

The utility is transferable within a coalition because of the continuous power allocation by water-filling (we will show in the next section that water-filling is still optimal under a relaying scenario). For example, if a RS $r_i$ gives up a subcarrier for RS $r_j$ (both are in the same coalition), then the increase in SNR of one subcarrier $r_j$ will result in the increase of the water level and thus benefits all the users that are relayed by $r_j$.

There are potentially many coalitions in the network. We further simplify the problem by assuming if two RSs $r_i$ and $r_j$ relay for the same UE, then they have exactly the same subset of UEs to relay for, i.e., $U_i = U_j$. Under this assumption, a RS only belongs to one coalition. Thus, each coalition treats itself as a “larger” player and competes with other coalitions for resource. Any benefit to a coalition can be transferred to its members. If the assumption does not hold, each coalition will not be able to purely consider other coalitions as a competitive relationship because some of its member may also belong to other coalitions. This will further complicate the situation and is beyond the scope of this paper.

In the following sections, we will firstly discuss how the utility shall be shared within the coalition and how the power shall be allocated. We will then discuss both non-cooperative and cooperative solutions for resource sharing among coalitions.

### IV. Resource Allocation within Coalitions

The resource allocation, especially the subcarrier allocation problem, is a combinatorial matching problem that is known to be NP-complete. Before we introduce the detailed distributed resource allocation algorithm, we firstly show several theorems which can greatly reduce the complexity of the resource allocation strategy.

The following theorem is cited from our previous work in [9].

**Theorem 2:** In the general situation, for a relaying network where the number of subcarriers $K$ is much larger than the number of RSs $L$, and $L$ is a small integer value, there is approximately only one RS $l$ among all $L$ RSs that needs to relay for any subcarrier transmitted from the BS. The optimal power allocation is water-filling by considering power from other RSs as noise.

Theorem 2 states that, within a coalition $S$, for any subcarrier from the BS to a UE that is dedicated to this coalition, the optimal solution only requires one RS to relay for this subcarrier among all the RSs in the coalition $S$. It is obvious that the optimal power allocation scheme is actually a Nash equilibrium. However, the theorem has an assumption that the RSs strictly use different spectrum and that they strictly do not interfere with each other.

We assume in this paper that RSs may share, and most likely do share the same spectrum, especially for the RSs in the same coalition (due to the possible location proximity). We extend the above theorem as follows.

**Theorem 3:** The maximum capacity $C_{2\text{max}}$ under the assumption of RSs using the same frequency spectrum for transmissions is equal to the maximum capacity $C_{1\text{max}}$ under the assumption of RSs using different frequency spectrum for transmissions; and their power allocation schemes are the same, provided that the subcarriers have already been allocated in the same way for both scenarios.

**Proof:** Due to space limitation, we only give a simple proof based on intuition. It is natural to see that in the assumption that RSs use different spectrum, maximal ratio combining (MRC) can be applied at the UEs. Thus, given the same subcarrier and power allocations, it will always be

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[a]: Although it is more natural to define the players as the set of UEs $\mathcal{U}$, allowing UEs to bargain among each other is not efficient for a power-limited device and the fairness is also hard to control with the possibility of malicious players.
superior to the assumption that RSs use the same spectrum. However, since the optimal subcarrier allocation assumes only one RS relays for a subcarrier, it is applicable to both scenarios. Therefore, their optimal subcarrier allocations are the same. The power allocations are also the same.

Similar to the case in [9], when considering user fairness, Theorem 3 still holds for any single UE. The RSs in a coalition can simply adjust the power allocated to each user to control any fairness scheme such as max-min fairness [4] or proportional fairness [6].

A simple and efficient greedy algorithm CoalitionAllocate() based on the previous analysis is described as follows. The aim of the algorithm is to allocate subcarriers and power at each RS to maximize the capacity of all the UEs serviced by the RSs in a coalition; user fairness can also be optionally maintained.

1) Assume power is proportional to the number of subcarriers allocated to the relay for a UE at each RS.

2) For each subcarrier from the BS to the UE, find the best subcarrier among all which have not been allocated for the UE by comparing a combination of both water-level and channel condition at each subcarrier.

3) According to the fairness scheme, iteratively adjust the power and recalculate the information rates for each UE, until the maximum step has been reached or the fairness criteria is matched.

The algorithm has a running time proportional to the number of UEs serviced by the coalition, as well as the amount of adjustments for fairness maintenance. The key idea is to greedily match the best RS to the UE using Theorem 3. Note that this algorithm is a centralized algorithm. It requires a selected RS to act as a coalition decision maker. The RS needs to overhear the information feedback from UEs to other RSs in the same coalition before the algorithm can be executed.

V. NON-COOPERATIVE RESOURCE ALLOCATION

In this section, we consider the non-cooperative resource allocation among coalitions. The algorithm can be run on the selected coalition decision makers for distributed spectrum sharing of coalitions. As mentioned in previous sections, the relationship among coalitions is competitive. The objective of the non-cooperative scheme is to reach the Nash equilibrium in a non-cooperative game where no RS can change its subcarrier allocation alone to benefit himself.

For a distributed system, such an equilibrium has to be reached in rounds. Due to space limitation, we only describe the basic idea below. Each coalition distributively and locally allocates the resource (the algorithm described in CoalitionAllocate()) using known channel conditions based on all other coalitions’ decision so far in turn. After a coalition makes a decision, it informs other coalitions about its decision for information update. The process terminates when either a predefined maximum running round or an equilibrium has reached.

Due to the discrete nature of the subcarrier allocation, this algorithm may not reach the equilibrium. However, when the total bandwidth of the RSs is not smaller than the bandwidth of the BS, and when the channel conditions of all RSs are good (which is most likely satisfied due to the multiuser diversity), the algorithm converges fast. This is because if a coalition $S_1$ uses a frequency band, it is not likely for another coalition $S_2$ who makes decision later to make use of the same frequency band, unless the interference caused by the coalition $S_1$ is negligible. In the later case, at the next round of execution of coalition $S_1$, the decision from coalition $S_2$ in the previous round is unlikely to affect $S_1$’s decision due to small levels of interference. Note that this algorithm may give supreme priority to the coalition that starts first. Fairness among coalitions are hardly obtained.

VI. COOPERATIVE RESOURCE ALLOCATION

We now discuss the cooperative resource allocation among coalitions. The aim of cooperation is to maximize the total capacity of UEs in each coalition while keeping fairness among the coalitions.

We choose to make use of the NNBS, which maximizes $\prod_{i=1}^{n}(x_i - d_i)^{w_i}$ and maintains weighted proportional fairness among the coalitions. For each coalition $i$, we define its weight $w_i$ as the percentage of the subcarriers or UEs that it relays over the total number of subcarriers from the BS (or the total number of UEs). A coalition relaying for more UEs should have higher priority. When the weighted fairness is maintained among the coalitions and each coalition also maintains its own fairness among UEs as shown in CoalitionAllocate(), the fairness among UEs is also maintained.

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We propose the algorithm in Figure 2. The key idea of this algorithm is to let the coalitions who lead the bound most “help” those who lag the bound most by exchanging and reallocating their subcarriers. Each coalition shall locally calculate its own dictatorial solution. The system initialization can be either randomized, or using non-cooperative resource allocation shown in Section V.

Since this is a distributed algorithm, the step that reallocates subcarriers among the two sets of coalitions may require a large number of information exchanges. There are two ways to reduce such complexity. The coalitions that are not adjacent to each other (thus have negligible interference to each other) may be neglected from such subcarrier reallocation. Alternatively, as proposed in [10], a solution based on the Hungarian method firstly finds a matching pair and then lets only the two pairs exchange and reallocate subcarriers.
Cooperative Resource Allocation
for each coalition $S_i$
  call CoalitionAllocate()
  calculate the weighted dictatorial solution $w_i \rho^D(F, d)$
initialize the system
while maximum adjustment step has not reached
  form a set of coalitions such that they are
    the top in exceeding their weighted dictatorial solution
  form a set of coalitions such that they are
    the last in exceeding their weighted dictatorial solution
  do subcarrier reallocation between these two sets
for each coalition $S_i$
  call CoalitionAllocate()

Fig. 2. Cooperative Resource Allocation Algorithm

(a) System capacity  (b) Fairness performance

VII. SIMULATION

We set up the simulations as follows. In the relaying downlink transmissions, the total bandwidth is chosen to be 5MHz, and the number of data subcarriers is 300, which are parameters used in the LTE OFDMA downlink system [16]. The wireless channel is modeled as a frequency-selective channel consisting of six independent Rayleigh fading paths. For simplicity of simulation, we assume the total bandwidth of all the RS is the same as that of the BS. We evaluate both the performance of non-cooperative and cooperative solutions by varying the size of the coalitions and the numbers of RSs and UEs inside each coalition.

We also assume that the BS does not apply any resource allocation to compare the effectiveness of the algorithms on the RSs. The RSs (either in the same coalition or in different coalitions) are set to have the same path fading to all UEs. The resource allocation inside each coalition has a fairness criteria such that each UE in the coalition has the same priority. Jain’s fairness index [17] is adopted to measure fairness of difference resource allocation schemes.

The simulation results for 2 coalitions and 3 UEs for each coalition are shown in Figure 3 (varying the number of RSs in each coalition). Although the RSs have individual power constraints, their total power is kept constant for fair comparison. It can be seen that both non-cooperative and cooperative subcarrier allocation strategies outperform the random resource allocation strategy in system capacity. The non-cooperative algorithm outperforms random allocation by approximately 50% in system capacity with 3 RSs in each coalition. The cooperative algorithm has approximately 5% loss in system capacity comparing with the non-cooperative algorithm, but achieves significant gain in fairness performance.

VIII. CONCLUSION

In this paper, resource allocation strategies have been investigated in an OFDMA-based relaying network. The system was modeled as a game where RSs with the same objectives are grouped into coalitions. The detailed algorithms on how the subcarriers and power can be allocated within each coalition, and how the subcarriers can be shared among the coalitions, were discussed. Both non-cooperative and cooperative solutions based on the NNBS were proposed for inter-coalition resource allocation. Simulation results showed that the cooperative algorithm had a 5% loss in system capacity comparing with the non-cooperative algorithm with 3 RSs in each coalition, but achieved a significant gain by approximately 25% in terms of fairness performance.

REFERENCES