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OPEN SHAPE MORPHING HONEYCOMBS THROUGH KIRIGAMI

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ABSTRACT
This work presents an “open” and deployable honeycomb configuration created using kirigami-inspired cutting and folding techniques. The open honeycomb differs from traditional “closed” honeycomb by its reduced density and its increased flexibility. The exploitation of these characteristics for multifunctional applications is the focus of this work. Potential fields in which the open honeycomb could find application include sandwich panel manufacturing, morphing, and deployable structures.

NOMENCLATURE
\( h \)  h-wall width
\( l \)  l-wall width
\( b \)  wall length
\( t \)  sheet material thickness
\( \theta \)  internal cell wall angle
\( \alpha \)  hinge opening angle
\( H \)  unit cell width
\( L \)  unit cell length
\( T \)  unit cell thickness
\( A_{\text{mat}} \)  area of sheet material in unit cell
\( \rho_{\text{mat}} \)  density of sheet material
\( t_{\text{mat}} \)  thickness of sheet material

INTRODUCTION
Kirigami is the ancient Japanese art of folding and cutting paper. Nojima and Saito developed a method of creating honeycombs using this process [1]. Slitting, corrugation, and folding operations are used to form a 2D sheet material into a 3D cellular structure. Mating faces can be bonded together to create a closed honeycomb or left unbonded to create an open honeycomb. The geometry of the honeycomb can be varied by altering the pattern of slits, giving a design flexibility otherwise not available with other techniques. By varying the spacing of slits on the sheet it is possible to produce a honeycomb with a varying thickness profile [2,3]. It is also possible to produce different cell shapes by changing the shape of the moulds used and adapting the slitting pattern accordingly [4]. Including circles in the slitting pattern gives holes in the cell walls. Such holes have previously been used for ventilation [5] but could also be used for (wire) access or actuation. Such features would be difficult to machine out of traditional honeycomb without causing damage [6]. We describe how the mechanical and actuation properties of the open honeycomb change with the Kirigami pattern and the fold angle. Finite Element Analysis (FEA) has been used to investigate the effect of fold angle and fold stiffness on the mechanical properties of the open honeycomb. A mechanical testing programme is planned to validate the FEA models. Subsequent work will focus on the multifunctionality of the structure. Of particular interest is the use of smart hinges made with Shape Memory Polymer (SMP), and the inclusion of ventilation or access holes with tuneable wires to create classes of deployable smart honeycombs using the Kirigami process.

KIRIGAMI MANUFACTURING
The kirigami process for creating honeycombs [1], converts a 2D sheet of material into a 3D cellular structure using four processing steps, as shown in Figure 1. An open honeycomb is produced by simply omitting step 4 and leaving the honeycomb unbonded. The kirigami process is flexible in terms of input materials and processing techniques; all that is required is that the sheet material can be cut, folded, and bonded to itself. This affords the user some choice in terms of materials selection. In this case, Victrex PEEK film was used for its good formability and mechanical properties[7].

The slits were made using a Blackman & White ply cutter. The corrugations were created by thermoforming the PEEK film between semi-hexagonal moulds using a hot press at 200°C. The slits in the sheet line up with the edges of the moulds as shown in Figure 2; this alignment is important because it allows the corrugated sheet to be folded back on itself to produce a hexagonal geometry.
After thermoforming, the corrugated sheet is folded back on itself repeatedly such that the slits (which are now semi-hexagonal) open up into hexagonal holes, resulting in the open honeycomb geometry shown in Figure 3. This is done at room temperature, by hand, and simply creates plastic deformation in the material where folded.

Holes or other such features can be included in the slitting pattern, which will later form part of the honeycomb, as shown in Figure 4.

If the slitting pattern is modified, a variant on the open honeycomb can be created with some parallelogram-shaped walls, as shown in Figure 5. To distinguish between the open configurations they shall henceforth be called “open rec” and “open par”, where “rec” and “par” are short for rectangular and parallelogram, respectively, and refer to the shape of the walls.
Figure 5: The relationship between cutting patterns and honeycomb geometries. The black shapes on the lowest cutting pattern represent material removed. The brown material on the cutting pattern becomes the brown “cell” on the right. The dotted box shows the bounding volume of the cell. The cell edges which touch the bounding box are shown in blue. Grey replica cells are shown to illustrate the tesselation of the structure.

VOLUMETRIC ANALYSIS

The two distinguishing features of the open honeycomb are its increased (and directional) flexibility and its reduced density. In this section the unit cell and the variable density of the open honeycomb are examined.

Figure 6 shows the unit cell of the closed configuration.

Dimensions $h$, $l$, $b$, and $\theta$ are determined by the moulds and cutting pattern. Dimensions $H$, $L$, $T$ and sheet material area $A$ are calculated as follows:

\[
H_{\text{closed}} = 2h + 2l \sin \theta \tag{1}
\]

\[
L_{\text{closed}} = l \cos \theta \tag{2}
\]

\[
T_{\text{closed}} = b \tag{3}
\]

\[
A_{\text{mat \ closed}} = 2bh + 2bl \tag{4}
\]

Figure 7 shows the unit cell of the open rec honeycomb, with a side view along the 1-direction.

\[
H_{\text{rec}} = 2h + 2l \sin \theta \tag{5}
\]

\[
L_{\text{rec}} = l \cos \theta \cos \alpha + b \sin \alpha \tag{6}
\]

Figure 7: The unit cell of the open rec configuration. The view on the right is a view along the 1-direction.

This structure is essentially the same structure, rotated about the 1-axis by angle $\alpha$ from the vertical. The dimensions and area $A$ are calculated as follows.
\[
T_{\text{rec}} = l \cos \theta \sin \alpha + b \cos \alpha
\]  
(7)

\[
A_{\text{rec}}^{\text{mat}} = 2bh + 2bl
\]  
(8)

Figure 8 shows the unit cell of the open par honeycomb, with a side view along the 1-direction.

The dimensions and area \( A \) are calculated as follows.

\[
H_{\text{par}} = 2h + 2l \sin \theta
\]  
(9)

\[
L_{\text{par}} = l \cos \theta \cos \alpha + b \sin \alpha
\]  
(10)

\[
T_{\text{par}} = b \cos \alpha
\]  
(11)

\[
A_{\text{par}}^{\text{mat}} = 2bh + 2bl
\]  
(12)

To isolate the effect of angle \( \alpha \) on the density of the honeycombs, dimensions \( l, h, b, \) and \( \theta \) must be fixed. The absolute density is of little interest; instead the density of the open configurations will be compared relative to the density of the closed configuration. The density of each configuration is given by:

\[
\rho = \frac{m}{V} = \frac{A_{\text{mat}} \rho_{\text{mat}} t_{\text{mat}}}{HLT}
\]  
(13)

It can be seen that \( H \) and \( A_{\text{mat}} \) are the same for all configurations. If the same sheet material is used to make all configurations then \( \rho_{\text{mat}} \) and \( t_{\text{mat}} \) are also constant. As a result these variables cancel out when the relative density is calculated as follows.

\[
\frac{\rho_{\text{open}}}{\rho_{\text{closed}}} = \frac{L_{\text{closed}} T_{\text{closed}}}{L_{\text{open}} T_{\text{open}}}
\]  
(14)

By substituting in equations (2-3,6-7,10-11) and rearranging, the following two expressions are obtained.

\[
\frac{\rho_{\text{rec}}}{\rho_{\text{closed}}} = \frac{1}{C} \left( \frac{b^2}{2bl \cos \theta} \right) \sin 2\alpha + 1
\]  
(15)

where \( C = \left( \frac{l^2 \cos^2 \theta + b^2}{2bl \cos \theta} \right) \)

And

\[
\frac{\rho_{\text{par}}}{\rho_{\text{closed}}} = \frac{1}{D} \left( \frac{b^2}{2bl \cos \theta} \right) \sin 2\alpha + 1
\]  
(16)

where \( D = \left( \frac{b^2}{2bl \cos \theta} \right) \)

When \( \alpha = 45^\circ \), \( \sin 2\alpha = 1 \) and the relative density of the open configurations is at a minimum with a value of \( 1/(C+1) \) or \( 1/(D+1) \) depending on the configuration, as shown in Figure 9.

The magnitudes of these minima are dependent on the dimensions \( l, h, b, \) and \( \theta \), and thus will vary between different size honeycombs, but the shape of the curve will not change qualitatively. For \( l > 0, \ b > 0, \) and \( 0^\circ < \theta < 90^\circ, \ C > D, \) meaning that for a given \( \alpha \) the open rec configuration will always have lower relative density than the open par configuration. For the honeycombs produced in this work (\( h = 5\text{mm}; \ b = 10\text{mm}; \ \theta = 30^\circ \)) \( C = 1.37 \) and \( D = 1.15 \), giving the following relative densities at \( \alpha = 45^\circ \):

\[
\frac{\rho_{\text{rec}}}{\rho_{\text{closed}}} = 0.42
\]  
(17)
\[ \frac{\rho_{\text{par}}}{\rho_{\text{closed}}} = 0.46 \]  

(18)

### FINITE ELEMENT ANALYSIS

FEA models were created to predict the mechanical properties of the various honeycomb configurations. Of particular interest is the shear performance of the open honeycombs when \( \alpha = 45^\circ \), since shear in many cases resolves itself as diagonal tension and compression which in this case would be aligned with the h-walls, and also because this coincides with the density minimum.

The aim of this FEA was to investigate the effect of several variables on the mechanical performance of the honeycomb configurations. The input variables were angle \( \alpha \), hinge (or fold) stiffness \( k \), and the boundary conditions, which were used to simulate the presence or absence of sandwich panel skins. For each open configuration loading case, \( \alpha \) was varied from 5° to 80° with intervals of 5°. \( k \) was given the values 1e-10, 0.1, 1, 1e10. The crude spread of this variable was intended to save computational time while still giving some indication of the effect of \( k \) across the entirety of its possible range, from \( k = 0 \) (frictionless hinge) to \( k = \infty \) (totally rigid joint). Any observed effects could then be investigated in more detail later.

The output variables were the flatwise modulus \( E_3 \) and the transverse shear moduli \( G_{13} \) and \( G_{23} \). \( G_{13} \) and \( G_{23} \) were not measured with sandwich skins absent, since shear loading by definition requires some kind of skin to introduce the load. Figure 10 shows a schematic of the test matrix, with each dot representing an individual model run. ABAQUS/CAE FEA software was used throughout, and a Python script was used to iteratively populate the test matrix.

<table>
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<th>CONFIG</th>
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<th>SKINS</th>
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<td>( E_3 )</td>
<td>( E_3 )</td>
</tr>
<tr>
<td>OPEN REC</td>
<td>( E_3 )</td>
<td>( E_3 )</td>
</tr>
<tr>
<td>OPEN PAR</td>
<td>( E_3 )</td>
<td>( E_3 )</td>
</tr>
</tbody>
</table>

Figure 10: Schematic of the FEA test matrix. Each black dot represents one model run, including the dots on the plots. NOTE that the lines on the plots do not show real trends and are only intended to illustrate the parameter space for each configuration.

A double-size unit cell was used to allow the hinge to be included in the model geometry. For the open configurations the model consisted of two separate strip instances connected by hinge elements along the relevant edges. Figures 11, 12, and 13 show schematics of how the unit cell models were constructed for the various configurations.

Shell elements (S4R) were used throughout the models. Following a convergence study an element size of 0.25mm was selected (for reference, the dimensions of the unit cell are given by \( h = l = 5\text{mm}; b = 10\text{mm}; \theta = 30^\circ \)).

Hinge connectors were used to connect the strip instances in the open configuration models. Connectors are an ABAQUS feature which allow the user to couple multiple Degrees Of Freedom (DOF) of two or more nodes, such that those nodes behave as if connected by a joint. Of the available joint types, the hinge was the most suitable for these models. The hinge connector has the option of including a spring stiffness (D44) and it is this variable that was used to implement the hinge stiffness \( k \).

The presence of sandwich skins was modelled as follows. On the bottom face of the honeycomb all 6 DOF were restrained using Boundary Conditions (BCs). On the top face, All rotations were restrained with BCs, and all 3 translations were coupled such that the top face nodes moved as one; these constraints were chosen to represent the cell walls being embedded in adhesive.
Figure 12: Schematic of the open rec configuration unit cell model. A simplified mesh is shown for convenience. The blue lines represent couplings between edge nodes. The green lines mark the coupling between the top face nodes. The pink lines represent hinge connections. The red arrow marks the load introduction point. The black triangles mark the nodes on the bottom face constrained by BCs.

For the cases in which the skins were absent, only flatwise compression was simulated. The bottom face of the honeycomb was restrained in $U_3$ using BCs, leaving the other 5 DOF free. One node on the bottom surface was restrained in $U_1$ & $U_2$ to prevent the entire model from sliding. The top face nodes were coupled in $U_3$ to represent a flat loading platen compressing the structure.

To simulate the effect of a continuous honeycomb, the outermost nodes were coupled to their counterparts on the other side of the cell using equations to couple their DOF. This was done on a node-by-node basis, using a for loop in the Python script, so that the wall edges were not made rigid. This is illustrated in Figures 11, 12, and 13.

Load was introduced through a single point, and the rest of the nodes on the top face were coupled to this point to ensure load redistribution. A load of 1N was used for all models. This low load was to ensure that the structure behaved in a linear fashion.

RESULTS

The results of the FEA modelling are shown in Figures 14-19. The moduli of the open and closed configurations are compared in two ways. The absolute modulus and the specific modulus are compared between configurations. The comparison of specific moduli takes into account the change in density due to variations in $\alpha$. 
Figure 14: Absolute modulus $E_3$ compared between the open and closed configurations.

Figure 15: Specific modulus $E_3/\rho$ compared between the open and closed configurations.

Figure 16: Absolute modulus $G_{13}$ compared between the open and closed configurations.

Figure 17: Specific modulus $G_{13}/\rho$ compared between the open and closed configurations.

Figure 18: Absolute modulus $G_{23}$ compared between the open and closed configurations.

Figure 19: Specific modulus $G_{23}/\rho$ compared between the open and closed configurations.

**DISCUSSION**

It can be seen that the effect of the hinge stiffness is negligible. The plots in Figures 14-19 show data series for the four different hinge stiffnesses, but it is mostly impossible to distinguish the individual lines because they are so close together. The hinges’ lack of influence on the moduli is not unexpected; in these loading cases the hinge is either totally...
restrained by the sandwich skins, or the loading is such that the hinge behaviour doesn't play a big part in the deformation mechanism of the structure. It is anticipated that the hinges will have the greatest effect on the bending behaviour of the structure.

Figure 14 And Figure 15 show that the presence or absence of sandwich panel skins has a significant effect on the compressive moduli. However, the effect of the absence of sandwich skins is not as great as expected. For an open configuration without skins, it should be possible for the strips to simply splay outwards under a compressive load, with only the stiffness of the hinge to resist this motion. This is one of the cases where the hinges were expected to have a significant effect on the structure's behaviour. This was not observed in the models, and this may be due to the side constraints which simulate a continuous honeycomb; these may have prevented the cell from expanding as it normally would.

As expected, the open honeycombs' absolute moduli are lower than that of the closed honeycomb. This is to be expected because for the open configurations the same amount of material has been spread over a greater volume. However, it is also important to note that there is overall less connectivity between the cell walls in the open configurations. In the closed configuration, all cell walls are supported by two other walls at the joints, and this, along with the the back-to-back h-walls, provides good buckling support. This is not true for the open configurations; especially the open rec configuration in which every cell wall has at least one free edge even when sandwich skins are present. The open honeycombs' specific moduli are also mostly lower than that of the closed honeycomb, but in some cases the specific moduli of the open par configuration approach that of the closed configuration. For both open configurations, the variation of alpha produces strongly nonlinear changes in moduli. This presents the possibility of tailoring the moduli and density for a specific purpose, by controlling the dimensions of the honeycomb. As shown in equations (17-18) the open configurations can have significantly lower density compared to their closed counterpart. This suggests that open honeycombs could be used for applications which are weight-critical but not stiffness-critical.

It can be seen that the open par configuration consistently outperforms the open rec configuration. This is due to the greater contact area between the open par honeycomb and the load introduction surface, which allows better load distribution in the structure and fewer stress concentrations. Indeed, the poor load distribution in the open rec configuration is what inspired the open par configuration. Figure 20 shows how the open rec configuration develops large stress concentrations in the l-walls in response to a compressive load. The h-walls, which touch each sandwich skin with one corner and provide a continuous path for the shear force to be transmitted. Instead the shear force is resisted almost entirely by the l-walls, which touch each sandwich skin with one corner and develop large stress concentrations.

![Image](https://via.placeholder.com/150)

Figure 20: The open rec configuration under flatwise compression loading. The contour plot shows Von Mises stress and reveals the stress concentrations at the corners of the l-walls. Note: this is an early model and has a different mesh density from the models used in this paper.

It is interesting to note that the closed configuration model agrees with Zhang & Ashby's analysis of a honeycomb in compression with doubly-thick h-walls [8]. The stress in the doubly-thick walls is double that in the single walls.

It should be noted that after alpha reaches about 60 degrees, the open par configuration becomes impractical, because the unit cell becomes so stretched that it would be difficult to manufacture.

**POTENTIAL APPLICATIONS**

**Manufacturing**

When $\alpha$ is small, the open par configuration's specific moduli are close to those of the closed configuration, as can be seen in Figures 15, 17, and 19. A honeycomb like this could potentially be used as a “drapeable” honeycomb – slightly inferior mechanical properties but much easier to form into complex/tight curvatures due to its greatly reduced bending stiffness. Such a honeycomb could easily form cylindrical shapes because the deformation of the hinges produces none of the secondary curvature normally observed in honeycombs in bending.
**Morphing**

As previously mentioned, holes can be easily included in the cell walls of the honeycomb. These have the potential to create a morphing structure when wires are fed through the channels created. Figure 21 shows a demonstrator, which shows how tensioning the wire can produce different, deformed shapes depending on the wire's location in the honeycomb.

Figure 21: A morphing demonstrator created by threading cables through the channels in a honeycomb. The knots at the ends of the cables allow them to exert force on the honeycomb when pulled.

**Deployable Structures**

Due to the honeycomb's increased flexibility and its reliance on the hinges for bending stiffness, this geometry could be used to create a deployable structure using shape memory materials. Figure 22 shows a possible concept whereby shape memory hinges control the expansion of the honeycomb. This could be useful for parts requiring a specific curvature.

Figure 22: A concept for a deployable honeycomb created using shape memory material for the hinges (purple) and strips (grey)

**CONCLUSIONS**

In this paper two open honeycombs are presented and analysed using analytical methods and FEA. These honeycombs display a significantly different behaviour from that of the traditional closed honeycombs. The particular deformation mechanism shown by open honeycombs could be exploited for applications related to the manufacturing of cellular structures, morphing, and deployable structures.

Future work will include a mechanical testing programme to validate the results of the FEA. Subsequent work will focus on developing the multifunctional aspects of the open honeycombs concept.

**ACKNOWLEDGMENTS**

Special thanks go to Mr. Ian Chorley (ACCIS Technician) for help and advice manufacturing and testing of the PEEK honeycombs.

The Authors are also grateful to Dr. Alan Wood from Victrex plc for the advice provided about the processing of the PEEK films.

The Authors would also like to thank Dr. Ian Farrow for encouraging the investigation into open honeycombs.

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