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A parametric investigation on the effects of inertia on the stability of power systems

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Abstract—The increase of non-synchronous generation sources on a power network are changing the system dynamics and this can lead to problems with stability. One of the contributions to the change in system dynamics is the variation in system inertia. By approximating a traditional stability metric, the critical clearing time (CCT) in an energetic framework we conduct a parametric investigation of the effect of inertia on power system stability. A set of solid three phase to ground faults are considered on a small but not trivial power system, the two-machine infinite bus network. The performance of the approximated CCT is compared to the true CCT.

Index Terms—Critical clearing time, direct methods, variable inertia, two machine infinite bus

I. INTRODUCTION

The complex dynamics of electric power systems has long been the subject of intense research, particularly in the area of stability. A power system is in a stable state when the total power supplied is equal to the total power consumed [1]. A fault on a power network, such as a solid three phase to earth fault, introduces a power imbalance, which causes some generators to temporarily accelerate away from the system frequency. Currently, there are practical developments on power systems that promise to radically change their dynamic behaviour. For example, the replacement of large, synchronous machines by wind turbines and the increasing use of high voltage DC to transfer power between regions will, together, dramatically alter the systems apparent mechanical inertia [2], [3]. Generator inertia helps to mitigate the effect of a fault on a power network by slowing the rate of change of generator frequency. As a consequence, there is value in developing new methods and articulating metrics that exploit theoretical, if simplified, descriptions of the system that can provide a deep understanding of the impact of new plants on the system. This approach has the potential to inform the design of control systems to better manage interactions between new devices on the system.

The system inertia is a function of the generator units that are operating on the network. As demand changes, the power dispatch plan can connect or disconnect units with different inertias which changes the total amount of inertia available to the system. The problem of low inertia in power systems has been studied in the control literature [2]–[8] with some research considering the effect of inertia on transient stability [3], [9], [10]. The swing equations [1], [11] have an explicit inertia parameter (for each machine) which we can consider as variable by using an aggregated model of a power network. This approach (also used in [3]) is adopted such that a decrease in the value of inertia in a particular region of the network reflects an increased penetration of renewable sources of generation, where renewable sources are assumed to contribute zero inertia. At the time of writing, the authors are not aware of any work which studies the effect of incremental changes in inertia over a wide range of values, which can provide a more rigorous study of its effect on system stability.

The effect of inertia on power system stability can be tested by subjecting the system to the same fault for different values of machine inertias. We use the so-called critical clearing time (CCT) metric [1], [12] to quantify the effect of faults on the stability of a power network. From the time a system suffers a fault, the rotor angles begin to diverge. Typically, the control systems in the exciter and governor of a generator do not have a significant effect on the dynamics of the rotor angles for a couple of seconds once a fault has been cleared. Therefore, we use the first swing criterion [13] to characterise system stability after a fault. We define a system as first swing unstable if the generator rotor angles continuously accelerate after a fault. The CCT is the upper bound on the duration of a short circuit on a power network (before the fault is removed - 'cleared' - by the action of protection mechanisms to isolate the faulted area) so that if the fault is on-line for longer, some generator rotors continuously accelerate after the fault has been cleared. This provides a worst-case scenario for stability.

The CCT provides a useful stability metric for power system design and it is generally computed using numerical
integration methods. However, in order to investigate the effect of many different varying system parameters on stability, a faster method is desirable to reduce the number of critical scenarios that must be analysed in detail. Alternative techniques have been developed to study stability of power systems, these include synchronisation [14], non-linear dynamics [15], bifurcation theory [16], passivity-based methods [17] and the computation of basins of attraction [18], [19]. Direct methods [20] consider the swing equations in an energetic framework. Despite their issues regarding the modelling of linear loads [21], [22] and scalability, they provide a well-defined conservative boundary for stability in terms of a critical system energy, which can be computed in reasonable time for small systems. In the literature, there are techniques for approximating the CCT using direct methods [23], [24], we present a new method for approximating the CCT by recasting the critical energy boundary.

A suitable strategy is required to observe the effect of a change in system parameters on the system stability without having to simulate the post-fault dynamics. Despite their drawbacks, direct methods are well-suited for use in parametric investigations of power systems because they provide a simple, well-defined stability boundary. This avoids the computational cost of numerically integrating the system after the fault is cleared for each change in the value of the inertia parameter. We will test the effectiveness of using direct methods against the more familiar first swing stability metric on a 9 bus, 3-generator power network found in [13].

II. BACKGROUND AND FAULT ANALYSIS

A. Model description

We consider the classic swing equation model [1], [12] to describe the stability effects of transient faults on a power system with synchronous generation. The generators are modelled as voltage sources behind reactances and the loads are of constant impedance. This model can be written as a set of coupled one-dimensional ordinary differential equations (ODEs), which describe the dynamics of the rotor angles of each synchronous generator $i \in \{1, \ldots, n\}$ in a network by considering Newton’s second law. In vector form the equation is

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$$

(1)

where

$$\mathbf{x} = [\delta^T; \omega^T]^T$$

and

$$\mathbf{F}(\delta, \omega) = \begin{pmatrix} \omega \\ (A(\delta, \omega)) \end{pmatrix}.$$ 

The vectors $\delta = [\delta_1, \ldots, \delta_n]^T$ and $\omega = [\omega_1, \ldots, \omega_n]^T$ are the generator rotor angles and angular speeds respectively, and the elements of the vector function $A(\delta, \omega)$ are

$$A_i(\delta, \omega) = \frac{1}{M_i} \left( P_{mi} - P_e(\delta) - P_{di}(\omega_i) \right),$$

where $M_i = \frac{2H_i}{\omega_0}$ is a lumped parameter, $\omega_0 = 2\pi f$ (where $f$ is the grid frequency: 50 Hz in Europe), $H_i$ is the inertia constant, $P_{mi}$ is the mechanical power input, $P_e(\delta)$ is the electrical power output and $P_{di}(\omega_i)$ is the power lost due to damping, modelled as in [1] by $P_{di} = D_i\omega_i$, where $D_i$ is a constant of proportionality.

The loads on the power system are assumed to be constant impedance loads such that Kron reduction [25] can be applied to the network. Therefore, the swing equations describe the dynamics of a reduced network comprising of constant voltage sources connected through a network of impedances [12]. The total load at generator $i$ is given by

$$P_i(\delta) = E_i^2 G_{ii} + \sum_{k \neq i} |E_i||E_k|G_{ik} \cos(\delta_i - \delta_k),$$

(2)

where $E_i = |E_i|e^{j\delta_i}$ is the internal voltage of generator $i$, $G_{ik}$ is the conductance between generators $i$ and $k$, $G_{ii}$ is the shunt conductance at bus $i$. The total electric power leaving generator $i$ is

$$P_{ei}(\delta) = P_i(\delta) + \sum_{k \neq i} \bar{P}_{ik} \sin(\delta_i - \delta_k),$$

(3)

where $\bar{P}_{ik} = |E_i||E_k|B_{ik}$ is the maximum active power flow between generators $i$ and $k$. $B_{ik}$ is the susceptance of the network connection between node $i$ and node $k$. The admittances $Y_{ik} = G_{ik} + jB_{ik}$ are the elements of the symmetric reduced bus admittance matrix $\mathbf{Y}_{\text{red}} \in \mathbb{C}^{n \times n}$. Kron reduction is fundamentally a matrix operation that constructs $\mathbf{Y}_{\text{red}}$ from a larger bus admittance matrix $\mathbf{Y}_{\text{BUS}} \in \mathbb{C}^{N \times N}$ (where $N \geq 2n$) which contains the full topology and load distribution (including the synchronous reactance) of a power network with $n$ synchronous generators.

B. Fault analysis

The traditional definition for the critical clearing time, as defined in the Introduction is also known as the first swing stability [13]. We assume, without loss of generality, that the moment a power system suffers a short-circuit is at time $t = 0$ and the fault is cleared at time $t_{cl}$. These two points in time define three distinct regimes in order to analyse the dynamics of a fault on a power system. These are (i) $t < 0$ (pre-fault), (ii) $0 \leq t < t_{cl}$ (fault-on) and (iii) $t \geq t_{cl}$ (post-fault). Each regime has a different reduced admittance matrix $\mathbf{Y}_{\text{red}}$ which will change some of the parameter values in the vector function in (1). Therefore, three separate sets of equations of the form (1) are required to model the power system for all time, which are denoted using the subscripts ‘pre’, ‘on’ and ‘post’.

Pre-fault, a power system is assumed to be balanced and therefore we assume that (1) is located at a stable (‘s’) equilibrium point $\mathbf{x}_{\text{pre}} = [\delta_{\text{pre}}^T, 0^T]^T$ where $|\delta_{\text{pre},i}| < \pi/2$, such that $\mathbf{F}_{\text{pre}}(\mathbf{x}_{\text{pre}}) = 0$, at time $t < 0$. The dynamics for $t \geq 0$ are computed by numerically integrating (1) and we write the solutions to (1) formally as

$$\mathbf{x}_{\text{on}}(t) = \Phi_{\text{on}}(t; \mathbf{x}_{\text{on}}(0) = \mathbf{x}_{\text{pre}}), \quad 0 \leq t \leq t_{cl}$$

(4)
during the fault and
\[ x_{\text{post}}(t) = \Phi_{\text{post}}(t, x_{\text{post}}(0) = x_{\text{in}}(t_{cl})), \quad t \geq t_{cl} \] (5)
after the fault. From these definitions we can define the CCT, denoted \( t_{\text{CCT}} \), formally as the minimum value of \( t_{cl} \) such that in the post-fault trajectory (5) some generators continuously accelerate [20].

III. A CRITICAL CLEARING TIME FROM DIRECT METHODS

A. An energetic approach

In general, a conservative metric for the local stability of systems of the form (1) can be found by considering an undamped system \( P_{di} = 0 \) for all \( i = 1, \ldots, n \) and constructing a suitable Lyapunov function. Direct methods use so-called ‘energy’ functions [11], which can also serve as Lyapunov functions, to measure the stability of such systems. A stability boundary is constructed in terms of a critical system energy \( E_c \) in the post-fault regime and a power system is classified as unstable when the total system energy surpasses this critical energy.

The total system energy can be measured when a power system (1) is modelled as a Hamiltonian system. However, the power consumed by the loads \( P_i(\delta) \) is a path-dependent quantity and cannot be modelled exactly by a conservative system. Many attempts have been made to approximate this term [21] so that an appropriate Hamiltonian system can be used. The most accepted technique [12, p.231] models the power consumed by the loads as a constant \( P_{ai} = P_i(\delta^0) \) where the point \( x^0 = [\delta^0, \omega^0]^T \) is a stable stationary point in the post-fault regime with \( |\delta_i^0| < \pi/2 \).

The so-called closest UEP (unstable equilibrium point) method [20] is used to find the critical system energy \( E_c \) in this work because, although it is the most conservative method (compared to the controlling UEP method or the potential energy boundary surface method [20]) it can be applied to any power system. Note that, in the presence of large linear loads in the network, the use of direct methods might lead to overestimates of the actual stability boundary.

The Hamiltonian function
\[ \mathcal{H}(x) = E_{\text{kin}}(\omega) + E_{\text{pot}}(\delta). \] (6)
quantifies the energy of a power system with \( n \) generators. It is the sum of the kinetic energy \( E_{\text{kin}}(\omega) \) and the potential energy \( E_{\text{pot}}(\delta) \) where
\[ E_{\text{kin}}(\omega) = \sum_{i=1}^{n} \frac{1}{2} M_i \omega_i^2, \] (7)
and
\[ E_{\text{pot}}(\delta) = - \sum_{i=1}^{n} (P_{mi} - P_{ai}) \delta_i - \sum_{i=1}^{n} \bar{P}_{Ik} \cos(\delta_i - \delta_k). \] (8)

An approximation of the CCT, denoted \( \dot{t}_{\text{CCT}} \), can be found by integrating the dynamics of the system during a fault until the system energy reaches the critical boundary \( E_c \). More specifically, such an estimate can be obtained by solving for \( t \) the equation
\[ \mathcal{H}(x_{\text{in}}(t)) = E_c. \]
The CCT approximation \( \dot{t}_{\text{CCT}} \) is much faster to compute than the traditional CCT because the dynamics of the post-fault system (4) do not need to be computed. The dynamics during the fault are numerically integrated until the critical system energy has been reached and this, in general, takes a short amount of time. Therefore, the effect that varying different parameters has on the CCT can be studied in reasonable time by using small incremental changes to the parameter values over a given range [26].

B. An example network

The specific network we choose to study is the so-called two-machine infinite bus (TMIB) system, previously studied in [18] using non-linear dynamics techniques and more recently in [27] and [22]. The equations that model the dynamics of the two synchronous generators in the TMIB system during post-fault are
\[ \dot{\delta}_1 = \omega_1 \]
\[ \dot{\delta}_2 = \omega_2 \]
\[ \dot{\omega}_1 = \frac{1}{M_1} \left[ (P_{m1} - P_{a1}) - \bar{P}_{13} \sin(\delta_1) - \bar{P}_{12} \sin(\delta_1 - \delta_2) \right] \]
\[ \dot{\omega}_2 = \frac{1}{M_2} \left[ (P_{m2} - P_{a2}) - \bar{P}_{23} \sin(\delta_2) - \bar{P}_{12} \sin(\delta_2 - \delta_1) \right] \] (9)
where damping has been neglected. A schematic of the TMIB network is found in Fig. 1 with annotations to explain the power flow terms in (9). The full power network (pre Kron reduction) is a 9-bus, 3-generator taken from [13] where the generator with the largest inertia is modelled as an infinite bus. A network diagram is included in Fig. 2 and the parameter values can be found in [13].

In Fig. 3 we illustrate the behaviour of rotor angle \( \delta_1 \) in a TMIB system for different fault clearing times \( t_{cl} \). We consider a solid three-phase to earth fault at a network location near Fig. 1. Schematic of the two machine infinite bus (TMIB) reduced power network modelled by (9). The network is composed of two synchronous generators and an infinite bus that models the rest of a larger power network. Each synchronous generator has a mechanical input power \( P_m \), a constant power dissipation \( P_a \) and inertia parameter \( H \). The infinite bus is a theoretical bus which has infinite inertia and therefore constant rotor angle which is set to zero.
generator 1. For \( t_{c1} \leq t_{\text{CCT}} \) (blue line) we observe oscillatory behaviour for all time after the fault has been cleared. For clearing times \( t_{\text{CCT}} < t_{c1} \leq t_{\text{CCT}} \) (green line) the rotor angle oscillates for a short duration before losing stability. As the clearing time is further increased to \( t_{c3} \geq t_{\text{CCT}} \) (red line) the rotor angle does not oscillate before losing stability and instead accelerates continuously after the fault.

Fig. 2. Schematic of the full two machine infinite bus (TMIB) power network adapted from [13] with all parameter values as they are in the reference. The network is composed of six buses with three generators, one of which is an infinite bus.

Fig. 3. An example of the evolution of the rotor angle \( \delta_1(t) \) when a TMIB system suffers a solid three phase to earth fault at a network location near generator 1. The dynamics are shown for three different fault clearing times indicated in the legend.

IV. EFFECT OF VARIABLE INERTIA ON STABILITY

A. Implementation details

The TMIB power network introduced in Section III-B is used for this study. In order to compare the effectiveness of the approximate CCT calculated from direct methods and the traditional CCT, we consider four different three-phase to earth faults on the network. The fault locations can be found in Table I. The clearing action in each case is to isolate the affected line. As the reader will recall, we use an aggregated model [12, Chapter 14] of a power system in a similar way to [3], where a network is split into regions. The specific regions that a power system is divided into is arbitrary but usually considers geography, population density and predominant power flows across region boarders.

In a TMIB network there are three regions, it is assumed that the dynamics for each region can be approximated by a single synchronous machine with parameters informed by the individual machines within the region. This technique has been previously used in [28], [29] for studying the dynamics of the GB power network. If the dynamics from a given region are assumed to be from a synchronous machine, we stipulate that the majority (> 50%) of power produced from each region is from synchronous generation. The increasing proportion of non-synchronous generation (e.g. from wind or photovoltaic generation) within a predominately synchronous region of generation is modelled by decreasing the inertia parameter of the large synchronous machine for that region.

B. Results

In Figures 4 and 5 we present our results for the effect of inertia variation for two different faults on the network in Fig. 2. The faults are on line 5-7 close to bus 7 and on line 8-9 close to bus 9 respectively. In each figure, there is a panel for each of the two CCT metrics we use. The dependence of the CCT approximation \( t_{\text{CCT}}(H_1, H_2) \) and the actual CCT \( t_{\text{CCT}}(H_1, H_2) \) on the machine inertias \( H_1 \) and \( H_2 \) is plotted using a colour chart. The machine inertias \( H_1 \) and \( H_2 \) are plotted on the \( x \) and \( y \) axes, and the CCT is plotted as a colour. The colour bar at the side of each plot gives a numerical value of time corresponding to the colour plotted and have the same scale for a given fault such that the two metrics can be directly compared.

From observation we note that, for each fault, the general dependence of the system stability as a function of the inertias as measured by \( t_{\text{CCT}} \) is captured by the CCT approximation \( \hat{t}_{\text{CCT}} \). In Fig. 4 both CCT metrics show a stronger dependence on inertia \( H_1 \) for any value of \( H_2 \), with the \( t_{\text{CCT}} \) metric showing an increased dependence on \( H_2 \) for larger values. In Fig. 5, although the exact curvature of the colour charts is different, they convey the same qualitative trend being that the stability is dependent on the machine with the largest inertia.

Linear regression is used to compare the estimate for the CCT approximation \( \hat{t}_{\text{CCT}} \) with the actual CCT \( t_{\text{CCT}} \). The colour charts in Figures 4 and 5 suggest that the direct method estimate follows the actual CCT with some offset. We use a linear regression formula of the form

\[
t_{\text{CCT}}(H_1, H_2) = \alpha \hat{t}_{\text{CCT}}(H_1, H_2) + \beta + e(H_1, H_2) \tag{10}
\]

to capture the offset, where \( \alpha \) and \( \beta \) are constant parameters and \( e(H_1, H_2) \) is the error. The parameters \( \alpha \) and \( \beta \) are chosen in order to minimise the mean square error of \( e \) over all values of \( H_1, H_2 \). For two colour plots that have the same profile with some offset \( \beta \), the parameter \( \alpha = 1 \). The bias of the direct estimate \( \hat{t}_{\text{CCT}} \), is given by \( (\hat{t}_{\text{CCT}} - t_{\text{CCT}}) \) and indicates whether the direct method underestimates (negative bias) or overestimates (positive bias) the CCT on average.
In Table I, we can see that all faults have values of $\alpha$ that are relatively close to one. This suggests that the error in the CCT approximation is relatively independent of the traditional CCT for given values of inertia. The largest and smallest deviation from $\alpha = 1$ are for the faults on line 5-7 and on line 8-9 respectively. These are plotted in Figures 4 and 5. The biases of the estimated CCT suggest that the CCT approximation derived from direct methods tends to underestimate the traditional CCT values for our system; the only exception is the fault on line 8-9 where the direct method produces a conservative estimate for CCT on average.

The TMIB power system suffers a solid three-phase to earth fault at bus $9$, near to the end of the line 8-9. The post-fault network has the line 8-9 switched out.

Fig. 4. A colour chart of the effect that variable inertia has on (a) the CCT approximation $\hat{t}_{CCT}(H_1, H_2)$ and (b) the first-swing CCT $t_{CCT}(H_1, H_2)$. The TMIB power system suffers a solid three-phase to earth fault at bus 7, near to the end of the line 5-7. The post-fault network has the line 5-7 switched out.

### V. CONCLUSIONS

We have presented a technique to measure the effect varying inertia in a system has on the CCT for a set of faults on a network. This technique extracts an approximation for the CCT from direct methods which can be computed faster than the traditional CCT metric. However, in general, direct methods do not give a conservative stability estimate for networks with high linear loads. High loading in a network increases the dependence on the path-dependent terms (2) of the swing equations resulting in the Hamiltonian model being less effective. Despite this drawback, it is clear that the approximate CCT metric performs well with regards to identifying the qualitative effect of the variation of the inertia parameters on stability. Therefore, we suggest that this technique can be used as an initial probe to discover the general effects of changes to generator inertias on system stability before more detailed models are employed.

In general, the method we used to evaluate the critical energy boundary relies on the fact that the position of the closest UEP does not change as the inertia parameters are varied. However, the position of the closest UEP is dependent on all the remaining system parameters and must be recalculated if these parameters are varied. Therefore, new techniques are required to improve the application of energetic methods to
investigate the effects of parameters other than inertia [26].

Finally, an aggregated model of a power network has been used to justify the variation of the inertia parameters of synchronous machines. It was assumed that by keeping synchronous generation as the predominant power producer within a region, the dynamics of an entire region could be modelled by a single synchronous machine. This is a reasonable assumption for low penetration levels of renewable generation but as the penetration level increases, the specific effect of the dynamics from power converters and control systems in renewable sources are not catered for in our model. Therefore, the extent to which a synchronous machine can adequately capture the dynamics of a particular region under higher penetration of renewable generation is a topic of further research.

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REFERENCES