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A NOVEL APPROACH TO PRODUCING SPECTRUM COMPATIBLE GROUND MOTIONS USING VOLTERRA SERIES

Nicholas A ALEXANDER1, Andrew A CHANERLEY2, Adam J CREWE3 and S. BHATTACHARYA4

Abstract: In this paper we discuss the application of Volterra series in the process of morphing of real ground motion accelerograms into ones that are spectrum compatible. The aims are, firstly that the morphed accelerogram should have a response spectrum that is very near to the target spectrum. Secondly, it should be very similar to the original seed accelerogram; specifically it should have a similar general envelope, with main pulses and peaks in similar temporal locations in the timeseries. Thus, the morphed record should visually and qualitatively appear like a real accelerogram. Thirdly the morphed record should allow for reasonably stable numerical integration so that the ground velocity and displacement timeseries can be obtained. Additionally we provide a Matlab toolbox (GUI) which implements this Reweighted Volterra Series Algorithm (RVSA)

Introduction
In any seismic analyses of structural artefacts there is large uncertainty in the input ground motions. Obtaining robust statistics, that include ground motion timeseries estimates, for future seismic events is a significant problem. Hence, this unknown seismic loading places bounds on the credibility of any sophisticated nonlinear dynamic system analyses (Nazri and Alexander 2014). Determining and bounding this unknown seismic loading has for most design codes around the world concentrated on satisfying a response spectrum of some kind. From a structural engineering perspective the attractiveness of employing a response spectrum was that it was subsequently possible to avoid all time-history analyses. In a sense the response spectrum attempts to characterise the unknown multivariate statistics of seismic ground motion timeseries when convolved through a structural system.

Nevertheless, the feasibility and economic benefit of nonlinear time-history analyses of structural/geotechnical systems has become more widespread in design practice. A performance based design philosophy has led to much more interest in levels of damage at a range of design limit states. However, time-history analysis requires ground motion timeseries rather than a response spectrum. So, the problem is that design codes have captured estimates of the unknown multivariate statistics of seismic ground motion in a response spectrum but a response spectrum does not mathematically imply a unique timeseries. Consequently we are presented with the inverse problem of conjecturing a ground motion timeseries that when it is convolved through a single degree of freedom system results in a given target, code specified, response spectrum.

Some advocate the use of real ground motions carefully selected such that their sample statistics are credible in the context of designing the structural artefact in a prescribed geographical location. This approach is not without its merits however, there are two main problems with this approach (i) the number of timeseries used must be large enough to ensure the robustness of the inferred sample statistics. It is worth considering that this is problematic since we have a limited spatio-temporal database of ground motion timeseries. (ii) Employing

1 Senior Lecturer, University of Bristol, Bristol
2 Senior Lecturer, University of East London, London
3 Reader, University of Bristol, Bristol
4 Professor, University of Surrey, Surrey.
a large sample, of timeseries, in the case of a complex, nonlinear, structural artefact places an excessively onerous computational effort on the analyst/designer. It is worth underlining the repetitive and evolutionary nature of design analyses given the input of various key stakeholders, e.g. client, Engineer, Architect and local public planning authorities. That is to say the structural form may evolve throughout the design phase and at each stage re-analyses may be required.

As an alternative, many (Rizzo, Shaw et al. 1975, Lihanand and Tseng 1988, Abrahamson 1992, Ostadan, Mamoon et al. 1996., Mukherjee and Gupta 2002, Bazzurro and Luco 2006, Hancock, Watson-Lamprey et al. 2006, Cacciola 2010, SeismoSoft 2012, Cecini and Palmeri 2015) advocate the generation of artificial timeseries that match a given spectrum. This approach is easy to criticise, (Priestley 2003, Al Atik and Abrahamson 2010), as it is considered overly conservative since these artificial timeseries have a flat and smooth broadband response spectra that unlike most real earthquakes. Nevertheless, the perceived attractiveness of spectrally matched records is that fewer accelerograms need be used in nonlinear time-history analyses. As an approach, it can be viewed as less sensitive to sample selection as all matched record should approximate the population statistics of site specific earthquakes, in terms of their spectra.

Using Volterra Series
There have been many proposed algorithms for obtaining a spectral matched timeseries. However, we seek an algorithm that modifies a real ground motion seed, so that its non-stationary envelope is maintained while matching some target spectrum. The early approaches based on the Fourier transform do not achieve this. We are also looking for an algorithm that is stable regardless of input seed record or the target spectrum; this is not always the case with some of the later approaches. Finally, we are looking for an excellent match to the target spectrum. The proposed algorithm in this paper is an improvement on what has gone before and is described in detail in (Alexander, Chanerley et al. 2014) along with a comparison with (SeismoSoft 2012). The Matlab toolbox RVSAmatch (Alexander 2015), see Figure 1, has been provided to disseminate the algorithm more generally.

Figure 1, RVSAmatch (Matlab toolbox) version 1.1 (Alexander 2015)
The Volterra series originates as a generalisation of the Taylor series expansion of a function. It describes a general nonlinear process that transforms one timeseries into another. In (Alexander, Chanerley et al. 2014) we conjecture a nonlinear process that transforms a timeseries $\ddot{x}(t)$ (which is some ground motion acceleration timeseries) into $\ddot{x}_M(t)$ (a response spectrum matching acceleration timeseries). A Volterra model is arbitrary as it models a generic nonlinear process. In this paper the wavelet transform is employed in order to make a discrete estimate the Volterra kernels of the series expansion (of $\ddot{x}(t)$) that maps $\ddot{x}(t)$ into an infinite set of $\ddot{x}_M(t)$. Given that $\ddot{x}_M(t)$ is some Volterra series expansion of $\ddot{x}(t)$ it inherits, to some degree, the shape and form of $\ddot{x}(t)$. Consequently, we seek a particular member of this set of $\ddot{x}_M(t)$ that has a response spectrum that matches some target response spectrum. Therefore this process can be modelled using a discrete Volterra series expansion, equation (1), on a wavelet basis $\phi_i(t)$ of $\ddot{x}(t)$ (equation (2)). The wavelet decomposition of the seed record $\ddot{x}(t)$, in equation (2), is achieved by using the stationary wavelet transform (SWT). The SWT is better than the discrete wavelet transform (DFT) as it is shift invariant and does not alias. In order to maintain a linear phase characteristic the Biorthogonal wavelet family is recommended.

$$\ddot{x}_M(t) = \sum_{i=1}^{m} \beta_1 \phi_i(t) + \sum_{i=1}^{m} \beta_2 \phi_i(t) \phi_j(t) + \sum_{i=1}^{m} \beta_3 \phi_i(t) \phi_j(t) \phi_k(t) + \cdots$$ (1)

$$\ddot{x}(t) = \sum_{i=-1}^{m} \phi_i(t)$$ (2)

where $m$ is the total number of wavelet levels (approximation plus detail) and $\beta_1$, $\beta_2$ and $\beta_3$ are time-invariant coefficients (weights) for Volterra kernels 1, 2 and 3. In this case, these coefficients can be considered to represent a discrete approximation of the continuous Volterra kernels. Hence, we re-express the Volterra series, equation (1), as the following linear combination of $\phi_i(t)$ and all positive multinomial combinations of $\phi_i(t)$, thus

$$\ddot{x}_M(t) = \mathbf{B}^T \mathbf{\Psi}, \quad \mathbf{\Psi}, \mathbf{B} \in \mathbb{R}^{m \times q}$$ (3)

where vectors $\mathbf{B}$ and $\mathbf{\Psi}$ are vectors (with $q$ rows) defined as follows

$$\mathbf{B}^T = \begin{bmatrix} \beta_1, \cdots, \beta_m, \beta_{11}, \cdots, \beta_{1m}, \beta_{21}, \cdots, \beta_{2m}, \cdots \end{bmatrix}, \quad \mathbf{\Psi}^T = \begin{bmatrix} \phi_1, \cdots, \phi_m, \phi_1 \phi_2, \cdots, \phi_1 \phi_m, \phi_2 \phi_m, \cdots \end{bmatrix}$$ (4)

where subscripts $i, j, k \in [1, m]$. This Volterra series expansion, e.g. $\ddot{x}_M(t)$ in equation (3), is expressed simply as linear combinations of functions within $\mathbf{\Psi}$. It is worth considering the implication of expressions (3) and (4). It is clear that each $\phi_i$ shares a similar non-stationary envelope with its progenitor signal $\ddot{x}(t)$. Hence all multinomial combinations, i.e. $\phi_1\phi_2$, $\phi_2\phi_3$ and so on, also share this similar non-stationary envelope. Thus, re-weighting (i.e. changing) the $\mathbf{B}$ coefficients in (3) allows for a modification of frequency and phase content of the signal while keeping the general form of the non-stationary nature of the signal $\ddot{x}(t)$. That is to say $\ddot{x}(t)$ and $\ddot{x}_M(t)$ can have qualitatively similar non-stationary features e.g. very similar envelopes and pulses. Therefore our objective here is to identify optimal values of these time-invariant weights $\mathbf{B}$ such that $\ddot{x}_M(t)$ is response spectrum matching.
Now consider a response spectrum \( s(\ddot{x}_w(t), f) \) produced from a signal \( \ddot{x}_w(t) \) at a discrete set of \( r \) frequencies in vector \( \vec{f} \). Here, the original signal \( \ddot{x}(t) \) and morphed signal \( \ddot{x}_w(t) \) are accelerograms. Thus, the problem of morphing a real accelerogram into a spectrum compatible one can now be expressed as a nonlinear least squares. We seek to minimise the difference between the spectrum \( s(\ddot{x}_w(t), f) \) and some target spectrum \( s_f(\vec{f}) \) by selecting the optimal and re-weighted coefficients \( \vec{\beta} \). An objective function \( v(\vec{\beta}) \) for this nonlinear least squares/norm problem can be stated as follows

\[
\min_{\vec{\beta}} \left\| v(\vec{\beta}) \right\|, \quad \text{where} \quad v(\vec{\beta}) = \frac{s(\ddot{x}_w(t), \vec{f}) - s_f(\vec{f})}{s_f(\vec{f})}
\]

Equation (5) represents \( r \) nonlinear algebraic equations with unknown coefficients \( \vec{\beta} \). We seek to minimise the Euclidian norm of \( v(\vec{\beta}) \) in order to determine the optimal spectrum matching morphed \( \ddot{x}_w(t) \). So the morphed timeseries is based on optimal and re-weighted coefficients \( \vec{\beta} \). This problem can solved using the Levenberg-Marquardt algorithm or Trust-Region-Reflective approach. Levenberg-Marquardt algorithm as it is able to deal with both over-determined (least square) problem i.e. \( q < r \) and under-determined ones (least norm) i.e. \( q > r \). Least norm problems are not such a good idea in this case. Thus, it is recommended that the number of frequency points in the target response spectrum is greater than \( q \). The Matlab RVSA toolbox will re-sample the target spectrum such that \( q < r \).

The solutions obtained from the Levenberg-Marquardt or Trust-Region-Reflective algorithms are only local optima and not a guaranteed global optimum. The searching algorithm that would be required for a global optimum is extremely costly in terms of computational resources i.e. time and memory, thus it has not been explored in this paper. We present only the first local optimum obtained from the given initial start and show that it is sufficiently good for our purposes. The initial start we have employed is the original record \( \ddot{x}(t) \) which is defined by the following coefficients \( \beta_i = 1, \quad i \in [1, m] \) and all other coefficients \( \beta_j = \beta_{jk} = \cdots = 0 \). That is to say, initially, all coefficients of the first order Volterra kernel are one and all coefficients for higher order Volterra kernels are zero. Thus, at the start of the optimisation \( \ddot{x}_w(t) \) equals \( \ddot{x}(t) \).

In summary, we define a nonlinear process that transforms a timeseries \( \ddot{x}(t) \) into \( \ddot{x}_w(t) \). This general processes is defined in terms of a Volterra series expansion of \( \ddot{x}(t) \). From this general expansion we search for a particular case where \( \ddot{x}_w(t) \) has a response spectrum that approximately matches some target response spectrum. This particular \( \ddot{x}_w(t) \) is determined by adjusting the weights (coefficients) of the Volterra series using nonlinear optimisation.

**Examples of using Re-weighted Volterra Series Algorithm (RVSA)**

We selected records from the PEER-NGA ground-motion database (PEER 2000) in (Alexander, Chanerley et al. 2014, Alexander 2015). These stratified sample of records were from events of magnitude 6 to 7.5 and be from ground of shear wave velocity \( V_s \geq 800 \text{m/s} \). The scaling of these records was performed by PEER-NGA such that the geometric mean of this record set is a reasonable match to the target spectrum. Although this amplitude scaling is completely unnecessary as was demonstrated previously in (Alexander, Chanerley et al. 2014). In this previous study we demonstrated the robustness and effectiveness of RVSA for a reasonable large sample of records. When 3 kernels are include the mean error (the misfit
error in spectrally matching, 20 records, using 3 Volterra kernels) was just 0.04% with a mean maximal error of 7.81%. These values are clearly very small and much better than other algorithms in the literature.

In this paper we shall demonstrate the performance of RVSA, in more detail, with an exemplar record from this set. As a heuristic case an EC8 type I horizontal elastic spectrum, on soil class A, with 5% percentage of critical damping, is employed in this paper. This total acceleration response spectrum was defined from 0.02s to 4s using \( r = 200 \) divisions on a logarithm scale (this should includes all points of slope discontinuity). However, as discussed previously if the number of coefficients in the Volterra series expansion \( q \) is greater the \( r \) then we resample the response spectrum such that the new number of division defining the target response is \( r = 1.2q \) and hence \( r > q \). This feature was not discussed previously but is part of the Matlab toolbox (Alexander 2015).

![Figure 2](image2.png)

**Figure 2**, an example response spectra of RVSA matching NGA296FN seed record

![Figure 3](image3.png)

**Figure 3**, an example timeseries of RVSA spectrum matching NGA296FN seed record
Seed record NGA296FN is used which is Irpinia (Italy) 1980 6.2M\text{\textsubscript{b}} event at station recorded at Bagnoli Irpinio. It was then processed by PEER and with Fault Normal component obtained by a classical rotational transformation. Even though, as we have already mentioned, it is unnecessary to scale this record for RVSA to work successfully, we scale the record for the purposes of graphical presentation and comparison. We scale $\ddot{x}$ by $\alpha$ such that we minimise the difference between its response spectrum and the target. To achieve this we use the following least square regression result: $$\alpha = \frac{\sum s_r(f)s(\ddot{x},f)\sum s(\ddot{x},f)^2}{25}.$$ 

Figure 2 depicts the origin scaled record $\alpha\ddot{x}$ and the matched records $\ddot{x}_M$ for 1 to 4 Volterra kernels. In this case we employ the biorthogonal mother wavelet ‘bior1.3’. Two Volterra kernels produce a good match with and mean misfit error of -0.08% and a standard deviation of 2.78%. Three and four kernels produce excellent matches with (mean error -0.03%, standard deviation 1.47%) and (mean error -0.03%, standard deviation 1.56%) respectively.

Figure 3 shows the timeseries of these spectral matched records. These plot demonstrate that the general envelope and pulses of the original seed record are maintained. What is surprising is that the timeseries of the original seed requires only small modifications to produce a spectral matched record. The differences between three and four kernels underlines that the mapping of timeseries to response spectrum is not uniquely invertible.

Figure 4 plots the power spectra (using Welch’s window and average method) for the Matched records and the original seed record. It is instructive to observe that the power content between 1 to 10 Hz is largely similar. At frequencies greater than 10Hz we see an increase in power content. This can also be observed in the timeseries plots. While this is perhaps not idea, the magnitude of these higher frequency components is still very low (remember the power axis is a logscale). At the low frequency (below 0.3Hz) we observe some more problematic increase in power content.

Figure 5 shows the cumulative energy vs time for original and spectrum matched record, NGA296 Fault Normal (3 kernels). Again this demonstrates that the modifications to the seed record (in terms of energy distribution in time) are small.
Stability of integrated acceleration timeseries
The previous analyses have demonstrated the efficacy of using RVSA for modifying a recorded accelerogram into a spectrum matching one. However, it is important to review the stability and plausibility of the first and second integral of the acceleration i.e. the velocity and displacement timeseries. The question of numerically integrating accelerograms is a problematic one (Graizer 2010) Integration is fundamentally a low-pass filter that attenuates the high frequency content and amplifies to very low frequency content of a signal (Chanerley,
Alexander et al. 2009, Chanerley and Alexander 2010, Berrill, Avery et al. 2011, Chanerley, Alexander et al. 2012, Chanerley, Alexander et al. 2013). The very low frequency content of an accelerogram is frequently corrupted by noise and ground pitching and rolling degrees of freedom (often termed ‘tilts’) (Graizer 2006). Regularly, as in the PEER-NGA database, this low frequency content is removed by filtering before and possibly after integration (Converse 1992) to remove these troublesome artefacts. However, Chanerley (2010, 2013) pointed out that this can remove very low frequency ‘fling’ components of the ground motion. Hence the resultant ground motion displacement timeseries is stable but erroneous in its low frequencies.

Hence, with spectrum matched records produced by RVSA we could (i) adopt the algorithm in Chanerley (2010, 2013) or (ii) we could integrate and filter out the low frequency components. In this paper all records have been processed and corrected by PEER and have already been low-cut filtered, thus in this case adopting option (i) is not consistent. Thus, we filter the matched accelerogram below 0.2Hz using a 4th order zero-phase Butterworth filter before and after integration to obtain stable velocity and displacement timeseries. An example of these is shown in Figure 6

![Figure 7](image7.png)

**Figure 7**, comparing matched spectra by RVSA using difference mother wavelet

![Figure 8](image8.png)

**Figure 8**, comparing matched timeseries by RVSA using difference mother wavelet
Influence of mother wavelet

Figure 7 and Figure 8 show the influence of using different mother wavelets in obtaining the basis functions $\phi(t)$ for the Volterra series. Both these figures are for the case of employing 3 Volterra kernels. The difference in the goodness of the fit are negligible (‘Bior1.3’ -> mean error 0.03%, Std 1.48%, ‘Bior3.3’ -> mean -0.02%, Std 1.26%, ‘Bior6.8’ -> mean -0.06%, Std 2.61%).

The biorthogonal family are filters with low number of coefficients (‘bior1.3’) to large number of coefficients (‘bior6.8’). The number of wavelet levels $m$ in the decomposition of the seed record is a function of record size and the number of coefficients in the mother wavelet. Smaller number of coefficients (in the mother wavelet filter bank) results in more levels of decomposition. This in turn results in more coefficients in the Volterra series expansion per kernel. Therefore $q$ is larger for ‘bior1.3’ than for ‘bior6.8’. Therefore we solve a slightly different order numerical problem for each case. If computational time is important then ‘Bior6.8’ is preferable.

Conclusions

In this paper we present a novel algorithm (based on state of the art signal processing and optimisation techniques) for modifying a given ground acceleration timeseries such that its response spectrum matches a given target response spectrum.

The Re-weighed Volterra Series Algorithm (RVSA) demonstrates stability and robustness. Unlike some approaches, it generally appears to converge to some useful record that meets the objectives of the spectral matching process. It can be used regardless of which code based spectral shape is chosen as a target spectrum. Additionally it can be classed as insensitive to the seed record selected; i.e. its magnitude or noise level. That is to say it converges to any given and plausible response spectrum from any given and plausible seed timeseries. This includes enveloped Gaussian noise as a given seed timeseries.

As the number of Volterra kernels employed increases up to 3 we observe an increase in the goodness of fit to the spectrum at the expense of computational time. Employing more than 3 kernels was found to be ineffective. Thus, spectrum matched timeseries shows an excellent fit to the target spectrum over the entire structural frequency range (and much better than was previously available) while maintaining a record that keeps a qualitatively similar appearance to the seed record.

References


