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Hybridisation of the FDTD technique

by:

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Outline

1. What is the problem? - Structures with both fine detail and large electrical size

2. For example, predicting PCB behaviour can be done by:
   1. Partial Element Equivalent Circuits (PEEC)
   2. Finite Difference Time Domain (FDTD)

3. The best of both worlds - hybridisation

4. Results

5. Onwards
An example of the problem?

- Printed Circuits and becoming more complex, dense and fast.

- They operate in complex environments.

- Issues such as signal integrity, interference and crosstalk have become key parts of circuit and system design.

- CAD tools have to keep up with improvements in manufacturing capability.

A typical modern PCB
Can use standard FDTD but...

- If the structure contains fine detail or boundaries which do not conform to the grid, a very fine mesh is needed.

Strip width \(\approx \lambda/100\!\)!

Can use PEEC but...

- If the structure size is a significant fraction of a wavelength then retardation effects must be included.

- This seriously complicates the method and can lead to late time instability which is challenging to get rid of.
... there is a better way

- Extend existing “thin wire formalisms” to allow for general wire and microstrip circuits.

- Let the formalisms take care of the detail, let the FDTD algorithm take care of the long range interactions.

- The final algorithm can be viewed as a hybrid between FDTD and PEEC.

What are thin wire formalisms?

- In standard FDTD, metals are treated by enforcing field boundary conditions. The currents are not explicitly calculated.

- With thin wire formalisms, the currents in the wire are explicitly treated using extra differential equations.

- This allows the singularities of the fields to be accounted for and allows many wires to be placed within a single FDTD cell.
Wire bundles embedded within an FDTD mesh

Consider a bundle of wires in the FDTD mesh. Two wires of the bundle are shown here.

The E field, tangential to the wires, at a point, \( r \), can be expressed in terms of the potentials as follows:

\[
E_z(r) = -\frac{\partial}{\partial z} \phi(r) - \frac{\partial}{\partial t} A_z(r)
\]

Where, the potentials round an *infinite* bundle may be approximated as:

\[
A_z(r) = \sum_j \frac{\mu_j}{2\pi} \ln|r - r_j| \quad \phi(r) = \sum_j \frac{\lambda_j}{2\pi\varepsilon} \ln|r - r_j|
\]
Wire bundles embedded within an FDTD mesh

Therefore the E field, tangential to the wires, at a point, \( r \), can be expressed in terms of the E field on the \( i^{th} \) wire as follows:

\[
E_z(r) = \frac{\partial}{\partial \zeta} (\phi(r_i) - \phi(r)) + \frac{\partial}{\partial t} (A_z(r_i) - A_z(r)) + E_z(r_i)
\]

Wire bundles embedded within an FDTD mesh

Following Ledfelt[1] we choose a set of weighting functions, \( w_i(r) \), to be non-zero on a circular shell centred on the \( i^{th} \) wire and zero elsewhere.

Wire bundles embedded within an FDTD mesh

Now multiply each side of the equation by each of the weighting functions, \( w_i(r) \) in turn and integrate over all space. This leads to a set of equations, one for each wire:

\[
\frac{\partial I}{\partial t} = L^{-1}X - c^2 \frac{\partial \lambda}{\partial z}
\]

where:

\[
L_{ij} = \langle A_j, w_i \rangle - A_j(d_{ij}) \quad X_i = \langle E, w_i \rangle + \frac{V_{si}}{\Delta}
\]

\( V_{si} \) is a voltage source if present.

These can be discretised in space using central differences:

\[
\frac{\partial I}{\partial t} = -c^2D\lambda + L^{-1}X \quad \frac{\partial \lambda}{\partial t} = -D^T I^n
\]

Where, for a wire:

\[
D = \frac{1}{\Delta} \begin{pmatrix} 1 & -1 & 0 & \vdots & 0 \\ 0 & 1 & -1 & \vdots & 0 \\ 0 & 0 & 1 & \vdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & 1 \end{pmatrix}
\]

\[
C = \frac{1}{\Delta} \begin{pmatrix} \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{pmatrix}
\]
Circuits embedded in the FDTD mesh

The approach used in the thin wire formalism can be readily extended to deal with this situation.

The “in-cell” mutual inductances

For example, the “in-cell” mutual inductance between two segments in the x-z plane and orientated in the z direction can be calculated by direct integration like this:

\[ A_1(x, y, z) = \frac{\mu}{4\pi} \iint_{\text{segment1}} \frac{1}{\sqrt{(x-x')^2 + (y-y_1)^2 + (z-z')^2}} dx' dz' \]

\[ L_{21} = \frac{1}{2\pi} \oint_{\text{circle2}} A_1(x_2 + r_o \cos(\phi), y_2 + r_o \sin(\phi), z_2) d\phi - A_1(x_2, y_2, z_2) \]
The wire update equations

The “in-cell” mutual capacitances can be calculated similarly and the update equations are given by:

\[
\frac{\partial \mathbf{I}}{\partial t} = \mathbf{L}^{-1}(\mathbf{X} - \mathbf{C}\mathbf{P}\lambda) \quad \frac{\partial \lambda}{\partial t} = \mathbf{C}^T \mathbf{I}^n
\]

where, \( C \), is the connection matrix and \( P \) is the inverse capacitance matrix.

Comparison with PEEC methods

In the PEEC method the self and mutual inductances between segments are used in an equivalent circuit.
The PEEC mutual inductances

The mutual inductance between two segments in the x-z plane can be calculated like this:

\[
L_{21} = \int_{segment\ 2} \int A_1(x_2, y_2, z_2) dx dz
\]

compared with the “in-cell” mutual inductance:

\[
L_{21} = \frac{1}{2\pi r_o} \int_{circle\ 2} A_1(x_2 + r_o \cos(\phi), y_2 + r_o \sin(\phi), z_2) d\phi - A_1(x_2, y_2, z_2)
\]

Comparison of mutual coupling

Hybrid vs. PEEC
Comparison of mutual coupling

- In PEEC, it has been shown [1] that mutual coupling effects are significant at distances of up to $5\lambda$.

- Retardation effects seriously complicate the method [2].

- In the hybrid approach mutual coupling is very low at distances greater than the size of the FDTD cell.

- Long range interactions are dealt with by FDTD


Example results

1. Microstrip low pass filter

2. Microstrip band pass filter
The low pass filter geometry

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<td>0.26</td>
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<td>Track 5</td>
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<td>Track 6</td>
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<td>Patch 2</td>
<td>3.09</td>
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<tr>
<td>Patch 5</td>
<td>3.46</td>
<td>2.35</td>
</tr>
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Substrate height 0.635mm
Box size: 30x60x6mm
Substrate $\varepsilon_r = 10.5$

The low pass filter and the FDTD mesh

Segment size: 1mm;
FDTD mesh size: 1mm*0.635mm*1mm (x*y*z);
Width of excitation pulse: 200 picoseconds;
Number of iterations: 8200
Results for the low pass filter

![Graph showing frequency response of the low pass filter with measured and calculated values.]

The bandpass filter geometry

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</tr>
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<td>Width: 0.356mm</td>
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</tr>
<tr>
<td>Track 6</td>
<td>Width: 0.356mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- $\Delta 1$: 0.2946mm  
- $\Delta 2$: 0.2744mm  
- $\Delta 3$: 0.2921mm  
- $\Delta 4$: 0.2768mm  
- $\Delta 5$: 0.2947mm
The bandpass filter and the FDTD mesh

- Segment size: 0.45mm;
- FDTD mesh size: 1mm*0.4mm*1mm (x*y*z);
- Width of excitation pulse: 20 picoseconds;
- Number of iterations: 8100.

Results using the hybrid method

shell radius = 3mm

Results using the hybrid method

shell radius = 1.5mm

![Graph](image)


Conclusions

- It has been shown that an extended wire formalism allows treatment of complex circuits within the FDTD mesh.

- Because the mutual inductance becomes very small when the wire separation is equal to the circle radius, long range effects are not a problem. Retardation is not necessary to be included.

- FDTD takes account of long range interactions PEEC takes account of the fine detail.

- Can be extended to include active components and networks.