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ENHANCING CLASS DISCRIMINATION IN KERNEL DISCRIMINANT ANALYSIS

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ABSTRACT
In this paper, we propose an optimization scheme aiming at optimal nonlinear data projection, in terms of Fisher ratio maximization. To this end, we formulate an iterative optimization scheme consisting of two processing steps: optimal data projection calculation and optimal class representation determination. Compared to the standard approach employing the class mean vectors for class representation, the proposed optimization scheme increases class discrimination in the reduced-dimensionality feature space. We evaluate the proposed method in standard classification problems, as well as on the classification of human actions and face, and show that it is able to achieve better generalization performance, when compared to the standard approach.

Index Terms— Kernel Discriminant Analysis, Optimized Class Representation, Nonlinear data projection

1. INTRODUCTION
Kernel Discriminant Analysis (KDA) is a well-known algorithm for supervised feature extraction and dimensionality reduction. It aims at the determination of an optimal subspace for nonlinear data projection, in which the classes are better discriminated [1, 2, 3, 4, 5, 6, 7]. It exploits data representations in an arbitrary-dimensional feature space determined by applying a non-linear data mapping process (and exploiting the so-called kernel trick [8, 9, 10]). After the determination of the data representation in the arbitrary-dimensional feature space, a linear projection is calculated, which corresponds to a non-linear projection of the original data. The adopted class discrimination criterion is the ratio of the between-class scatter to the within-class scatter in the reduced-dimensionality feature space, which is usually referred as the Fisher ratio.

KDA optimality is based on the assumptions of: a) normal class distributions with the same covariance structure in the kernel space and b) class representation by the corresponding class mean vector (determined in the kernel space). Under these assumptions, the maximization of the Fisher ratio leads to maximal class discrimination in the reduced-dimensionality feature space. Under the assumption of normal class distributions in the kernel space, the assumption that each class should be represented by the class mean vector seems reasonable. However, the normality assumption is restrictive and difficult to be met. Recently, by observing that the between-class and within-class scatter matrices employed for linear data projection in Linear Discriminant Analysis (LDA) can be considered to be functions of the adopted class representation, it has been shown that, when the two aforementioned assumptions are not met, the adoption of class representations different from the class mean vectors leads to increased class discrimination in the reduced-dimensionality feature space [11]. In addition, it has been proven that, given a data projection matrix determined by maximizing the criterion adopted in LDA, the optimal class representations can be analytically calculated. In order to determine both the optimal data projection matrix and the optimal class representations for the case of LDA, an iterative optimization scheme has been proposed [11]. The outcomes of [11] have also been verified in [12], where Particle Swarm Optimization-based Fisher ratio maximization has been employed for the maximization of the LDA criterion.

In this paper, we formulate an optimization problem that exploits a non-linear data mapping process to an arbitrary-dimensional feature space, in which optimized class representations are determined. By employing such optimized class representations, a linear data projection from the arbitrary-dimensional feature space to a reduced-dimensionality feature space of increased discrimination power is subsequently calculated. We prove that the determination of the optimal class representation in the arbitrary-dimensional feature space has a closed form solution and formulate an iterative optimization scheme for the determination of both the optimal class representations and the optimal nonlinear data projection, in terms of Fisher ratio maximization. The proposed method is evaluated on standard classification problems, as well as on two computer vision problems problems, i.e., the recognition of human actions and face. Experimental results show that the proposed method is able to enhance class discrimination and achieve better performance.

The rest of the paper is structured as follows. The proposed method is described in detail in Section 2. Experimental results on two human action recognition and three face recognition datasets are provided in Section 3. Finally, con-
conclusions are drawn in Section 4.

2. PROPOSED METHOD

Let us denote by $x_{ij} \in \mathbb{R}^D, i = 1, \ldots, C, j = 1, \ldots, N_i$ a set of $D$-dimensional data, each belonging to one of $C$ classes. The number of samples belonging to class $i$ is equal to $N_i$.

In order to determine a nonlinear data projection, the input space $\mathbb{R}^D$ is mapped to an arbitrary-dimensional feature space $\mathcal{F}$ (having the properties of Hilbert spaces) [8, 9, 10] by employing a function $\phi(\cdot) : x_{ij} \in \mathbb{R}^D \to \phi(x_{ij}) \in \mathcal{F}$ determining a nonlinear mapping from the input space $\mathbb{R}^D$ to the arbitrary-dimensional feature space $\mathcal{F}$.

Let us denote by $\Phi_i \in \mathbb{R}^{D \times N_i}$ a matrix containing the samples belonging to class $i$ (represented in $\mathcal{F}$). By using $\Phi_i$, $i = 1, \ldots, C$ we can construct the matrix $\Phi = [\Phi_1, \ldots, \Phi_C]$ containing the representations of the entire data set in $\mathcal{F}$. The so-called kernel matrix $K \in \mathbb{R}^{N \times N}$ is given by $K = \Phi^T \Phi$. Let us denote by $K_i \in \mathbb{R}^{N \times N_i}$ a matrix containing the columns of $K$ corresponding to the samples belonging to class $i$. That is, $K = [K_1, \ldots, K_C]$, where $K_i = \Phi^T \Phi_i$.

In the proposed method, each class $i$ is represented by a vector $\phi(\mu_i)$. We do not set the assumption that the class representation must be the class mean vector in $\mathcal{F}$. $\phi(\mu_i)$ can be any vector enhancing class discrimination in the projection space $\mathbb{R}^d$. In order to determine both the optimal data projection matrix $P$ and the optimal class representations $\phi(\mu_i), i = 1, \ldots, C$, we propose to maximize the following criterion with respect to both $P$ and $\mu_i$:

$$
\mathcal{J}(P, \mu_i) = \frac{\text{trace}(P^T \tilde{S}_w(\mu_i) P)}{\text{trace}(P^T \tilde{S}_b(\mu_i) P)},
$$

(1)

where the matrices $\tilde{S}_w(\mu_i), \tilde{S}_b(\mu_i)$ are given by:

$$
\tilde{S}_w(\mu_i) = \sum_{i=1}^C \sum_{j=1}^{N_i} \left( \phi(x_{ij}) - \phi(\mu_i) \right) \left( \phi(x_{ij}) - \phi(\mu_i) \right)^T,
$$

(2)

$$
\tilde{S}_b(\mu_i) = \sum_{i=1}^C N_i \left( \phi(\mu_i) - \phi(m) \right) \left( \phi(\mu_i) - \phi(m) \right)^T.
$$

(3)

$\phi(m)$ is the mean vector of the entire dataset in $\mathcal{F}$. In the following, we assume that the data set is centered in $\mathcal{F}$. This can always be done by using $\phi(x_{ij}) = \phi(x_{ij}) - \phi(m)$, leading to a centered version of the kernel matrix given by $K = \frac{1}{N} K_1 - \frac{1}{N_1} 1K + \frac{1}{N_1} 1K1$, where $1 \in \mathbb{R}^{N \times N}$ is a matrix of ones.

The maximization of (1) leads to the determination of a data projection that can be used to map the original data to a reduced-dimensionality feature space $\mathbb{R}^d$, where the data dispersion from the corresponding class vector $\mu_i = P^T \phi(\mu_i)$ is minimized, while the dispersion of the class vectors belonging to different classes from the total mean is maximized. In order to determine both the optimal data projection $P$ and the optimal class vectors $\phi(\mu_i)$ we employ an iterative optimization scheme formed by two processing steps. In the following, we describe them in detail.

2.1. Calculation of $P$

In order to determine the optimal data projection matrix $P$ we work as follows [4]. Let us denote by $p$ an eigenvector of the problem $\tilde{S}_b(\mu_i) p = \lambda \tilde{S}_w(\mu_i) p$ with eigenvalue $\lambda$. $p$ can be expressed as a linear combination of the data (represented in $\mathcal{F}$) [8, 9, 10], i.e., $p = \sum_{i=1}^C \sum_{j=1}^{N_i} a_{ij} \phi(x_{ij}) = \Phi a$, where $a \in \mathbb{R}^N$. In addition, we can express $\phi(\mu_i)$ as a linear combination of the samples belonging to class $i$, i.e., $\phi(\mu_i) = \sum_{j=1}^{N_i} b_j \phi(x_{ij}) = \Phi_i b_i$, where $b_i \in \mathbb{R}^{N_i}$. By setting $K a = u$, the aforementioned eigenproblem can be transformed to the following equivalent eigenproblem:

$$
B(b_i) u = \lambda W(b_i) u,
$$

(4)

where $B(b_i) = \text{blockdiag}(N_1 b_1 b_1^T, \ldots, N_C b_C b_C^T)$ and $W(b_i) = \text{blockdiag}(W_1, \ldots, W_C)$, with $W_i = I_{N_i} - 1N_i 1N_i^T - b_i 1N_i^T - b_i^T 1N_i$. Both $B(b_i), W(b_i) \in \mathbb{R}^{N \times N}$. Thus the maximization of (1) can be approximated by applying a two step process:

- **Solution of the eigenproblem** $B(b_i) u = \lambda W(b_i) u$. By keeping the eigenvectors corresponding to the $d$ maximal eigenvalues, a matrix $U = [u_1, \ldots, u_d]$ is obtained.

- **Calculation of the projection matrix** $A = [a_1, \ldots, a_d]$, where $K a_j = u_j$. In the case where $K$ is non-singular, the vectors $a_j, j = 1, \ldots, d$ are given by $a_j = K^{-1} u_j$. When $K$ is singular, the vectors $a_j, j = 1, \ldots, d$ can be approximated by $a_j = (K + cI)^{-1} u_j$, where $c$ is a small positive value and $I \in \mathbb{R}^{N \times N}$ is the identity matrix.

After the calculation of $A$, a vector $x_t \in \mathbb{R}^D$ can be projected to the discriminant space $\mathbb{R}^d$ by applying $y_t = A^T k_t$, where $k_t \in \mathbb{R}^{N}$ is a vector given by $k_t = \Phi^T \phi(x_t)$.

2.2. Calculation of $\phi(\mu_i), i = 1, \ldots, C$

In order to maximize (1) with respect to the class vectors $\mu_i, i = 1, \ldots, C$, we also exploit that $p = \Phi a$ and $\phi(\mu_i) = \Phi_i b_i$. The optimization problem in (1) can be transformed to the following equivalent optimization problem:

$$
\tilde{J}(A, b_i) = \frac{\text{trace}(AABA^T)}{\text{trace}(AWW^T)},
$$

(5)

where $B = \sum_{i=1}^C N_i \Phi^T \Phi_i \Phi_i b_i^T \Phi_i^T \Phi_i$ and $W = \sum_{i=1}^C \left( K_i K_i^T - K_i 1N_i b_i^T K_i^T - K_i^T b_i 1N_i K_i^T + N_i K_i b_i b_i^T K_i^T \right)$. 

By solving for $\nabla b_i \left( \tilde{\beta} \right) = 0$ we obtain:

$$b_i = \frac{\gamma}{N_i} 1_{N_i}.$$  \hspace{1cm} (6)

In the above, $1_{N_i} \in \mathbb{R}^{N_i}$ is a vector of ones. $\gamma$ is given by:

$$\gamma = \frac{\text{trace} \left( \sum_{i=1}^{C} A K_i K_i^T A^T \right)}{\text{trace} \left( \sum_{i=1}^{C} \frac{1}{N_i} A K_i 1_{N_i} 1_{N_i}^T K_i^T A^T \right)}.$$ \hspace{1cm} (7)

After the calculation of $b_i, i = 1, \ldots, C$, class $i$ is represented in $\mathcal{F}$ by using $\phi(\mu_i) = \sum_{j=1}^{N_i} b_{ij} \phi(x_{ij})$.

### 2.3. Iterative Optimization Scheme

Taking into account that $A$ is a function of $b_i, i = 1, \ldots, C$ and that $b_i$ is a function of $A$, a direct maximization of $\mathcal{J}$ with respect to both $A$ and $b_i$ is difficult. In order to maximize $\mathcal{J}$ with respect to both $A$ and $b_i$, we employ the following iterative optimization scheme. Let us denote by $b_{i,t}, i = 1, \ldots, C$ the class vectors calculated at the $t$-th iteration of the optimization scheme. By using $b_{i,t}$, the data projection matrix $A_t$ can be calculated by following the process described in subsection 2.1. After the calculation of $A_t$, $b_{i,t+1}$ can be calculated by using (6). The above described process is initialized by using the class mean vectors, i.e., $b_{i,0} = \frac{1}{N_i} 1_{N_i}, i = 1, \ldots, C$ and is terminated when $(J(t+1) - J(t))/J(t) < \epsilon$, where $\epsilon$ is a small positive value, equal to $\epsilon = 10^{-6}$ in our experiments.

### 3. EXPERIMENTS

In this Section we describe experiments conducted in order to compare the performance of the proposed method with that of KDA [4] employing the class mean vectors for class representation. We have applied the two algorithms on standard classification problems, as well as on human action and face recognition problems. Experiments conducted on standard classification problems are described in Subsection 3.1. Experiments conducted on publicly available action and face recognition databases will be described in Subsections 3.2 and 3.3, respectively. In all the experiments we have employed the proposed method and KDA-based data projection in order to map the data to the corresponding discriminant subspace $\mathbb{R}^d$. Subsequently, classification is performed by using the class mean vectors for the KDA-based classification scheme. For the proposed classification scheme, classification is performed by using the class reference vectors.

#### 3.1. Experiments on Standard Classification Problems

We have conducted experiments on eight publicly available classification datasets coming from the machine learning repository of University of California Irvine (UCI) ([13]).

<table>
<thead>
<tr>
<th>Dataset</th>
<th>KDA</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abalone</td>
<td>52.85 (±0.69)</td>
<td>54.19 (±0.4)</td>
</tr>
<tr>
<td>German</td>
<td>70.65 (±1.18)</td>
<td>72.16 (±1.25)</td>
</tr>
<tr>
<td>Glass</td>
<td>67.66 (±3.6)</td>
<td>68.36 (±3.39)</td>
</tr>
<tr>
<td>Indians</td>
<td>72.24 (±1.04)</td>
<td>74.61 (±1.66)</td>
</tr>
<tr>
<td>Iris</td>
<td>80.53 (±2.79)</td>
<td>85.07 (±2.71)</td>
</tr>
<tr>
<td>Spect</td>
<td>79.59 (±1.9)</td>
<td>81.09 (±1.17)</td>
</tr>
<tr>
<td>SpectF</td>
<td>77.87 (±1.73)</td>
<td>79.14 (±1.46)</td>
</tr>
<tr>
<td>TeachAss</td>
<td>56.03 (±6.35)</td>
<td>58.28 (±2.84)</td>
</tr>
</tbody>
</table>

On each dataset, the 5-fold cross-validation procedure has been performed by using the same data partitioning for the two classification schemes. The mean classification rate over all folds has been used to measure the performance of each algorithm in one experiment. Ten experiments have been performed for each data set. The mean classification rate and the observed standard deviation over all experiments have been used to measure the performance of each algorithm. In all these experiments we have employed the RBF kernel function $[K]_{j,l,m} = \exp \left( -g \| x_l - x_m \|^2 / 2 \right)$. The value of parameter $g$ has been automatically chosen in each fold from a set $g = 10^r, r = -6, \ldots, 6$, by applying 5-fold cross-validation on the corresponding training set. The mean classification rates and the observed standard deviations over all experiments for each data set are illustrated in Table 1. By observing this Table, it can be seen that the proposed method outperforms KDA in all datasets, enhancing its performance by 1%–5%.

#### 3.2. Experiments on Human Action Recognition

We have conducted experiments on two publicly available action recognition datasets, namely the Hollywood2 and the Olympic Sports datasets. A brief description of the datasets and the experimental protocols used in our experiments is given in the following. We have employed the Bag-of-Words (BoW)-based video representation by using HOG, HOF, MBHx, MBHy and Trajectory descriptors evaluated on the trajectories of densely sampled interest points [14]. Following [14], we set the number of codebook vectors for each descriptor type equal to $D_k = 4000$ and employ the $\chi^2$ kernel function $[K]_{j,l,m}^k = \exp \left( \frac{1}{2} \sum_{r=1}^{D_k} \frac{(x_{lr} - x_{mr})^2}{x_{lr} + x_{mr}} \right)$. The value of parameter $\sigma_k$ has been determined by applying 5-fold cross validation on the training vectors of descriptor $k$ using the values $\sigma = 2^r, r = 0, \ldots, 3$. Different descriptors are finally combined by exploiting a multi-channel approach [15], i.e., $[K]_{j,l,m} = \prod_{k=1}^{C} [K]_{j,l,m}^k$.

The Olympic Sports dataset consists of 783 videos depict-
The performance obtained by applying the two classification schemes on each data set is illustrated in Table 2. By observing this Table, it can be seen that the proposed method outperforms KDA in both databases, enhancing its performance by 2 – 3%.

### 3.3. Experiments on Face Recognition

<table>
<thead>
<tr>
<th>Dataset</th>
<th>KDA</th>
<th>Proposed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Olympic Sports</td>
<td>81.54 ± 0.6</td>
<td>83.35 ± 0.4</td>
</tr>
<tr>
<td>Hollywood2</td>
<td>58.63 ± 0.4</td>
<td>61.22 ± 0.5</td>
</tr>
</tbody>
</table>

Table 2. Performance (%) on Human Action Recognition.

In this paper, we described an optimization scheme aiming at optimal nonlinear data projection, in terms of Fisher ratio maximization. By optimizing the Fisher ratio with respect to both the data projection matrix and the class representation in the projection space, the optimal discriminant projection space is obtained. Experimental results on standard classification problems, as well as on human action and face recognition problems show that the adopted approach increases class discrimination, when compared to the standard KDA approach.

### Acknowledgment

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5. REFERENCES


