A practice-oriented approach for the assessment of brittle failures in existing reinforced concrete elements

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ABSTRACT

A practice-oriented approach was used to assess shear failures in existing reinforced concrete (RC) elements. A simple tool, in form of non-dimensional domains, is obtained considering the capacity models suggested by European and Italian codes. The reliability of failure domains depend strictly on the reliability of the shear capacity model employed; thus, a critical review of code and literature analytical formulations was also carried out. Sezen and Moehle’s experimental database was, then, used to compare the different shear capacity models considered. The code and literature review of shear capacity models emphasizes differences and affinities of the analytical approaches followed in different countries. The domains carried out can be used as a practical instrument aimed at checking shear-flexure hierarchy in existing RC elements and contextualized in the framework of preliminary assessment given the character of input information required. Preliminary applications of the domains are also provided, and emphasize the effectiveness of the new tool for detailed and large scale assessment of existing RC structures.

Keywords: shear capacity model, shear-flexure hierarchy, fast assessment, existing RC elements

1. INTRODUCTION

Brittle failures are a typical problem for existing substandard reinforced concrete (RC) buildings. These kinds of buildings have been generally designed for very low lateral force resistance, if any. As a result, they are expected to develop significant inelastic action, even under a moderate earthquake. To sustain it, they should have considerable ductility, at both local and global level. However, potential plastic hinge regions are not detailed for ductility in these RC buildings. Further, structural members are not capacity-designed against pre-emptive brittle failure in shear. So, it is more by coincidence than by design that an existing substandard element shows a ductile failure mode [1].

Non-ductile failures due to poor detailing of members or connections in RC buildings are plenty in reconnaissance reports, in-field campaigns and case-studies (e.g., [2-6]). Columns without

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engineered earthquake resistance have often been designed only for gravity compression with a nominal eccentricity. Thus, they are not only undersized and poorly detailed, but also have low flexural and shear resistance against lateral load. By contrast, beams of seismically deficient buildings normally have substantial flexural and shear resistance, thanks to their design for gravity loads. So, unlike column failures, which abound, beam failures are rare [1].

When a building is capacity designed (e.g., according to Eurocode 8 part 1, (EC81) [7] or recent Italian seismic code [8]), it cannot be characterized by brittle failures. Capacity design prevents brittle failures or other undesirable failure mechanisms by deriving the design action effects of selected regions from equilibrium conditions. It is assumed that plastic hinges, with their possible overstrengths, develop at the end sections of the element and the design shear demand is evaluated as the maximum shear that can be registered because of flexural behavior.

In existing RC buildings, brittle failures often represent a significant proportion of the retrofitting costs. Brittle failures occur in beam-column joints and columns but rarely in beams.

Analytical formulations for shear capacity can largely differ from each other. The latter is an effect of the complexity of the physical phenomena involved in the mechanism of shear resistance. Given the differences in the analytical formulations and the discrepancies between shear capacities evaluated with different approaches, it is worth investigating geometrical characteristics of elements and material properties for which such discrepancies arise more evidently. To this aim, a general review of the shear capacity formulations according to Italian, European and American guidelines [7-14] is provided herein. Furthermore, Sezen and Moehle’s experimental database [15] is employed for a numerical versus experimental comparison of the different analytical formulations, provided in the literature review. Every numerical versus experimental comparison is always limited to the completeness of the database available in terms of geometry, reinforcement ratios and mechanical properties of the elements. Thus, a model comparison is pursued herein in normalized form. The latter allows investigating the performances of the different analytical shear capacity formulations considering wider ranges of parameters controlling elements characteristics.

Given the key role played by brittle failures in the assessment and retrofitting of existing reinforced concrete structures, it can be helpful to implement a practice-oriented tool to check shear-flexure hierarchy in RC elements. This can lead to a rough prediction of the potential occurrence of shear failures in a given structure or in a given population of buildings. This kind of approach can be contextualized in the framework of preliminary or fast assessment procedures for RC buildings, such as the screening procedures adopted during recent years (e.g., [16-17]). Some methods require dimensions, orientation and material properties of the lateral load-resisting structural system, as well as the quality of workmanships and materials [16-17].
The practice-oriented tool, provided herein, is carried out in the form of normalized domains in which the curve equalizing the maximum shear demand and the shear capacity represents the boundary separating brittle and ductile failures. In terms of brittle or ductile behavior, the approach provides a preliminary classification of the elements.

In the case of detailed assessment of a single building, the potential failure classification can lead to a rough evaluation of the local retrofitting necessary for the elements against shear failures. This issue can be crucial given the high economical impact caused by local retrofitting for brittle failures for substandard RC buildings. The preliminary assessment of brittle failures through the normalized domains can also be useful as a comparative tool between obsolete design approaches and code regulations. It can provide a preliminary evaluation to check whether a specific design approach, based on obsolete design prescriptions, can be more likely characterized by brittle failures. A qualitative example is provided, herein, for the specific case of old Italian codes regulations. Furthermore, the effect of the variability of the mechanical properties of steel and concrete can be included in the evaluation. The above possible employments of the domains are all outlined in the following by means of qualitative and quantitative examples.

2. SHEAR CAPACITY MODELS IN CODES AND LITERATURE

Within the framework of evaluating shear-flexure hierarchy in existing RC elements, the choice of a reliable shear capacity model becomes necessary. Nonetheless, literature on shear capacity models and codes can be quite different to each other depending on whether analytical theories or experimental databases are being used. While the evaluation of flexural strength is almost identical in all codes over the world, shear failure mechanisms are physically more complex. Thus, different interpretations of the mechanical phenomenon led to different theories and models. The complexity of the phenomena is also the reason why some of the most recent shear models follow the regression approach on experimental data; in analogy with the approach followed for the evaluation of chord rotation capacities (e.g., [18,19]).

2.1. Design shear capacity formulations

Throughout the twentieth century, truss models have influenced design procedure for shear. Truss models in reinforced and prestressed concrete structures are behavioral tools used to study the equilibrium between loads, reactions, and internal forces in concrete and reinforcement. The first use of truss models in reinforced concrete beams was presented by Ritter [20]. In his original truss model for shear, the compression diagonals were inclined at 45 degrees. Mörsch [21], later, suggested the possibility of having angles of inclination different from 45 degrees and also introduced the use of truss model for torsion. This approach gave conservative results when
compared to testing evidence. These pioneer works received new impetus in the period from the 1960s to 1980s when a concrete contribution was added to shear capacity. Attention was focused on the truss model with variable angle of inclination for shear and torsion in reinforced and prestressed concrete beams, [22, 23]. Collins and Mitchell [22,23] further developed the truss model for beams by introducing a compatibility condition for strains in the transverse and longitudinal steel and the diagonal struts. Of note, the variable strut inclination model in its enhanced versions is essentially calibrated for beams under static load, (e.g., [24]). The research carried out at the University of Toronto over the last 35 years developed shear design provisions suggested by 2004 Canadian provisions [25] and adopted by Model Code 2010 [14].

The cracked web of a reinforced beam transmits shear in a relatively complex manner. As the load increases, new cracks form while preexisting cracks spread and change inclination. Because the section resists bending moment as well as shear, the longitudinal strains and the crack inclinations vary over the depth of the beam [26]. The early truss model by Ritter [20], approximated this behavior by neglecting tensile stresses in the diagonally cracked concrete and by assuming that shear would be carried by diagonal compressive stresses in the concrete, inclined at 45 degrees to the longitudinal axis. According to the 45 degree truss analogy, the shear capacity is reached when the stirrups (α=90 degree) yield and corresponds to the capacity in equation (1). In this equation, $A_{yw}$ is the area of shear reinforcement within distance $s$, $f_{yw}$ is the yielding strength of the transversal reinforcement, $\alpha$ is the inclination of the reinforcement, and $d$ is the distance from the compression fiber to centroid of longitudinal tension reinforcement, assuming $d'=0.9d$ as internal lever arm.

$$V_{R,45^\circ} = \frac{A_{yw} \cdot f_{yw} \cdot d^* \cdot (\sin \alpha + \cos \alpha)}{s}$$  \hspace{1cm} (1)$$

Given the conservative shear design obtained with the 45 degree truss analogy, several design procedures were developed to economize on stirrup reinforcement. The first approach was based on the general idea of adding a concrete contribution term to the shear capacity obtained by equation (1), and the second approach was based on using a variable angle of inclination of the diagonals. Both approaches were able to account for the existence of aggregate interlock and dowel forces in the cracks. It should be noted that a combination of the variable-angle truss and a concrete contribution has also been proposed [24,27]; the concrete contribution for non-prestressed concrete members diminishes with the level of shear stress.

An implementation of the two aforementioned approaches was provided by Eurocode 2 (EC2) in 1991 [9], in fact, both the standard method and the variable strut inclination approaches were followed. The standard method, employing an additive formulation, was based on a fixed truss
angle of 45 degrees, see equation (1). The shear reinforcement was required to carry the excess of shear above the concrete contribution provided by members without shear reinforcement, equal to \( V_{Rd1} \), see equation (2). In equation (2), \( b \) is the width of the section, \( \tau_{Rd} \) is equal to 0.25 of the design tensile stress of the concrete, \( \rho_l \) is the longitudinal reinforcement ratio in tension, accounting for dowel action, \( \sigma_{cp} \) is the average longitudinal stress and \( k=1 \) for members where more than 50\% of the bottom steel has been curtailed, otherwise \( k=(1.6-d) \) where \( d \) is in meters.

\[
V_{Rd,EC2-1991} = b \cdot d \left[ \tau_{Rd} k (1.2 + 40 \rho_l) + 0.15 \sigma_{cp} \right] \tag{2}
\]

In the variable truss angle method, according to old version of EC2 [9], all the shear is given to the transversal reinforcement, but the truss angle \( \theta \) can take any value between 68.2° and 21.8° (cot\( \theta \) respectively equal to 0.4 and 2.5). The variable strut angle approach is considered to be the most rigorous of the two methods and also the most economical in design. In the actual version of EC2 [10], only the variable strut inclination model is employed; the limiting values of cot\( \theta \) are, in this code, equal to 1 and 2.5. According to EC2 [9,10], the shear strength of a concrete member is evaluated as the minimum between the capacity based on the shear reinforcement \( V_{Rsd} \), and the capacity based on the strength of the compression strut \( V_{Rcd} \), see equation (3), (4) and (5). In equation (5), \( f_c \) is the concrete compression strength, \( \nu_1 \) is a strength reduction factor for concrete cracked in shear, and \( \alpha_{cw} \) is a coefficient taking into account the state of stress in the compression chord. The value of \( \nu_1 \) and \( \alpha_{cw} \) are suggested in EC2 [10], although the value may change according to the National Annex of each country. The recent Italian code [8], which provides the variable strut inclination model as capacity model for design, assumes \( \nu_1 \) equal to 0.5 and \( \alpha_{cw} \) value equal to the suggested value in EC2 [10].

\[
V_{Rd} = \min \left( V_{Rsd}, V_{Rcd} \right) \tag{3}
\]

\[
V_{Rsd} = \frac{A_{cw} \cdot f_{yw} \cdot d^*}{s} \cdot (\cot \alpha + \cot \theta) \cdot \sin \alpha \tag{4}
\]

\[
V_{Rcd} = \alpha_{cw} \cdot \nu_1 \cdot f_c \cdot b \cdot d^* \cdot \frac{(\cot \alpha + \cot \theta)}{\left(1+\cot^2 \theta\right)} \tag{5}
\]

Then again, the employment of variable strut inclination model, as presented in Eurocodes [9,10] is open to misinterpretations. EC2 seems to suggest that the designer may select any strut angle he chooses between the specified limits. This concept of free choice does not reflect the behavior of a beam. Beams will fail in a manner corresponding to a strut angle of roughly 21.8° (cot\( \theta =2.5 \)), unless constrained by the detailing or the geometry of the system to fail at some steeper
angle. In other words, the maximum shear strength corresponds to the situation in which the capacity based on the shear reinforcement \( V_{Rsd} \) just equals the capacity based on the strength of the strut \( V_{Rcd} \). Consequently, the inclination \( \theta^* \) have to equal the two contributions \( V_{Rsd} \) and \( V_{Rcd} \), see equation (6), being \( \omega_{sw} \) the mechanical transversal reinforcement ratio, defined in equation (7). In the case in which \( \cot \theta^* \) is not within the mandatory ranges, the closest endpoint of the interval \([1, 2.5]\) is assumed, and shear strength is the minimum between \( V_{Rsd} \) and \( V_{Rcd} \).

Sections in which \( \cot \theta^* \) exceeds 2.5 can be referred to as “lightly transversal reinforced” sections, since \( V_{Rsd} \) rules equation (3). If the second member of equation (6) is equalized to the limiting value of 2.5, it is possible to compute the values of \( \omega_{sw} \) and normalized axial force (for the definition of \( \alpha_{cw} \)) that define the field of lightly transversal reinforced sections. In particular, for \( \nu_1=0.5 \) and \( \alpha_{cw}=1.0 \) (the value to be assumed in the case of absence of axial force), the limiting \( \omega_{sw} \) is 0.069. Such a limiting \( \omega_{sw} \) increases with the increasing of axial force of the section up to a limiting value of \( \omega_{sw} \) equal to 0.086.

On the contrary, the sections in which \( \cot \theta^* \) is lower than 1 can be referred to as “strongly transversal reinforced” sections, since \( V_{Rcd} \) rules equation (3). Similarly, if the second member of equation (6) is equalized to the limiting value of 1.0, it is possible to compute the limiting value of transversal mechanical reinforcement ratio that bounds strongly transversal reinforced sections. In the case of \( \nu_1=0.5 \) and \( \alpha_{cw}=1.0 \), \( \omega_{sw} \) is equal to 0.25, increasing with axial force up to a value of \( \omega_{sw} \) equal to 0.3125.

\[
\cot \theta^* = \sqrt{\frac{V_1 \cdot \alpha_{cw}}{\omega_{sw}}} - 1 \quad (6)
\]

\[
\omega_{sw} = \frac{A_{sw}}{b \cdot s} \cdot \frac{f_{yw}}{f_c} \quad (7)
\]

Since 1995, American provisions for design shear strength [30], provide a truss model with 45° constant inclination diagonals supplemented by a concrete contribution. The additive approach is still employed in the actual regulation [12], and concrete contribution is evaluated according to equation (8), in which \( A_g \) is the gross cross-sectional area.

\[
V_{c,ACI318} = 0.166 \left(1 + \frac{N}{13.8A_g} \right) \sqrt{f_c \cdot b \cdot d} \quad (8)
\]

Model Code 2010 [14] provides a variable strut inclination model with concrete contribution. Shear strength for static loads at ultimate limit state can be evaluated at three different levels. Each level increases the accuracy of the evaluation. For all three levels, an additive formulation is
provided; a variable strut inclination approach is considered for stirrup contribution, the same as shown in equation (4). Furthermore, a concrete contribution according to equation (9) is provided, in which \( z \) is the internal lever arm. The value of the strut inclination (\( \theta \)) and the coefficient \( k_v \) assumes different values according to the level of approximation. The levels of approximation differ in the complexity of the applied methods and in the accuracy of results. Level I is meant for conception and design of new structures, level II is meant for design and brief assessment, while level III is meant for the design of members in a complex loading state or more elaborate assessments of structures. \( k_v \) coefficient is evaluated as a function of the geometrical percentage of transversal reinforcement (\( \rho_w \)) for level I, equal to zero for level II, and a function of \( \rho_w \) and a longitudinal strain at the mid-depth of the member (\( \varepsilon_x \)), [25].

\[
V_{c,\text{ModelCode 2010}} = k_v \frac{f_{ck}}{\gamma_c} b \cdot z
\]  

(9)

2.2. Shear capacity formulations under cyclic loads

The shear transfer mechanisms, already complex under static loads, become even more complex in the case of cyclic loads. Seismic loads ask for a modeling approach that in some cases can differ strictly from the models described in section 2.1. Another difference can be made between the case of design or assessment. In fact, while in the context of design of new buildings requirements for minimum reinforcements or local detailing can overpass the possible lack of accuracy of analytical models calibrated for static loads; in the case of assessment, more accurate formulations are necessary. Thus, it is very frequent identifying shear capacity formulations meant for cyclic loads as assessment models.

Studies on columns’ failures under cyclic loads highlight the possibility of brittle failures after yielding the element. Such types of failures are described as limited ductility failures and indicate a degradation effect on shear capacity caused by cyclic loads after yielding [e.g., 15,31-34]. The degradation effect of shear capacity is ruled by ductility demand. The analytical evaluation for shear strength degradation under cyclic loads uses a regression approach. In literature, different regression models accounting for shear strength degradation have been reported [15,31-34]. All of them additive and calibrated on an experimental basis. The most recent regression models for the evaluation of shear strength under cyclic loading [15,34] have been adopted respectively in the American [13] and European [11] assessment codes.

It is worth noting that, according to experimental tests, the variable strut inclination approach is not well suited cyclic loading [34]. Nevertheless, European provisions for design under seismic loads [7] adopt the same variable strut inclination model provided in [10]. In EC8 1 [7], the shear
design is pursued according to capacity design rule; thus ensuring a shear-flexure hierarchy that prevents any brittle failure. The strut inclination is fixed at 45 degrees only in the case of beams designed in high ductility class (DCH), neglecting any concrete contribution. Thus, the classical Ritter’s model is employed for elements that are meant to experience high ductility demand. However, in critical regions of any primary element, code mandated transversal reinforcement details always rule the design procedure. Thus, even if the variable strut inclination model is not always suitable for cyclic loads, the shear design procedure according to EC8 results in elements and, consequentially, in buildings that comply with the safety requirements according Eurocode 0 [35].

The lack of reliability of the variable strut inclination for cyclic loads is recognized in Model Code 2010 [14]. According to Model Code 2010, a limit value to the maximum \( \cot \theta \) to be employed for cyclic shear resistance at the ultimate limit states in seismic design of members with shear reinforcements is provided (par. 7.4.3.5 in second volume of Model Code 2010). \( \cot \theta \) is assumed equal to 1, if ductility demand in the element exceeds the value of 2, while it is assumed equal to 2.5 in the case of zero plastic rotation \((\theta<\theta_p)\); linear interpolation is considered in-between these values. This latter provision recalls shear strength degradation based on ductility demand.

Priestly and colleagues (1994) created one of the first models to account for shear strength degradation [31]. The model calculates the shear strength as the summation of three contributions: 1) a concrete contribution, 2) a truss contribution in which the inclination \( \theta \) was considered equal to 30° and 3) an arch mechanism contribution, because of axial load. In their model, the strength degradation affects only the concrete contribution by means of a reduction factor \( k \), evaluated as a function of displacement ductility. An enhancement of this model was successively developed [32,33], leading to significant improvements in accuracy with respect to experimental results. Based on Priestley’s pioneering works [31-33], recent regression models [15,34] address shear strength degradation to both concrete and transversal reinforcement contribution.

Biskinis and colleagues (2004) created a regression model [34] which employed a database of 239 elements. The analytical formulation changes in the case of shear failure is controlled by diagonal compression or by diagonal tension. Both the empirical formulas account for shear strength degradation through the plastic ductility factor \( \mu_{Apl} \), equal to the chord rotation demand over the yielding chord rotation minus 1, \((\theta/\theta_p-1)\). The shear strength degradation because of cyclic loads varies linearly between \( \mu_{Apl} \) equal to 0 and 5. \( \mu_{Apl} \) equal to 5 is the value at which the maximum degradation is attained. The regression model by Biskinis and colleagues is employed in Eurocode 8 part 3 (EC8_3) [9] for existing buildings. According to EC8_3, in the case of elements characterized by a shear span ratio \((L_V/h)\) lower or equal to 2, shear failure is controlled by diagonal
compression and the first regression model by Biskinis applies, see equation (10). On the contrary in the case of shear span ratio higher than 2, shear failure is controlled by diagonal tension and the second regression model applies, see equations (11) to (14). In both the analytical formulations provided by EC8₃, the coefficient \( \gamma_{el} \) is equal to 1.15; \( \gamma_{el} \) accounts for uncertainties in the fit of experimental data.

According to EC8₃, the shear capacity is the minimum value obtained by the one of the regression models in equations (10) and (11) and the variable strut inclination according to EC8₁ [7]. However, in most practical cases, the regression model represents the minimum, as it will be shown in section 3.

\[
V_{EC8,squat} = \frac{1}{\gamma_{el}} \frac{1}{4} \left(1-0.02\min(5;\mu_{splat})\right) \left(1+1.35\frac{N}{A_{f_c}}\right) \left(1+0.45\cdot100\rho_{tot}\right) \sqrt{\min(f_c;40)} \cdot b \cdot z \cdot \sin 2\delta
\]  
(10)

\[
V_{EC8,slender} = \frac{1}{\gamma_{el}} \left[V_N + V_c + V_w\right]
\]  
(11)

\[
V_N = \frac{h-x}{2L_v} \cdot \min(N;0.55A_{f_c})
\]  
(12)

\[
V_c = \left(1-0.05\min(5;\mu_{splat})\right) \left[0.16 \max(0.5;100\rho_{tot}) \cdot \left(1-0.16\min\left(5;\frac{L_v}{h}\right)\right)\sqrt{f_c} \cdot A_e\right]
\]  
(13)

\[
V_w = \left(1-0.05\min(5;\mu_{splat})\right) \cdot \left(\rho_w f_{yw} b \cdot z\right)
\]  
(14)

In the following the only regression model for elements that fail in diagonal tension, (\(L_v/h>2\)) is considered, given its more relevant practical interest for typical RC moment resisting frames. The regression model in equations (11) to (14) accounts for three contributions: 1) the classical 45-degrees truss model \(V_w\), 2) supplemented with concrete contribution \(V_c\), where both depend on cyclic displacement ductility demand, and, 3) the axial load contribution \(V_N\). All the symbols in equations (10) to (14) are the same employed in EC8₃ [9].

According to American provisions for existing buildings [13], shear strength shall be calculated with the additive formula provided by ACI 318 [12]. In the case of not properly detailed elements, specific provisions in yielding regions are considered, see section 6.3.4 in ASCE/SEI 41-06 [13]. For concrete moment frames, shear strength in columns can be evaluated according to Sezen and Moehle’ s regression model [15], see equations (15) to (16). This model accounts for shear strength degradation caused by ductility demand by means of the coefficient \(k\). Moreover, it is calibrated on an experimental database of 51 rectangular columns characterized by light transversal reinforcement. \(k\) coefficient is equal to 1 (no degradation) if the displacement ductility is less than or equal to 2, while it is equal to 0.7 in regions where displacement ductility is greater than or equal
to 6. \( k \) varies linearly for displacement ductility between 2 and 6. In EC8, and Sezen and Moehle’s model, both concrete and transverse reinforcement contributions are affected by degradation as a result of ductility demand [11,15,34].

\[
V_{\text{Sezen}} = k \cdot (V_w + V_c) \tag{15}
\]

\[
V_{\text{Sezen}} = k \cdot \left[ \frac{A_{yw}}{s} f_{yw} \cdot d + \left( \frac{0.5\sqrt{f_y}}{L_v/d} \cdot \sqrt{1 + \frac{N}{0.5 \cdot \sqrt{f_c} \cdot A_s}} \right) \cdot 0.8A_s \right] \tag{16}
\]

### 2.3. The Italian misprint issue

In the Italian code [8] the design of new elements in both seismic and non-seismic regions is made according to the variable strut model, following the same recommendations employed in EC8 [10]. As previously mentioned, the Italian code for the value of \( \nu_1 \) is assumed equal to 0.5.

For the assessment, the code itself [8] does not provide any specific rule, providing only general criteria, whereas the explicative documents to the code [36] generally follows the same prescription of EC8. Regarding shear capacity models to the assessment, the explicative document to the Italian code [36] provides some suggestions without providing any explicit capacity model. In [36] it is written that “the approach of design in non-seismic regions should be followed (so, the variable strut inclination model), accounting for a concrete contribution \( V_c \) at most equal to the one for elements without transversal reinforcement”. Now, these suggestions are meaningless, unless an additive formula is assumed; as it used to be according to the old Italian code prescriptions [37]. In fact, during that time, the standard method using the old Eurocode 2 [9] was suggested by the Code [37]. The only solution that complies with these suggestions is to apply the classical Ritter’s model, discard any concrete contribution, and employ the variable strut inclination model (\( \cot \theta = 1 \)). Such an approach can be, evidently, very conservative, and it does not fit with the general idea that assessment procedures should lead to a realistic evaluation of the capacity of a structure.

Interestingly, the reason of such an ambiguous prescription is caused by a misprint in the documents. In fact, most of the explicative documents to the actual code [36] was taken by another document that was released after the 2003 Umbria-Marche earthquake [38]. The latter document, in turn, was taken mostly from EC8. The OPCM 3274 [38] took into account the fact that the main code at the time used to prescribe an additive formula [37]. The prescriptions in the explicative document of the new code [36] were not changed with respect to OPCM 3274 provisions, and, at the same time, the new version of the code [8] had switched to variable strut inclination model. The issues related to this misprint are now going to be solved, since a new upgraded version of both the code [8] and its explicative document [36] is going to be released.
3. EXPERIMENTAL RESULTS VERSUS ANALYTICAL FORMULATIONS: MODELS’ COMPARISONS

The models described in the previous subsection according to European, Italian and American codes are now investigated with respect to experimental tests. The database considered is composed of 51 rectangular columns characterized by light transversal reinforcement. Experimental data are fully available in [15,39]. The database is characterized by columns whose inelastic behavior tends initially to be dominated by flexure but whose ultimate failure and deformation capacity appears to be controlled by shear mechanisms (i.e., limited ductility); and emphasizes shear capacity degradation because of plastic demand in the element. The main characteristic of the database is the lightly transversal reinforcement. This is characteristic of old design approaches adopted in the Mediterranean area and United States (e.g. [4,15,40,41]). Other databases and other analytical formulations for shear capacity under cyclic loading have been calibrated. For example, the database by Biskinis and colleagues [34], (characterized by a larger number of specimens, 239 tests), includes most of the columns considered in [15,39]. Biskinis and colleagues database is not specifically oriented to the case of elements characterized by light transversal reinforcement.

The main properties of the columns in the database considered in this study [15,39] are shown in Figure 1 to 4. Materials’ mechanical properties distributions are shown in Figure 1; frequency distribution of reinforced concrete compressive strength ($f_c$) varies from 15 MPa to 45 MPa, so covering a wide range that can be representative of both existing and new reinforced concrete properties. Steel yielding strength frequency distribution of the longitudinal reinforcement ($f_y$) varies in the range 300 to 600 MPa. A large amount of specimens is characterized by 350 and 450 MPa. Thus, regarding longitudinal reinforcement, these two values can be good representatives of the typical yielding strengths of existing steel (350 MPa) and more recent steel (450 MPa), [42]. Steel yielding strength frequency distribution of the transversal reinforcement ($f_{yw}$) is on average higher than the strength of the longitudinal steel and also characterized by a wider variability.

Figure 2 shows frequency distribution of shear span ratio ($L/V/h$), ductility ($\mu_{\Delta}$), and normalized axial force ($v$) in the database. Ductility is defined by the ratio of the ultimate displacement to yield displacement [15]. It is worth noting that shear span ratios are not equally distributed in the range [2;3.5] and most of the specimens are characterized by small shear span ratios. Ductility frequency distribution varies in the range of [1;8]. Finally, normalized axial force varies in the range of [0;0.6] but most of the values are comprised between [0;0.3]; the latter represents a realistic range for reinforced concrete buildings.

Figure 3 shows the frequency distribution of transversal ($\rho_{sw}$) and longitudinal ($\rho_{tot}$) reinforcement ratios. The database is characterized by percentage of longitudinal reinforcement...
ratio that ranges from 0.01 to 0.04 with a large amount of specimens characterized by 0.02. \( \rho_{\text{tot}} \) values can be considered high if the targets are existing buildings in the Mediterranean area. According to old seismic design codes, longitudinal reinforcement ratios could have reached higher values. For example, the old Italian code provisions [43] allowed a longitudinal reinforcement ratio up to 6%. However, data collected in reconnaissance campaigns suggest it is unlikely to find longitudinal reinforcement ratios that exceeds 2%. In most cases, especially in medium-low seismicity areas, (e.g. in the area struck by the recent 2009 L’Aquila earthquake), the average value of longitudinal reinforcement ratios is equal to 1%, [44]. Transversal reinforcement ratio \( \rho_w \) with the higher frequency in the database is 0.003. Data reconnaissance after L’Aquila earthquake and studies [4,44] showed transversal reinforcement in RC building that ranges between [0.001; 0.002]. Figure 4 shows the frequency distribution of transversal and longitudinal mechanical reinforcement ratios, \( \omega_{\text{trans}} \) and \( \omega_{\text{tot}} \), respectively.

![Figure 1](image1.png)  
Figure 1. Sezen and Moehle’s database, data frequency distributions: material properties, reinforced concrete compressive strength \( f_c \), steel yielding strength of longitudinal reinforcement \( f_y \) and transversal reinforcement \( f_{yw} \).

![Figure 2](image2.png)  
Figure 2. Sezen and Moehle’s database, data frequency distributions: shear span ratio \( (L_v/h) \), ductility \( (\mu_\Delta) \), and normalized axial force \( (\nu) \).

The analytical models described in the previous section are, herein, compared with the experimental results in the database. Table 1 shows the mean \( (\mu) \), the standard deviation \( (\sigma) \) and the coefficient of variation \( (\text{CoV}) \) of the ratio between the experimental and analytical shear capacities \( (V_{\text{exp}}/V_{\text{model}}) \). Results are shown for the models by Sezen and Moehle (Sezen), [15], Biskinis (Bisk) [34], for the variable strut inclination \( (NTC) \) [7,8], the classical Ritter model \( (45^\circ) \), [20], and finally the Eurocode 8 part 3 formulations \( (EC8) \) [11]. The only difference between Bisk and EC8 formulations is the presence of the coefficient \( \gamma_{el} \) equal to 1.15.
Figure 3. Sezen and Moehle’s database, data frequency distributions: transversal ($\rho_w$) and longitudinal ($\rho_{tot}$) geometrical reinforcement ratios.

Figure 4. Sezen and Moehle’s database, data frequency distributions: transversal ($\rho_w$) and longitudinal ($\rho_{tot}$) geometrical reinforcement ratios.

Table 1. Mean and standard deviation of the experimental over analytical ratio for the capacity models considered.

<table>
<thead>
<tr>
<th>$V_{exp}/V_{model}$</th>
<th>NTC</th>
<th>45°</th>
<th>ACI</th>
<th>Bisk</th>
<th>EC83</th>
<th>Sezen</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>1.04</td>
<td>2.59</td>
<td>1.02</td>
<td>0.96</td>
<td>1.10</td>
<td>1.06</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.47</td>
<td>1.20</td>
<td>0.21</td>
<td>0.19</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>CoV</td>
<td>0.45</td>
<td>0.46</td>
<td>0.20</td>
<td>0.19</td>
<td>0.19</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Statistics showed in Table 1 emphasize how the degrading models (Bisk, EC83, Sezen) provide a response closer to the experimental observations with mean values close to 1 and low dispersions. Evidently, Sezen and Moehle’s model which is calibrated on the database, has the best performances, while Biskinis and colleagues formulation (without the application of the correction coefficient as suggested in Eurocode 8) can only be unconservative analytical model. Of note, Biskinis and colleagues formulation is calibrated on a different database. ACI model is the only non degrading analytical approach that leads to reasonable dispersion and excellent mean result. Surprisingly, the variable strut inclination model shows a mean value even closer to 1 if compared to all the degrading models considered. On the other hand, variable strut inclination model
dispersion is very high. Classical Ritter’s truss is very conservative and highlights the fundamental role played by concrete contribution on the shear strength capacity. In Figure 5 and 6, shear strengths of all specimens of the database versus the analytical prediction are shown according to the different models.

Figure 5. Performance of the analytical models considered respect to Sezen and Moehle (2004) database: (a) Sezen and Moehle’s model (Sezen), (b) Biskinis et al.’ model (Bisk), (c) Eurocode 8 part 3 model (EC8).

Figure 6. Performance of the analytical models considered respect to Sezen and Moehle (2004) database: (a) ACI 318 model (ACI), (b) Ritter’s model (45°), (c) variable strut inclination model (NTC).

For a better characterization of the estimation trend of each single analytical model, the ratio between ‘experimental’ and ‘analytical’ strengths is plotted versus the $\omega_{sw}$ shown in Figure 7 and 8. Non degrading models are characterized by most of the error in the range of very low transversal reinforcement ratios. The variable strut inclination model becomes systematically unconservative for $\omega_{sw}$ higher than 0.06. The latter is very close to the boundary values of $\omega_{sw}$ characterizing lightly transversal reinforced sections, for which equation (6) ends up to a value of cot$\theta$ higher than 2.5.

In Figure 9, the value of the strut inclination angle is evaluated as a function of the mechanical transversal reinforcement ratio ($\omega_{sw}$) and the ductility demand ($\mu_\Delta$). The experimental value of $\theta$ angle ($\theta_{exp}$) has been evaluated from equation (4), equalizing the left member of equation (4) to the experimental shear capacity. The resulting $\theta_{exp}$ is then compared to the value computed according to Eurocode 2, Eurocode 8, and Italian Code provisions [7,8,10], ($\theta_{NTC}$). Model Code 2010 provisions regarding $\theta$ angle limitations as function of ductility demand for cyclic loads are also represented in Figure 9b. Further, the value of cot$\theta$ equal to 2.5 (corresponding to an angle of approximately 22°), in lightly reinforced sections, seems to be over-conservative for $\omega_{sw}$ ranging in the interval between [0;0.06] and significantly unsafe for $\omega_{sw}$ ranging in the interval between[0.06;0.1]. In this latter
case, $\theta_{\text{exp}}$ values are closer to 45°.

Thus, “concrete contribution” to shear capacity seems to decrease by means of increasing transversal reinforcement. The decrease is not captured by values obtained from the variable strut inclination model (calibrated for static loads). Model Code 2010 provisions [14] regarding the evaluation of $\theta$ as function of ductility demand can be conservative. In any case, it follows the experimental trend shown in Figure 9b.

Figure 7. Trend with transversal mechanical reinforcement ratio of the experimental over analytical ratio for the capacity models considered: (a) Sezen and Moehle’s model (Sezen), (b) Biskinis et al.’ model (Bisk), (c) Eurocode 8 part 3 model (EC8).

Figure 8. Trend with transversal mechanical reinforcement ratio of the experimental over analytical ratio for the capacity models considered: (a) ACI 318 model (ACI), (b) Ritter- Mörsch’s model (45°), (c) variable strut inclination model (NTC).

Figure 9. Trend of the strut inclination angle $\theta$ as function of $\omega_{\text{sw}}$. (a) and ductility ratio $\mu_\Delta$, (b). The angle $\theta$ is evaluated from experimental shear values ($\theta_{\text{exp}}$), and according Eurocode 2 provisions [10], $\theta_{\text{NTC}}$. Model Code 2010 provisions [14] regarding the value of $\theta$ under cyclic loads are also plotted as function of $\mu_\Delta$, (b).
4. NON DIMENSIONAL SHEAR STRENGTH: MODELS’ COMPARISON

The shear strength capacity models according to European codes [7,10,11] are now compared in non-dimensional form. It allows considering in a wider range of $\omega_{sw}$ (transversal mechanical reinforcement ratio) and $v$ (normalized axial force). In fact, once a model is adopted by a code, no restrictions on the characteristics of the sections are generally applied to the model; thus, it can be employed in general situations, discarding the limits of the experimental database on which it was calibrated or discarding the analytical hypotheses made to obtain it.

According to the results in the previous section, Eurocode 8 part 3 capacity model seems to be the most reliable solution for existing buildings and cyclic loads in the European code framework; not only because of the good performances it shows respect to the experimental database considered above, but also because of the experimental database on which it was calibrated. Biskinis and colleagues database is wider; so it covers a wider range of parameters governing the shear capacity in RC elements.

In this section, the comparison is made considering shear capacities of the classical Ritter’s model (employed in [7,8] for DCH beams), the variable strut inclination approach, employed in [7,8,10] and the Eurocode 8 part 3 (for slender columns) model, employed in [11]. The comparison is made normalizing the shear capacity according to each model by the ultimate axial force of the section ($bh_f$). Such a simplified approach requires some approximations.

$$\frac{V_{45°}}{bh_f} = \omega_{sw} \cdot \frac{d'}{h} = \omega_{sw} \cdot \frac{0.9(h-d')}{h} = \omega_{sw} \cdot k_2$$  \hspace{1cm} (17)

$$\frac{V_{NTC}}{bh_f} = \omega_{sw} \cdot k_2 \cdot \cot \theta'$$  \hspace{1cm} (18)

$$\frac{V_{EC8}}{bh_f} = \frac{1}{\gamma_d} \left[ \frac{vh}{2L_v} \left(1-1.25v\right)+(1-\beta) \cdot \left[ \frac{16}{0.9\rho_{tot}} \left(1-0.16 \frac{L_v}{h}\right) \frac{1}{\sqrt{f_{c}}} + \omega_{sw} \cdot k_1 \right] \right]$$  \hspace{1cm} (19)

Ritter’s model (45°) and variable strut inclination (NTC) have been simplified assuming the value of $k_2$ coefficient is equal to 0.8, see equation (17) and (18). Eurocode 8 model (EC8), see equation (19), requires additional hypotheses that do not affect the general reliability of the comparison. The value of the neutral axis (x) in the formula was assumed equal to $(vh/0.8)$. This x is obtained according to three main hypotheses, assuming the stress block stress-strain relationship for concrete: (i) the reinforcement is made of only two registers, (ii) the area of steel in compression and tension is equal and (iii) both steel in compression and tension have attained yielding, so that the stress in the steel is equal to $f_y$ (see the next section for further details). $k_1$ coefficient, equal to $(d-d')/h$, was fixed to 0.8. Moreover $v$ cannot exceed 0.55, the geometrical reinforcement ratio ($\rho_{tot}$)
is higher than 0.5% and $L_V/h$ is lower than 5.

As it was described previously, the maximum value of the shear strength degradation coefficient in Eurocode 8 model is equal to 0.75. Consequently, in equation (19), $\beta$ coefficient is equal to 0 in the absence of any degradation, and equal to 0.25 in the case of maximum shear strength degradation.

By means of the non-dimensional expressions of the shear capacity in equations (17) to (19), it is possible to show the ratio between the capacities of the models considered. Fixing the value of $\rho_{tot}$ to 0.01 and 0.02, the concrete compression strength ($f_c$) to 20 MPa (simulating an example of likely characteristics for an existing building [44]), the ratio between Eurocode 8 formulation ($V_{EC8}$) and Ritter’s model ($V_{45^\circ}$), can be plotted as function of $\nu$ and $\omega_{sw}$, respectively if $L_V/h$ equals to 3 and 5, see Figure 10 and 11, assuming $\beta$ equal to 0.25 (maximum shear strength degradation). The same plot, with the same hypotheses can be made when considering the ratio between $V_{EC8}$ and the variable strut inclination model ($V_{NTC}$), see Figure 12 and 13.

Figure 10. Ratio between EC8 shear capacity (for $\beta=0.25$, maximum strength degradation) and Ritter’s model in the case of $L_V/h=3$ for $\rho_{tot} = 0.01$ (a) and $\rho_{tot} = 0.02$ (b).

Figure 11. Ratio between EC8 shear capacity (for $\beta=0.25$, maximum strength degradation) and
Ritter’s model in the case of $L_v/h=5$ for $\rho_{tot} = 0.01$ (a) and $\rho_{tot} = 0.02$ (b).

The ratio $V_{EC8}/V_{45^\circ}$ shows how the maximum shear strength degradation in $V_{EC8}$ can lead to a capacity that is even lower than the value obtained with Ritter’s model. This effect is emphasized with the increases of the shear span ratio ($L_v/h$) and the decreases of $v$. In fact, the normalized axial force rules the weight of $V_N$ in the Eurocode formulation that represents the only contribution without degradation, see equations (11) to (14). The increasing $\rho_{tot}$ reduces this effect thanks to dowel action in the concrete contribution that appears in equation (11).

Figure 12. Ratio between EC8 shear capacity (for $\beta=0.25$, maximum strength degradation) and variable strut inclination model in the case of $L_v/h=3$ for $\rho_{tot} = 0.01$ (a) and $\rho_{tot} = 0.02$ (b).

Figure 13. Ratio between EC8 shear capacity (for $\beta=0.25$, maximum strength degradation) and variable strut inclination model in the case of $L_v/h=5$ for $\rho_{tot} = 0.01$ (a) and $\rho_{tot} = 0.02$ (b).

The ratio of $V_{EC8}$ with $V_{NTC}$ emphasizes how $V_{NTC}$ can lead to an overestimation of the shear capacity that cannot be acceptable in the assessment framework. This effect is observed in the range of $\omega_{sw}$ characterizing existing buildings, from approximately 0.02 to 0.1.

This result confirms the results shown in Figure 8c and Figure 9. The value of $\cot \theta$ equal to 2.5 is not well calibrated in the case of lightly transversal reinforced elements subjected to cyclic load,
in which $V_{Rsd}$ rules the minimum capacity, see equation (3). Figure 12 and 13 emphasize the fact that the regression formulation according to equation (11) provides a strength always lower than that provided by the variable strut inclination. The only exception is the case of very low transversal reinforcement ratio, approximately equal to 0.005 (increasing with increasing $\nu$ and $\rho_{tot}$, and with decreasing $L_V/h$); in which Eurocode 8 part 3 capacity is higher than that evaluated according to the variable strut inclination model.

5. A PRACTICE-ORIENTED APPROACH FOR THE ASSESSMENT OF SHEAR FAILURES

Shear failures can limit the global displacement capacity of existing RC buildings. A brittle failure, in fact, even if it represents a local event in a building, leads the structure to collapse. Capacity design rule prevents brittle failures imposing shear-flexure hierarchy. Thus, the behavior of all new designed elements can be always assumed as ductile.

An element can be defined as ductile when the maximum shear that can be registered because of flexural behavior ($V_{flex}$) does not exceed the shear capacity ($V_{shear}$); whereas an element can be defined as brittle when $V_{shear}$ is lower than the value of $V_{flex}$. In the case of shear capacity models that accounts for strength degradation, the so called limited-ductility behavior can be defined. It represents the case in which brittle failures limits the ductility capacity of the elements, see Figure 14.

While in new buildings ductile behavior can be taken for granted because of design, RC elements in existing buildings can likely show brittle failures or limited ductility behavior (if a degrading shear capacity model is employed). Thus, it is fundamental to classify element behaviors within the assessment framework. Herein, a simplified tool for the assessment of brittle failure in RC buildings is carried out employing an approximated classification procedure.

The general idea is to create a failure domain once the shear capacity model is chosen. Such a failure domain is obtained by means of an equation that equalizes the maximum shear demand ($V_{flex}$) with the shear capacity ($V_{shear}$), see equation (20). Maximum shear demand or plastic shear corresponds to the shear value for which both the maximum bending moments are attained at the
two end sections of the element. In particular, the plastic shear ($V_{flex}$), described below is specialized for the case of RC columns.

Thanks to different hypotheses described below, equation (20) can be represented in the 2D plane of transversal and longitudinal geometrical reinforcement ratios ($\rho_{sw} - \rho_{tot}$), normalizing both $V_{flex}$ and $V_{shear}$ by means of the maximum axial load of the section ($bhf_c$).

$$\frac{V_{flex}}{bhf_c} = \frac{V_{shear}}{bhf_c}$$ (20)

By using simplified expressions of shear capacities shown in section 4, it is possible to obtain the failure domains for each shear capacity model employed in European and Italian codes, Ritter’s model ($45^\circ$), variable strut inclination (NTC), and the hypotheses of absence ($\beta=0$) or maximum shear strength degradation ($\beta=0.25$) for the Eurocode 8 part 3 (EC8).

Since the second member of equation (20) was obtained for all the models described above, it is now necessary to define an approximate formulation for $V_{flex}$. The maximum shear flexural demand needs to be expressed in a simplified way assuming some conservative basic hypotheses:

a) the longitudinal reinforcement is made of two registers, ($A_{tot}=A_x + A_y$);
b) there is the same steel area in tension and compression, ($A_x = A_y$);
c) both the compression and tension reinforcement have attained yielding ($\sigma_s=f_y$).

The first two hypotheses, a) and b), are well suited for the typical longitudinal reinforcement of columns. In fact, they have symmetrical reinforcement with respect to the centroidal axes of the rectangular section. In particular, hypothesis a) is justified by two main aspects. The first aspect is that, according to typical gravity and old seismic design, a representative frame used to be chosen in the principal direction of the building and all the design procedure was based on it [45]. This approach, other than reducing the computational effort required, used to lead to uniaxial design of columns. The resulting reinforcement configuration in two registers used to optimize the quantity of steel employed, given the design bending moment. The second aspect is that, in any case, hypothesis a) is conservative; it results in a maximization of the moment capacity of the section, and consequently of $V_{flex}$. However, the knowledge of the exact position of steel bars in the section represents detailed information that goes beyond the approximate and practice-oriented approach of the methodology.

Hypothesis c) is clearly a non-rigorous approximation; however, it leads to acceptable accuracy in the evaluation of the moment capacity, as shown in [29]. The approximation of hypothesis c) has a slight impact in the case of low and high value of the axial force. Essentially, for low normalized axial force ($\nu=0.20$), the compression longitudinal reinforcement can be not yielded, and the
The flexural capacity is slightly underestimated. Conversely, for high normalized axial force (\(\nu \geq 0.45\)), the tension longitudinal reinforcement can be not yielded, and the flexural capacity is slightly overestimated.

Hypotheses a), b), and c) allow expressing \(V_{\text{flex}}\) in the form shown in equation (24), and detailed in equation (21) to (23). \(V_{\text{flex}}\) normalized by the maximum axial force of the section (\(bf_c\)), is shown in equation (25), assuming the value of neutral axis \(x=vh/0.8\) (as a consequence of hypotheses b) and c), see [29]) and the value of coefficient \(k_1=(d-d')/h\), equal to 0.8, again.

\[
N = 0.8xbf_c - A_yf_y + A_xf_y = 0.8xbf_c \tag{21}
\]

\[
x = \frac{N}{0.8bf_c} = \frac{vh}{0.8} \tag{22}
\]

\[
M_y = 0.8xbf_c \left(\frac{h}{2} - 0.4x\right) + f_yA_x(d-d') = \frac{Nh}{2}(1-\nu) + f_yA_x(d-d') \tag{23}
\]

\[
V_{\text{flex}} = \frac{M_y}{L_y} = \frac{1}{L_y} \left[ \frac{Nh}{2}(1-\nu) + f_yA_x(d-d') \right] \tag{24}
\]

\[
V_{\text{flex}} = \frac{y}{2L_y} \left[ \nu(1-\nu) + \frac{\rho_{\text{tot}}f_y}{f_y}k_1 \right] \tag{25}
\]

Equalizing the value of normalized \(V_{\text{flex}}\) and \(V_{\text{shear}}\), the latter was evaluated according to the three different models used in the European and Italian code [8,10,11]; the boundary of brittle and ductile failure can be defined in the 2D plane (\(\rho_{\text{sw}} - \rho_{\text{tot}}\)). Shear span ratio (\(L_v/h\)), normalized axial force (\(\nu\)), and material properties need to be defined.

For slender columns (\(L_v/h>2\)), equations (26) to (28) show brittle-ductile failure boundary in the case of Ritter’s model (45°), NTC and EC8 part 3 model. An equivalent definition of such domains can be made in the 2D plane of mechanical reinforcement ratios (\(\omega_{\text{sw}} - \omega_{\text{tot}}\)) as shown in [46, 47]. On the other hand, geometric reinforcement ratios allow a direct comparison with design prescriptions and building practice; thus leading to a straightforward evaluation of the elements that can be more likely characterized by brittle failures.

In the case of Eurocode 8 part 3 formulation, the error on the neutral axis, caused by the approximation (\(x=vh/0.8\)) affects both the evaluation of \(V_{\text{flex}}\) and \(V_{\text{shear}}\). In \(V_{\text{EC8}}\), see equations (12) and (19).

\[
\rho_{\text{tot,45°}} = \left[ \frac{2L_y}{h}k_2f_yv(1-\nu)f_y}{k_1} \right] \frac{1}{f_y} \tag{26}
\]
\[
\rho_{\text{tot, NTC}} = \left[ \frac{2L_v k_2}{h k_1} \rho_{\text{sw}} \cdot f_{yw} \cdot \cot \theta^* - \frac{v(1-v)f_c}{k_1} \right] \frac{1}{f_y}
\]

\[
\rho_{\text{tot, EC8}} = \left[ \frac{1}{2} \gamma_{cl} \cdot \nu + \left( \frac{\gamma_{cl} - 1.25}{2} \right) \nu^2 \right] \cdot f_c + \left( 1 - \beta \right) \frac{L_v}{h} \cdot \rho_{\text{sw}} \cdot f_{yw} \cdot k_1
\]

\[
\rho_{\text{tot, EC8}} = \left[ \gamma_{cl} \cdot \frac{k_1}{2} \cdot f_c - \left( 1 - \beta \right) \frac{L_v}{h} \left( 16 - 2.56 \cdot \frac{L_v}{h} \right) \right] \frac{f_c}{\nu}
\]

The failure domains point out the \( \rho_{\text{sw}} \) and \( \rho_{\text{tot}} \) values leading to brittle or ductile behaviors. Figure 15 and 16 show an example of the failure domains for the three shear capacity models assuming: \( L_v/h \) respectively equal to 3 and 5, the average value for concrete compressive strength equal to \( f_c = 25 \) MPa, and the yielding strength for both transversal (\( f_{yw} \)) and longitudinal (\( f_y \)) steel equal to 450 MPa. Decreasing \( L_v/h \) and increasing \( \nu \) values, as expected, decrease the ductile domain. \( L_v/h \) is the parameters that influences stricter the classification.

Figure 15. Fast assessment domains for Eurocode 8 shear capacity model without (solid lines) and with maximum (dashed lines) shear degradation in the case of \( L_v/h = 3 \) (a) and \( L_v/h = 5 \) (b), assuming \( f_c = 25 \) MPa and \( f_y = f_{yw} = 450 \) MPa.

Figure 16. Fast assessment domains for variable strut inclination (dotted lines) and Ritter-Mörsch (dotted-dashed lines) shear capacity models in the case of \( L_v/h = 3 \) (a) and \( L_v/h = 5 \) (b), assuming \( f_c = 25 \) MPa and \( f_y = f_{yw} = 450 \) MPa.

In the Eurocode capacity model, it can be observed that given the lack of details on ductility...
demand, the maximum shear strength degradation should be employed (\(\beta=0.25\)). Elements characterized by a limited ductility behavior should be classified as brittle; leading to a more conservative evaluation that better suits the approximate hypotheses made when obtaining the boundary equation in the \((\rho_{sw} - \rho_{tot})\) plane.

A first test bed for the practice-oriented approach carried out can be the classification of the 51 columns of the database employed in section 3. Their behaviour tends initially to be dominated by flexure, whereas the ultimate failure and deformation capacity appears to be controlled by shear mechanisms (limited ductility behavior); EC8 formulation can capture this type of failure. Assuming \(\beta\) equal to 0.25 and 0, the two bound values of \(\rho_{tot}\) \((\rho_{tot,\beta=0.25}, \rho_{tot,\beta=0})\), given the real \(\rho_{sw}\) of the columns, can be determined according to equation (28). If the real \(\rho_{tot}\) of the 51 columns is comprised in the interval \([\rho_{tot,\beta=0.25} ; \rho_{tot,\beta=0}]\), the practice-oriented approach would predict the failure observed through the experimental investigation. The practice-oriented approach leads to the following predictions: limited-ductility failure in 57\% of cases \((\rho_{tot,\beta=0.25} \leq \rho_{tot,\text{real}} \leq \rho_{tot,\beta=0})\), brittle failure in 27\% of cases \((\rho_{tot,\text{real}} > \rho_{tot,\beta=0})\), and ductile failure in 16\% of cases \((\rho_{tot,\text{real}} < \rho_{tot,\beta=0.25})\), see Figure 17. The ductile failures represents situations in which the prediction fails. The 16\% of cases, in which the prediction fails, is comparable to 19\% CoV of EC8 in Table 1. Removing a), b), and c) hypotheses from the evaluation of \(V_{\text{flex}}\), the predicted failure mode would still be ductile for that 16\% of cases. The latter observation emphasizes how, in the case of the experimental database in section 3, the hypotheses made to carry out the approximate formulation do not affect the final output of the approach.

![Figure 17](image-url)

Figure 17. \(\rho_{tot,\beta=0.25}\) (grey bars), and \(\rho_{tot,\beta=0}\) (white bars) evaluated according to equation (28) for the 51 columns of the database in [15], compared with the real value of \(\rho_{tot}\) (red dotted line) in the specimens. Columns identifiers are numbered according to the same criterion adopted in [15].

The failure domains obtained can be useful in fast or preliminary assessments, when the amount
of brittle failures is of concern because of its impact on retrofitting costs. Furthermore, such domains can be a useful tool also in large-scale assessments. The information necessary for the failure classification can be easily carried out. In fact, given the age of construction, from codes and practical design rules at the time, it is possible to obtain code mandated prescriptions regarding element section dimensions and percentages of longitudinal and transversal reinforcements; while material properties (concrete and steel) can be evaluated from codes and commentaries, or from databases available in the literature and data from in situ inspections (e.g. [42,48-49]). These kinds of information recruiting are also the basis for the definition of building portfolios according to analytical/mechanical vulnerability assessment procedures for existing reinforced concrete structures (e.g. [50-54]).

5.1. Practical examples

A first practical application of such domains has been pursued in a RC building in Italy, designed in the 80s’ according to old seismic prescriptions [47]. The output of the preliminary classification of failure mode of columns versus the results of the detailed assessment showed excellent accordance. The case study building in [47] emphasized also a general tendency towards brittle failure mode in columns of buildings designed according to old seismic design criteria. The brittle failure mode is justified by the shear design approach employed up to the period in which countries switched from allowable stresses to limit states approach (e.g. limit states approach was introduced in Italy in 1996 [37]).

Relevant code prescriptions [55-59] in different countries, employing allowable stress approach, previously suggested a threshold value for the maximum tangential stress $\tau$ equal to $\tau_0$. If the tangential stress did not exceed a minimum limit value $\tau_0$ (corresponding to diagonal cracking initiation), the shear was assigned to the concrete and the minimum transversal reinforcement was provided. If the tangential stress $\tau$ exceeded $\tau_0$, the transversal reinforcement ought to be designed. Some codes [56,58] prescribed to design the transversal reinforcement considering the complementary part of the total shear subtracting that taken by the concrete. Whereas, the majority of European codes [54,55,57] assigned the whole shear to the transversal reinforcement and they did not rely on the resisting mechanisms of the concrete alone. Furthermore, in order to avoid excessive compression in the concrete inclined struts, the tangential stress was bounded also by an upper limit $\tau_1$. The transversal reinforcement was designed adopting the Ritter’s model with constant inclination of the struts equal to 45°. For further details regarding old codes design provisions, please see [4,54].

Generally speaking, it would seem reasonable to assume that a building designed according to
old seismic criteria has better performances than a structure in which seismic forces were not considered at all. On the contrary, given the old code provisions for shear design in both seismic and non-seismic situations, described above, it is easy to recognize that brittle failures can more likely occur in the case of old seismic design. In fact, according to those old design criteria, seismic forces led to a higher percentage of longitudinal reinforcement in columns (higher $\rho_{\text{tot}}$ compared to gravity load design) and, at the same time, transversal reinforcement in columns was not designed for both cases of seismic and non-seismic design, since shear stresses seldom exceeded the value of $\tau_0$ prescribed by old codes.

For example let us consider a gravity designed column, characterized by a specific value of $\rho_{\text{tot}}$, $\rho_{\text{sw}}$ and $\nu$, in which $\rho_{\text{sw}}$ is ruled by the minimum transversal reinforcement prescribed by codes, since $\tau_0$ is not exceeded. Now, assuming the same section dimensions for a seismic designed column, the value of $\nu$ and $L_V/h$ are the same, $\rho_{\text{tot}}$ is higher to bear the bending moment resulting from horizontal loads and still shear stress demand does not exceeds $\tau_0$, resulting in the same $\rho_{\text{sw}}$. In such cases, from Figures 15 and 16, it is easy to recognize that the seismic designed column is more likely characterized by shear failure. Let us consider another example where section dimensions are higher in the case of the seismic designed element, so $\rho_{\text{tot}}$ can be equal respect to the case of non seismic design and, again, $\tau_0$ is not exceeded and $\rho_{\text{sw}}$ is equal in both cases, $\nu$ is lower in the seismically designed case but $L_V/h$ is smaller, finally resulting in a higher probability of occurrence of brittle failure in the seismic designed elements; please compare Figure 15a and 15b or Figure 16a and 16b. A strong qualitative example of the above observations can be found in reconnaissance reports after earthquakes [2-6], in which brittle failures in columns are reported frequently.

Data are available from school buildings in the region close to the area struck by the 2009 L’Aquila earthquake [44], a region considered seismically prone since the first decades of the twentieth century and in which most of the reinforced concrete structures used obsolete seismic design criteria [4]. The data available in [44] report frequent occurrences of shear failures and emphasize how a significant part of the retrofitting costs can be produced by the repair or the prevention of brittle failure. These data represents a good representative of buildings realized between 70s and 90s in medium seismicity areas in Italy. The median value of $L_V/h$ in columns is equal to 3 and ranges between 2 and 4; while $\rho_{\text{tot}}$ ranges between 0.5% and 2.5% with a median value equal to 1% and $\rho_{\text{sw}}$ ranges between 0.1% and 0.2% with a median value equal to 0.15%. In this sample[44], normalized axial force ($\nu$) in columns evidently depends on the number of stories of each building; on the other hand, the median value of $\nu$ for the first storey is equal to 0.25. Figure 18 shows the comparison of average data of columns in [44] with the fast assessment domains
evaluated for the European shear strength capacity model. The average values lead to a limited ductility element if Eurocode 8 part 3 model is employed, a brittle element if Ritter’s model is employed and a ductile element (actually very close to the boundary of the domain) if the variable strut inclination model is employed.

Figure 18. Fast assessment domains for Eurocode 8 shear capacity model without (solid lines) and with maximum (dashed lines), (a), and variable strut inclination (dotted lines) and Ritter-Mörsch (dotted-dashed lines) shear capacity models, (b), in the case of \( \text{Lv}/\text{h}=3 \) and \( \nu=0.25 \), assuming \( f_{c}=25 \text{ MPa} \) and \( f_{y}=f_{yw}=450 \text{ MPa} \) and compared with the average data of columns in [44].

The practice-oriented approach provides a classification of the potential failure mode of the columns. The demand of the columns can be in some cases lower than their capacity; it is the case of elements that are not involved in the plastic mechanism of a building (e.g., columns in upper storeys). Generally, if the detailed assessment of a building is made through a nonlinear analysis method, the plastic mechanism can be recognized and only the elements involved in it are retrofitted according to a capacity design approach. In contrast, if the analysis method is linear and no information is available regarding the plastic mechanism characterizing the building, it is good practice to retrofit according to a capacity design approach all the elements. So the potential failure mode of the columns becomes a significant information for the assessment.

5.2. **Material uncertainties**

Material properties are always characterized by uncertainties and probabilistic characterization of mechanical properties can help significantly within a practice-oriented fast assessment framework.

An example of the influence of material properties is shown in Figure 19 and 20. In the figures the domains are obtained in the 2D plane \( (\rho_{sw} - \rho_{tot}) \), assuming two normal probability density functions (pdf) for concrete compressive strength, \( f_{c} \), and for steel of transversal and longitudinal reinforcement, respectively \( f_{yw} \) and \( f_{y} \). In both figures, concrete compressive strength normal distributions are evaluated considering a mean value of 25 MPa. This value is also adopted in [52]
and it is comparable to in situ concrete compressive strength in Italian buildings between the 70s and 90s [49]. The coefficient of variation (CoV) was adopted equal to 0.12, according to the value employed in [50]. It is worth to note that such a value of the CoV is smaller than the value assumed in [49,52] ranging between 25% and 47%. In the opinion of the authors, these CoV values can be too high considering that experimental campaigns tend to collect together concrete cores coming from different buildings not grouped according to the allowable stress employed in design that can significantly differ. So, the latter approach estimates high CoV values that tend to overestimate the variability of concrete compressive strength in a single building or in a single group of buildings designed with the same allowable stress. The latter is the reason why CoV value in [50] was preferred.

Figure 19. Material uncertainties in the fast assessment domains for Eurocode 8 shear capacity model without (grey) and with (black) shear degradation. 16°, 50° and 84° percentiles are shown for AQ50 steel pdf [42,48] and concrete pdf evaluated [49,50,52].

For steel reinforcement pdfs the database presented in [42,48] was employed. In the case of steel strength the employment of databases is more reliable than the case of concrete. In fact, steel is not produced in situ, consequently it is more controlled, and it is characterized by a homogeneous properties along building stocks referring to specific periods and areas. Figure 19 and 20 refer respectively to the case of employment of smooth bars and ribbed bars. In Figure 19 the case of smooth reinforcement is considered; $f_y$ and $f_{yw}$ normal distributions are assumed considering an AQ50 steel in the period 1970-1980; the mean is equal to 371.4 MPa, the standard deviation ($\sigma$) is equal to 29.2, and the CoV is equal to 0.078. In Figure 20 the case of ribbed bars is considered; $f_y$ and $f_{yw}$ normal distributions are assumed considering an FeB44k steel in the period 1980-1990; the mean is equal to 511.7 MPa, the standard deviation ($\sigma$) is equal to 42.0, and the CoV is equal to
A Monte Carlo simulation on $f_c$, $f_y$ and $f_{yw}$ values was performed obtaining the percentiles shown in Figures 19 and 20. $f_y$ and $f_{yw}$ values were assumed with the same pdf, sampling independently their realizations. Results for $L_V/h$ equal to 3 and 5 and $\nu$ equal to 0, 0.25 and 0.5 are shown for the EC8 model in both the case of absence and maximum shear strength degradation.

EC8 model with maximum shear strength degradation ($\beta=0.25$) shows smaller variability than the case of absence of degradation ($\beta=0$). Since it is more conservative, the case of maximum degradation represents the better choice for fast assessment. The increasing $L_V/h$ reduces the effect of material uncertainties. The latter effect of $L_V/h$ is caused by the analytical form of equation (28). The structural counterpart of such an effect is that geometrical characteristics (e.g. slenderness of the element) becomes predominant respect to the effect of material properties on the evaluation of shear-flexure hierarchy.

Figure 20. Material uncertainties in the fast assessment domains for Eurocode 8 shear capacity model without (grey) and with (black) shear degradation. 16°, 50° and 84° percentiles are shown for FeB44k steel pdf [42,48] and concrete pdf evaluated according to [49,50,52].

In Figure 21, the resultant coefficient of variation (CoV) for Eurocode 8 model (in the case of $\beta=0.25$) is shown in both the cases of smooth and ribbed bars, referring to the data already shown in Figure 19 and 20. It is possible to recognize a constant trend of the CoV with $\rho_{sw}$ in the case of normalized axial force ($\nu$) equal to zero. The significant difference of the CoV registered in the case $L_V/h$ equal to 3, between smooth and ribbed bars, is caused by the lower average value of the steel yielding strength in the case of smooth bars, that becomes critical in the case in which slenderness of the elements is not ruling equation (28). The structural counterpart of such effect is that the variability of concrete compressive strength has a higher impact on the evaluation of shear-flexure hierarchy in the case of low values of steel yielding strength and low values of $L_V/h$ ratio. It is
worth noting that very small values of $\rho_{sw}$ lead to a numerical instability of equation (28) leading to a localized significant increase of the CoV. On the other hand, it is rare to find $\rho_{sw}$ values lower than 0.05% in existing buildings.

Figure 21. Trend of the coefficient of variation (CoV) with the geometric transversal reinforcement ratio ($\rho_{w}$), for Eurocode 8 shear capacity model with maximum shear strength degradation, for AQ50 steel (smooth) and FeB44k steel (ribbed) considered in Figures 19 and 20, respectively.

6. CONCLUSIONS

A practice-oriented approach for the assessment of shear failures in existing reinforced concrete (RC) elements was carried out. Such a practice-oriented tool asks for basic information on the element, such as, information on the total amount of longitudinal and transversal reinforcements, the value of the normalized axial force and, above all, the shear span ratio of the element. The information allows carrying out the boundary between brittle and ductile domains in the 2D plane of transversal and longitudinal reinforcement ratios. Such domains can be employed in assessment problems at different scales. Regardless of the type of employment of domains, uncertainties on material properties can play a significant role. Thus, brittle-ductile domains can be obtained by accounting for probability density functions of material mechanical characteristics and allow the user to choose the most suitable fractile according to the confidence given to material properties assumed.

The evaluation of shear-flexure hierarchy cannot ignore the issue regarding the most suitable shear capacity model to be employed for assessment. In fact, given the complexity of physic phenomena ruling shear strength in reinforced concrete elements, different theories and consequently different analytical approaches are available in the literature and guidelines. A detailed code review regarding shear capacity formulations and a consequent comparison in terms
of experimental and normalized results permits the selection of the capacity model to be employed in the assessment. The majority of the attention has been addressed to the European regulation given the upcoming adoption of Eurocode in all European countries.

REFERENCES


