Sensitivity of simplified pile settlement calculations to parameter variation in stiff clay
Sensibilité des calculs simplifiés de règlement des tas de variation de paramètre dans l'argile raide

P.J. Vardanega*1

1 University of Bristol, Bristol, United Kingdom
* Corresponding Author

ABSTRACT  Pile settlement is a key geotechnical design consideration. The serviceability limit state for deep foundations cannot be ignored and yet many design methods merely assume that large factors of safety are sufficient to prevent excessive settlements. A simple model, supported by previously published databases of load testing on bored piles founded in London clay, is used to make predictions of settlement for bored pile foundations in the same geological deposit. The results of a detailed sensitivity study of the key parameters that affect the performance of bored piled foundations are presented. The parameters studied include: the mobilisation factor; the mobilisation strain; the elastic modulus of the concrete; the undrained shear strength profile; the pile length and the pile diameter. Based on the preliminary results of this sensitivity study, design guidance is presented and a rank is attributed to the parameters in order of their influence on the settlement calculation result. The influence of soil non-linearity is also studied.

RÉSUMÉ  L’implantation des pieux est une considération de conception géotechnique majeure. L’état limite de service pour les fondations profondes ne peut être ignoré et encore beaucoup de méthodes de conception prennent simplement pour acquis que les grands facteurs de sécurité suffisent à prévenir les implantations excessives. Un modèle simple, soutenu par des bases de données sur les essais de chargement des pieux forés dans l’argile de Londres précédemment publiées, est utilisé pour établir des prédictions d’implantation de fondations en pieux forés dans le même dépôt géologique. Les résultats d’une étude détaillée de sensibilité des paramètres les plus importants affectait les performances des fondations en pieux forés sont présentés. Les paramètres étudiés incluent : le facteur de mobilisation, l’effort de mobilisation, le module élastique du béton, l’allure de la force de cisaillement non drainée, la longueur du pieu et le diamètre du pieu. Basé sur les résultats préliminaires de cette étude de sensibilité, design guidance is presented and a rank is attributed aux paramètres en fonction de leur influence dans les résultats des calculs d’implantation. L’influence du sol non - linéarité est également étudié.

1  INTRODUCTION

Simple methods for the estimation of pile settlement are useful for geotechnical engineers. Generally piled foundations in the UK are designed on the basis of collapse considerations as opposed to serviceability considerations (Vardanega et al. 2012a).

This paper presents a generalized version of the simple MSD-style calculation for pile settlement in stiff clay (Vardanega et al. 2012b and Vardanega 2012). MSD for piles is reminiscent of traditional p-y calculations for piled foundations (Bouzid et al. 2013). The calculation method used in this paper is inspired by the formulation of Randolph (1977). It makes use of the strain to mobilize half the undrained shear strength (also used in the early work of Matlock, 1970) and the power-law formulation for soil-stress strain presented and calibrated in Vardanega & Bolton (2011a). The method has been shown (in Vardanega et al. 2012b) to reasonably match the generalized pile-settlement curves for the database of tests in London clay presented in Patel (1992). The simple model also reasonably matched data from centrifuge model tests in kaolin (Williamson, 2014) and for two pile tests at a site in the Jurassic clay in the Moscow region (Kolodiy et al. 2015).

This paper presents the results of a sensitivity analysis aimed at assessing the relative influence of
the key design parameters using the previously published calculation method and the results of previously published databases. The baseline condition is considered to be a typical characterization of the relevant design parameters for the London clay deposit.

2 PILE SETTLEMENT MODEL

Equation 1 is the simple power-law model for strength mobilization that has been calibrated with a large database of tests on clays and silts (Vardanega & Bolton, 2011a).

\[
\frac{1}{M} = \frac{\tau}{c_u} = 0.5 \left( \frac{\gamma}{\gamma_{M=2}} \right)^b \quad 5 > M > 1.25 \quad (1)
\]

where, \( b \) = non-linearity factor determined from curve fitting analyses; \( c_u \) = undrained shear strength; \( \tau \) = mobilized shear stress; \( \gamma \) = shear strain; \( \gamma_{M=2} \) = shear strain to half mobilisation (0.5\( c_u \)) and \( M \) = mobilisation factor.

Equation 2 is Randolph’s approximate equation of radial reduction of shear stress (Randolph, 1977; Fleming et al. 2009)

\[
\tau = \frac{\tau_0 r_0}{r} \quad (2)
\]

where, \( \tau_0 \) = shear stress on the pile shaft; \( r_0 \) = pile radius and \( \tau \) = shear stress at radius, \( r \) (see Figure 1).

![Figure 1. Displacement of a single pile (plot adapted from Vardanega et al. 2012b)](image)

Noting that the downward displacement of the pile, \( w \) is equal to the integral of the shear strain with respect to the radii of concentric surfaces (Randolph, 1977) (Equation 3 and Figure 1), we can say

\[
w = \int_{r_0}^{\infty} \gamma \, dr \quad (3)
\]

Substituting in Equations 1 and 2 we get

\[
w = \int_{r_0}^{\infty} 2^{(1/b)} \gamma M = 2 \left( \frac{\tau_0 r_0}{r c_u} \right)^{(1/b)} \, dr \quad (4)
\]

where, \( \gamma M = 2 \) = mobilized shear stress; \( \tau = \frac{\tau_0 r_0}{r} \)

\[
\frac{w_{soil}}{D} = \eta \frac{\gamma M = 2}{M^{(1/b)}} \quad (5)
\]

where, \( w_{soil} \) = computed pile settlement (soil contribution) and \( D \) = pile diameter and

\[
\eta = \frac{1}{2} \left[ \frac{1}{b} - 1 \right] \quad (6)
\]

The estimation of the contribution of compression of the concrete component to the total pile settlement is given in Vardanega et al. (2012b).

Equation 7 shows the generalized form of the bored pile settlement equation shown in Vardanega et al. (2012b) (see also Williamson, 2014)

\[
\frac{w_h}{D} = \eta \left( \frac{\gamma M = 2}{M^{(1/b)}} \right) + \left( \frac{c_u}{M E_c} \right) \left( \frac{L}{D} \right)^2 \quad (7)
\]

where, \( w_h \) = computed pile head settlement (total); \( E_c \) = elastic modulus of the concrete pile and \( L \) = length of pile in the clay.

Equation 7 makes use of the assumed stress distribution in Figure 1 and the simple pile geometry and soil strength profile shown in Figure 2. It is acknowledged in Vardanega et al. (2012b) that a more rigorous non-linear model could make use of full load-transfer (e.g. Fleming et al. 2009) and in such analysis inclusion of the base resistance is possible.

Vardanega et al. (2012b) explain that for the collapse condition for bored piles (i.e ‘\( \alpha \)-method’) to hold then Equation 8 (which links the factor of safety \( F \) [shaft] to the adhesion value, \( \alpha \) and the mobilization factor, \( M \)) should also hold when using Equation 7 for settlement checks:

\[
F = \alpha M \quad (8)
\]
For example, if \( a = 0.45 \) (Skempton, 1959) then the lower ‘realistic’ limit of \( M \) is about 2.2 and if \( a = 0.6 \) (Patel, 1992) the lower realistic limit of \( M \) is about 1.7. This simple analysis shows why relatively large \( M \) values are needed to deal with both collapse and serviceability considerations in bored pile design.

3 BASELINE VALUES (LONDON CLAY)

3.1 Soil stress-strain variation

The \( \eta \) term in Equation 7 allows for varying values of \( b \) which has a mean value (\( \mu \)) of about 0.6, based on the database of 115 tests on 19 natural clays and silts presented in Vardanega & Bolton (2011a), with a standard deviation (\( \sigma \)) of about 0.15. Vardanega & Bolton (2011b) show that based on 17 reported tests on London clay \( b \) is on average 0.6 with a standard deviation of 0.12, i.e. very similar to the values from the larger database. The standard deviation of \( \gamma_M \) is much lower for the London clay tests – around 0.002 and this value will be used in this paper. The variation of \( \eta \) with \( b \) is given in Table 1.

Table 1. Variation of \( \eta \) with \( b \)

<table>
<thead>
<tr>
<th>( b )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( -2 \sigma )</td>
<td>1.93</td>
</tr>
<tr>
<td>( -1 \sigma )</td>
<td>1.96</td>
</tr>
<tr>
<td>mean</td>
<td>2.38</td>
</tr>
<tr>
<td>( +1 \sigma )</td>
<td>3.37</td>
</tr>
<tr>
<td>( +2 \sigma )</td>
<td>5.99</td>
</tr>
</tbody>
</table>

3.2 London clay \( c_u \)-profile variation

The values used in for the sensitivity of the \( c_u \)-profile (shown in Table 2 and plotted on Figure 3) are derived using the *Graphical Three-Sigma Rule* (e.g. Duncan, 2000) and the database of mean undrained strength profiles in London Clay collected by Patel (1992).

Table 2. Variation \( c_u \)-profile in London Clay (based on the database presented in Patel 1992) [\( d \) as defined in Figure 2]

<table>
<thead>
<tr>
<th>( c_u )-profile</th>
<th>( c_u ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>lowest conceivable</td>
<td>( c_u ) (kPa) = 5.7d + 25</td>
</tr>
<tr>
<td>( -2 \sigma )</td>
<td>( c_u ) (kPa) = 6.2d + 40</td>
</tr>
<tr>
<td>( -1 \sigma )</td>
<td>( c_u ) (kPa) = 6.7d + 55</td>
</tr>
<tr>
<td>( \mu )</td>
<td>( c_u ) (kPa) = 7.2d + 70</td>
</tr>
<tr>
<td>( +1 \sigma )</td>
<td>( c_u ) (kPa) = 7.7d + 85</td>
</tr>
<tr>
<td>( +2 \sigma )</td>
<td>( c_u ) (kPa) = 8.2d + 100</td>
</tr>
<tr>
<td>highest conceivable</td>
<td>( c_u ) (kPa) = 8.7d + 115</td>
</tr>
</tbody>
</table>

![Figure 2. Idealized soil strength profile and pile geometry](image)

![Figure 3. Results of ‘graphical three sigma’ construction (e.g. Duncan, 2000) to the database of Patel (1992). N.B. Only one site in the database has data that is close to the upper bound line (as drawn) at depth and is therefore considered to be an outlier](image)

3.3 Concrete elastic modulus variation

It is difficult to sensibly assign a population \( \mu \) and \( \sigma \) value to \( E_c \). In this paper simple range of values (10 to 30GPa) is used and a baseline value of 20GPa is adopted as was done in Vardanega et al. (2012b).

3.4 Mobilization factor

Vardanega et al. (2012a) show that for six codes of practice the factor of safety (\( F \)) demanded for bored pile design can range from about 1.7 to upwards of 3 with a typical value of 2.5. Noting that we are studying London clay (\( a = 0.6 \)), the range of \( F \) implied by
the range $5>M>1.7$ is $3>F>1$. This range of ‘sensible’ $M$ values will be used in the analysis, with $M=3$ taken as a baseline value (it is conceded that values of $\mu$ and $\sigma$ cannot be realistically assigned for the $M$ parameter). The influence of the pile base is neglected in this analysis.

4 SENSITIVITY ANALYSIS

For the analysis the listed baseline values in Table 3 are used. The aim (where possible) is to examine the influence of each parameter listed in Table 3 $\pm 2\sigma$.

### Table 3. Baseline values for sensitivity analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_c$, GPa</td>
<td>20</td>
<td>Vardanega et al. (2012b)</td>
</tr>
<tr>
<td>$M$</td>
<td>3</td>
<td>Vardanega et al. (2012a)</td>
</tr>
<tr>
<td>$c_u$, kPa</td>
<td>$70+7.2d$</td>
<td>Figure 3 and Table 2</td>
</tr>
<tr>
<td>$b$</td>
<td>0.6</td>
<td>0.12</td>
</tr>
<tr>
<td>$\gamma_{M-2}$</td>
<td>0.007</td>
<td>0.002</td>
</tr>
</tbody>
</table>

#### 4.1 Influence of pile diameter

Figure 4 shows the influence of pile diameter on the computed pile head settlement. As the pile lengthens the influence of pile diameter becomes more marked.

#### 4.2 Influence of $c_u$-profile variability

Figure 5 shows that as the soil undrained shear strength is increased the normalized pile settlement increases. This is because a pile founded in strata with a stronger $c_u$-profile can carry a higher load before the ultimate condition is reached. Therefore, at higher loads more concrete compression is expected, leading to more settlement. This trend becomes increasingly apparent as the $L/D$ ratio increases.

![Figure 5. Normalized pile head settlement ($w/D$) with increasing $L/D$ with a varying $c_u$-profile (a) $D = 0.6m$; (b) $D = 1.2m$](image)

#### 4.3 Influence of non-linearity factor

Figure 6 shows the influence of the non-linearity factor, $b$ on the computed normalized pile head settlement. Clearly as $b$ increases there is a marked increase in the computed settlements. Figure 6 shows the magnitude of the effect as remaining relatively consistent across the range of $L/D$ ratios.

#### 4.4 Influence of mobilization strain

Figure 7 shows that the mobilization strain parameter has a more limited effect on the computed normalized pile-settlement than the non-linearity parameter, $b$. The variation of normalized pile settlement shown on Figure 7 relates to changes of $\gamma_{M-2}$ of approximately $\pm 2\sigma$, and indicates that at least for London clay the variation does not have a significant effect. However, for other soil deposits where $\gamma_{M-2}$ can vary more significantly the trend shown on Figure 7 may not necessarily hold and the influence of $\gamma_{M-2}$ may be more considerable.
and influence of each parameter listed in Table 3. The aim (where possible) is to examine the influence of the pile base is neglected for the ultimate condition is reached. Therefore, at the influence of pile diameter becomes more marked.

Figure 4 shows the influence of pile diameter on the computed pile head settlement varying with pile length 

\[
\text{Normalized pile head settlement (} \frac{w_h}{D} \text{) with increasing } L/D \text{ with a varying } b \text{ and } \eta \text{ (a) } D = 0.6\text{m; (b) } D = 1.2\text{m}
\]

Figure 7. Normalized pile head settlement (\( \frac{w_h}{D} \)) with increasing \( \frac{L}{D} \) with a varying \( \gamma_{\text{c}} \). (a) \( D = 0.6\text{m} \); (b) \( D = 1.2\text{m} \)

4.5 Influence of concrete modulus

Figure 8 displays the influence of concrete elastic modulus on the computed normalized pile head settlement. The settlement ratio increases as the concrete modulus is reduced. This trend becomes especially apparent at high \( \frac{L}{D} \) ratios. Given that concrete modulus clearly can have a major influence on the computed settlements it is a parameter worthy of further study by construction and geotechnical engineers.

4.6 Influence of mobilization factor

As the mobilization factor is increased the computed settlements are seen to decrease (Figure 9). Factoring down the undrained shear strength means that less load is transferred to the concrete pile (less concrete compression) and there is also less soil straining and hence a lower computed \( w_{\text{soil}} \) component (Equation 5). The effect of an increasing \( M \) value is more marked as the pile lengthens but it is still significant for shorter piles. Adjusting \( M \) can be used to control foundation movements, as shown in Vardanega et al. (2012b).
The results of the parametric study reveal that especially as $L/D$ increases the mobilization factor and the concrete elastic modulus tend to dominate the settlement response of the pile. The influences of the $c_u$ profile and $b$ appear to have a more moderate influence while $\gamma_{M-2}$ tends to have a less significant effect on the computed settlements. The influence of $b$ and $\gamma_{M-2}$ are more pronounced as $L/D$ decreases. The results of the parametric study are shown in Table 4.

Table 4. Rankings of the studied factors (for London clay)

<table>
<thead>
<tr>
<th>Factor</th>
<th>Rank</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>1</td>
<td>most influence</td>
</tr>
<tr>
<td>$E_c$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$b$ (or $\eta$)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$c_u$-profile</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{M-2}$</td>
<td>5</td>
<td>least influence</td>
</tr>
</tbody>
</table>

5 SUMMARY

This paper has reviewed the simple calculation method proposed in Vardanega et al. (2012b) and shown the influence of variation of the London clay design parameters on the computed normalized head settlements of bored piles.

The results of the parametric study reveal that especially as $L/D$ increases the mobilization factor and the concrete elastic modulus tend to dominate the settlement response of the pile. The influences of the $c_u$ profile and $b$ appear to have a more moderate influence while $\gamma_{M-2}$ tends to have a less significant effect on the computed settlements. The influence of $b$ and $\gamma_{M-2}$ are more pronounced as $L/D$ decreases. The results of the parametric study are shown in Table 4.

Table 4. Rankings of the studied factors (for London clay)

<table>
<thead>
<tr>
<th>Factor</th>
<th>Rank</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>1</td>
<td>most influence</td>
</tr>
<tr>
<td>$E_c$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>$b$ (or $\eta$)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$c_u$-profile</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>$\gamma_{M-2}$</td>
<td>5</td>
<td>least influence</td>
</tr>
</tbody>
</table>

ACKNOWLEDGEMENTS

Thanks are due to S. Alogna for her assistance preparing the French abstract. Thanks are also due to Dr M. Williamson for his helpful suggestions and comments that helped improve the paper. The author also thanks Professor M. Bolton for supervising his doctoral studies.

REFERENCES


Kolody, E., Vardanega, P.J. & Patel, D.C. 2015. Settlement prediction of bored piles in stiff clay at a site in the Moscow region. (these proceedings).


