Low carbon warehouse management under cap-and-trade policy

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Abstract: Green warehouse management plays a significant part in developing a carbon efficient supply chain. This research examines the behaviour change in warehouse management decisions under the cap-and-trade emission policy and explores the role of green technology investment in managing the trade-offs between the economic and environment performances of warehousing operations. This study analyses the optimal decisions in warehouse management and technology investment under the cap-and-trade emissions policy to assist the practitioners in making efficient decisions. Moreover, this study also investigates the effect of initial carbon emission allowance and transaction costs of the unit carbon emission trading with the outside market, on the economic and environment performances of warehousing operations. The findings of this study provide useful insights in greening the warehousing operations and reducing the carbon emissions.

Keywords: Green warehouse management, Low carbon operations, Cap-and-trade emission policy, Green technology investment

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1 Introduction

In today’s world changing competitive landscape, volatile demand pattern, new environmental regulations and need for optimizing supply chain operations have altogether posed significant challenges to businesses. As a result, organizations are nowadays not only paying attention on improving their supply chain operations, but also looking for innovative ways to address environmental issues associated with their operations. Past literature has put forward the notion of ‘green supply chain management’ (GSCM) to combine the operational efficiency with environmental performance (Kumar et al. 2014). Van Hock (1999) and Zhu and Sarkis (2004) stressed that GSCM has emerged as an important new archetype for companies to gain market competitiveness by lowering their environmental risks and impacts, while raising their ecological efficiency. Green et al. (2012) reported that the adoption of GSCM practices by manufacturing organizations leads to improved environmental and economic performances, which, in turn, positively impact operational performance.

Many studies have attempted to address various issues surrounding the environmental impacts of supply chain operations. For instance, Blome et al. (2013) identified the antecedents and effects of green procurement and green supplier development on supplier performance. Mosgaard (2015) focused on improving the practices of green procurement of minor items. The analysis showed that changes in the purchasers’ practices are not as much dependent on whether they understand, but rather a matter of whether the purchasers actually put their knowledge into practice. Deif (2011) proposed a model on green manufacturing which provided a comprehensive qualitative answer to the question of how to design and/or improve green manufacturing systems as well as a roadmap for future quantitative research to better evaluate this new paradigm. Issues around green purchasing and distribution, and green transportation have received prominent attention by many researchers (Björklund 2011; Paksoy and Özceylan 2013; Chen and Wang 2016). However, research on green warehousing has received relatively little attention (Fichtinger et al. 2015). Warehousing management is an essential part of the supply chain management, as Kumar et al. (2011) pointed out that warehouses and distribution centres are the last points where productivity could be controlled and managed in the supply chain to further reduce the cost.
This paper therefore aims to address this research gap by focusing on green/low carbon warehouse management. The paper discusses the impact of the cap-and-trade emission policy as well as the significance of green technology investment on greening the warehouses. Rest of the paper is organized as follows; section 2 reviews the literature in green warehouse management, cap-and-trade policy and green technology investment; section 3 describes the proposed model and assumptions; and section 4 presents the basic model. Section 5 describes the model with green technology investment and section 6 presents the discussion. A case study is presented in Section 7, in which numerical examples are provided to give more management insights. Finally section 8 concludes this paper and presents area for future investigation.

2 Literature review

2.1 Green warehouse management

Past literature in warehouse management (Heragu et al. 2004; Gu et al. 2010; Mishra et al., 2011; Topan and Bayindir 2012; Yang et al., 2012; Fichtinger et al., 2015; Reaidy et al. 2015) has focussed on addressing different issues related to warehousing activities such as inventory management, order fulfilment, optimal space utilisation, operational efficiency, loading / unloading problems, material handling issues etc., however limited studies have attempted to understand the carbon footprint of different warehousing operations. While considering the issue of carbon emission in context of a supply chain, many past literature have given more attention to understanding the impact of procurement, manufacturing, transportation etc. on environmental, but limited discussion has been found in understanding the consequences of warehousing activities on environment (Marchant 2010). Moreover, limited research could be found in evaluating the principles of warehouse management in light of current policies and regulations, particularly under cap-and-trade policy. This section focuses on reviewing the literature on different sustainability issues and strategies in warehouse management.

While considering green warehouse management, many firms have focussed on the efficient and economic use of energy input, conventionally from fossil fuel, that provide power for material handling equipment, and regulated the optimum temperature, light, and
water usage in warehouses (Mckinnon et al. 2010). Marchant (2010) developed a three-stage warehouse sustainability model that addressed the business, economic, environmental and social aspect of warehousing, and considered a wide range of measures and actions where companies are seeking to achieve minimal impact over economic, resources, environmental, and ecological features. Tan et al. (2009) discussed the application of sustainability principles in the context of warehouse storage and distribution management, and developed a sustainability model for setting up of a warehouse or transformation of an existing warehouse. They argue that while sustainability is a core value to many businesses, they find it hard to implement in their current business setting particularly when third party logistics management system like warehousing and distribution is present in their supply chain network.

Moreover, Mckinnon and Piecyk (2012) discussed different approaches for the reduction of carbon emissions from logistics operations. They argued that mostly firms simply apply corporate-level targets to logistics, despite the fact that carbon abatement potential and cost–effectiveness vary by different logistical functions and activities. They further proposed different principles applicable to the decarbonization of logistics in practice. Tan et al. (2010) discussed the concept of sustainable enterprise simulation models in the context of a warehousing and distribution company, and explained the interconnectivity between disparate sustainability dimensions in practice. Similarly, literature such as Rai et al. (2011) and Validi et al. (2014) discussed the assessment of carbon emissions in a distribution system. Bouchery et al. (2012) also incorporated sustainability criteria into inventory models and examined the effectiveness of different regulatory policies in controlling carbon emissions. Further, Żuchowski (2015) argued that implementation of sustainable solutions for warehouse management reduces emission of greenhouse gases and resource consumption, and, in the long run, leads to a "green" warehouse.

Johnson (2008) discussed the issue of carbon emission in context of material handling equipment in warehousing. He further argued that there is insufficient information on carbon emission, and energy consumption rates in warehousing operations in practice. In practice, when it comes to the different choices of material handling equipment, managers tends to focus on a cost of ownership approach based on the equipment cost, fuel cost and maintenance cost rather than the wider evaluation of the total emission. Some other studies
have focussed on designing green building for warehouses to reduce the carbon emission (Mckinnon et al. 2010, Carbon Trust 2000). However, the range of building types, and the different operating conditions make it challenging to produce benchmarks on green standards in warehouse management (Johnson 2008, Marchant 2010). Organisations also showed indications of incorporating green strategies for the purpose of benefiting the company, if these investments were not too costly. From the literature review, it is evident that even though, a few studies attempted to address sustainability issues in context of warehousing operations, the discussion over the implication of carbon emission policies on warehouse management is limited in literature. Therefore, the focus of this paper is timely and relevant in addressing this relevant research gap in literature.

Resource re-usage and green technology are also gaining interest in supply chain management research. In recent years, new advancements in green and cleaner technology have encouraged different organisations to adopt green practices in their processes (Wiesenthal et al. 2012). However, cost associated with technology adoption poses significant barriers for the successful implementation. Businesses look for the opportunities where both economic and environmental performance can be improved at the same time, and therefore, quick return on investment is the key driver for implementing green technology in various supply chain processes (Wang 2015). Warehousing is often been considered as an energy intensive process. Investment in greener technologies to make these processes energy efficient would help to reduce the carbon footprint of the supply chain.

Technological investment is considered as strategic decisions for organisations to reduce carbon emissions and become environmental friendly. Firms view many of these technological investments as possible alternatives for gaining or maintaining a competitive advantage (Sarkis 2003). Zhu et al. (2006) highlighted that many research in past have proposed different prescriptive models for evaluation of green practices and technology in supply chain. The development and improved capabilities of information technology have changed the ways in which the supply chain operates. Chung and Wee (2010) examined the impact of the green product design and the information technology investment in business process considering remanufacturing. The results show that new technology evolution, reusable-item take-back ratio and direct shipment are the critical operational factors in green
product design and information technology investment.

Mackinnon et al. (2010) discussed that technologies with lower energy inputs and better operational performance could reduce the energy demand in warehouses. However, to engage more actively in developing a sustainable warehouse a move to renewable sources of energy should be encouraged. However, the suitability and potential applicability of the energy sources for an individual warehouse depends upon wide range of operational, cost, environmental, market and regulatory factors (Marchant 2010). Most companies see compliance to regulation as the principal motivation for change towards green warehousing; however lack of understanding of green regulations, and its implications on warehousing operations could be challenging to achieve the goal of sustainability within the warehouse sector.

2.2 Cap-and-Trade and warehouse management

Many carbon emission control policy schemes e.g. mandatory carbon emissions capacity, carbon emissions tax, cap-and-trade, and carbon offsets, have been implemented across different countries in the world (Jin et al. 2014; Chen and Hao 2015; Xu et al. 2016; Jiang and Chen 2016). Jin et al. (2014) analysed the impact of various carbon policies on selection of different transportation mode and designing the supply chain. They argued that different policies have different impacts on the cost and effectiveness of emission reduction and how to choose policy parameters is critical to the effective implementation of particular carbon policy. Among the popular policies, carbon tax schemes are often criticized for being overly explicit the costs associated with controlling carbon emissions that lead to higher operating costs and, as a result, make products more expensive (Metcalf 2009). In contrast, cap-and-trade provides a flexible market mechanism for the emissions control. Zakeri et al. (2015) presented a tactical planning model to manage supply chains under the carbon tax and emission trading policy schemes. They concluded that a carbon trading mechanism appears to lead to better environmental and economic performances of the supply chain. However, as emissions trading costs are dependent on many uncertain market conditions, a carbon tax may be more worthwhile from an uncertainty perspective. Hua et al. (2011) highlighted that the carbon emission trading is one of the most effective market-driven mechanisms to curb carbon emissions, and investigated how firms manage carbon footprints in inventory management.
under the carbon emission trading mechanism. They examined the effects of carbon trade, carbon price, and carbon cap on operational decisions and associated economic and environmental performance.

For a cap-and-trade policy, an overall cap is set on the overall amount of carbon emissions. Companies can sell the unused portion of their cap to others who are in need or keep the spare allowances to cover its future needs. Oppositely, they have to purchase additional emission allowance from the market if they exceed their allowance. Otherwise, heavy fines are imposed. Cap-and-trade policy instruments have been effectively considered in the number of environmental problems with varying success (Colby, 2000). For instance, the European Union Emissions Trading Scheme has been demonstrated as an important policy to address climate change (Zhu and Wei, 2013). The advent of tradable emission signals a paradigm shift to a new era of market based mechanism for emission transfer from one organisation to another. The shift from carbon tax to emission trading has increased the uncertainties over the value of carbon credits. So far, there has been a growing number of studies that focus on optimize the economic and environmental performances of different operations areas under the cap-and-trade policy including manufacturing (Zhang and Xu 2013; Chen et al. 2016; Luo et al. 2016), transportation (Chen and Wang 2016), and supply chain (Jaber et al. 2013; Du et al. 2015, 2016; Xu et al. 2016; Jiang and Chen 2016). However, as far as authors’ understanding, very little research has been done regarding warehouse management under the cap-and-trade policy. This paper is going to address this gap by considering the stochastic market demand for modeling the low-carbon warehousing management under this policy. The discussion presented in the literature review lead to a number of research questions which this paper aims to address. These are:

- How does the cap-and-trade policy affect the warehouse management decisions and performances?
- What effect does the green technology investment have on the warehouse management decisions and performances under the cap-and-trade policy?
- How to develop an appropriate carbon emissions reduction policy to achieve green/low carbon warehouse management?
3 Model descriptions and assumption

We consider a retailer who orders products from the supplier and sells to end-users with stochastic demand. Before the selling season, the retailer receives an initial allocation of emission allowance from the government. Warehouse carbon emissions during the retailing period include two sources: one is the initial warehouse carbon emissions during the retailing period, which is fixed, and the other is unit warehouse carbon emissions during the retailing period, which has a linear relationship with product quantity. The initial warehouse carbon emissions are the main source of warehouse carbon emissions. The retailer can invest green warehouse technology to reduce the initial warehouse carbon emissions during the retail period. Under the cap-and-trade policy, the retailer can also buy additional allowance from or sell them to the outside market. Then the retailer places an order. At the beginning of selling season, the retailer obtains and warehouses the products and then sells to the customers during the retailing period. Because of a long production time and short sale period, we assume that the retailer has no chance to make an order during the retailing period. After the selling season, the excessive product can be salvaged, and the retailer should not discharge more emissions than the allowance they hold. So, the retailer should decide the order quantity, green technology investment, and carbon emission trading quantity before the customers’ demands are arrived to meet the emissions requirement and achieve his maximum expected profit.

Throughout this paper, we use the parameters and variables using the notations presented in Table 1.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Descriptions</th>
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<tbody>
<tr>
<td>$D$</td>
<td>The stochastic market demand.</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>Probability density function for the stochastic market demand.</td>
</tr>
<tr>
<td>$F(x)$</td>
<td>Distribution function for the stochastic market demand, which is differentiable, invertible and strictly increasing.</td>
</tr>
<tr>
<td>$p$</td>
<td>Unit retail price of product.</td>
</tr>
<tr>
<td>$w$</td>
<td>Unit wholesale price of product.</td>
</tr>
<tr>
<td>$v$</td>
<td>Units salvage value of product.</td>
</tr>
</tbody>
</table>
Retailer’s unit penalty cost for demand that cannot be filled

$q$  
Retailer’s order quantity.

$K$  
Initial carbon emission allowance from government.

$k_0$  
Initial warehouse carbon emissions during retail period.

$k$  
Warehouse carbon emissions during retail period after green technology investment, $k < k_0$.

$e$  
Unit warehouse carbon emission during retail period.

$t$  
Retailer’s green technology investment cost coefficient.

$T$  
Retailer’s green technology investments, $T = t(k_0 - k)$ (Yalabik and Fairchild, 2011).

$b$  
Unit price of carbon emission buying from the outside market.

$s$  
Unit price of carbon emission selling to the outside market.

$E_o$  
Carbon emission trading quantities with the outside market.

$x^+$  
$max(x, 0)$.

$x^-$  
$min(x, 0)$.

In addition, we assume that the parameters satisfy the following conditions:

(1) $b > s$. This condition means that unit price of carbon emission bought from the outside market is higher than that selling to the outside market due to differences in the transaction cost. (Gong and Zhou, 2013; Toptal and Çetinkaya, 2015)

(2) $p > w > s > 0$. This condition states there is a positive profit margin for retailer to sell a product to the consumer market. Moreover, the salvage value is less than the ordering and transportation costs, which implies that there is a loss if a product is not sold.

4 The base model

First, we consider the basic model without carbon emissions policy. The retailer’s profit, denoted by $\pi_n(q)$, is

$$\pi_n(q) = p \cdot \min(q, D) + v(q - D)^+ - g(D - q)^+ - wq \quad (1)$$

The first term is retail revenue. The second term is the salvage value. The last two terms represent the shortage cost and purchase cost respectively.

The retailer’s expected profit in the base model, denoted by $E[\pi_n(q)]$, is

$$E[\pi_n(q)] = (p + g - w)q - (p + g - v) \int_0^q F(x) \, dx - g \int_0^{+\infty} x f(x) \, dx \quad (2)$$

Without carbon emissions policy, the problem faced by the retailer is a classic newsvendor model and the retailer’s optimal order quantity, denoted by $q_n$, is $q_n =
\[ F^{-1}(\frac{p+g-w}{p+g-v}). \] The retailer’s marginal profit, denoted by \( \theta(q) \), is \( \theta(q) = \frac{dE[\pi_n(q)]}{dq}. \)

With the cap-and-trade policy, the decision problem faced by the retailer is to decide the optimal ordering and carbon emission trading decisions to meet the initial carbon emissions cap set by the government and maximize their expected profit. Then, the retailer’s decision making model is

\[
\max_q E[\pi(q)] = \max_q \{E[\pi_n(q)] - bE_0^+ + sE_0^-\} \\
\text{s.t} \ eq + k_0 = K + E_0 (3)
\]

This constraint means that the retailer’s total carbon emission is equal to the initial carbon emissions cap set by the government and the trading quantity of carbon emission with the outside market. When the retailer needs additional carbon emissions quota, he will buy the shortage quota from the outside market at unit price \( b \), that is \( E_0 > 0 \). When the retailer does not use up its entire carbon emissions cap, he will sell his remaining quota to the outside market at unit price \( s \), that is \( E_0 < 0 \). When the retailer uses up its entire carbon emissions cap and does not need additional carbon emissions quota, he will not trade with the outside market, that is \( E_0 = 0 \).

As to the retailer’s optimal order quantity \( q^e \) and optimal carbon emissions trading quantities \( E_0^e \) in the model with cap-and-trade, the following proposition is obtained.

**Proposition 1**

(1) When \( K < eq_b^e + k_0 \), then \( q^e = q_b^e \) and \( E_0^e = eq_b^e + k_0 - K \).

(2) When \( eq_b^e + k_0 \leq K \leq eq_s^e + k_0 \), then \( q^e = q_s^e \) and \( E_0^e = 0 \).

(3) When \( K > eq_s^e + k_0 \), then \( q^e = q_s^e \) and \( E_0^e = eq_s^e + k_0 - K \).

Where 

\[
q_b^e = F^{-1}(\frac{p+g-w-be}{p+g-v}), \quad q_0^e = \frac{K-k_0}{e} \quad \text{and} \quad q_s^e = F^{-1}(\frac{p+g-w-se}{p+g-v}).
\]

The proof of Proposition 1 and other propositions are provided in the Appendix. This proposition indicates that with cap-and-trade, the retailer’s optimal order quantity \( (q^e) \) and optimal carbon emissions trading quantities \( (E_0^e) \) are existence and unique. This proposition also indicates that there are two important limits: \( q_b^e \) and \( q_s^e \). \( q_b^e \) is the lower limit for the retailer’s optimal order quantity \( (q^e) \), and \( q_s^e \) is the upper limit for the retailer’s optimal order quantity \( (q^e) \). That is, the retailer’s optimal order quantity \( (q^e) \) always falls into the interval \([q_b^e, q_s^e]\). Here, we define the retailer’s optimal inventory policy as a **two-side limit inventory**
policy.

With cap-and-trade, the retailer’s profit can be described by Fig. 1. When the initial carbon emissions cap set by the government is low, that is, \( K < eq_b^e + k_0 \), then the retailer’s optimal order quantity is the lower limit \( (q_b^e) \) and he will buy the shortage carbon emissions quota \( eq_b^e + k_0 - K \) from the outside market. When the initial carbon emissions cap set by the government is medium, that is, \( eq_b^e + k_0 \leq K \leq eq_s^e + k_0 \), then the retailer’s optimal order quantity is \( q_0^e \), which is equal to the case without the cap-and-trade policy, and he will not trade with the outside market. When the initial carbon emissions cap set by the government is high, that is, \( K > eq_s^e + k_0 \), then the retailer’s optimal order quantity is the upper limit \( (q_s^e) \) and he will sell the surplus carbon emissions quota \( K - eq_s^e - k_0 \) to the outside market.

From proposition 1, the following corollary is obtained.

**Corollary 1** \( q_b^e, q_0^e \) and \( q_s^e \) all are decreasing functions of \( e \).

This corollary means that under the cap-and-trade policy, if the unit warehouse carbon emission during the retail period \( (e) \) is high, the retailer will order less products for the carbon emission cap and gain less profits. In contrast, if the unit warehouse carbon emission during
retail period \((e)\) is low, the retailer will order more products and gain more profits.

Now we consider the case where the stochastic demand is normally distributed with mean \(\mu\) and demand variance \(\sigma\). Denote the cumulative distribution function (CDF) and probability density function (PDF) of the standard Normal distribution as \(G\) and \(g\), respectively. Regarding the lower limit for the retailer’s optimal order quantity \((q_b^e)\), let \(z^b = G^{-1}\left(\frac{p+g-w-b\delta}{p+g-v}\right)\), then \(q_b^e = \mu + \sigma z^b\), where \(z^b\) is the optimal quantile. Similarly, regarding the upper limit for the retailer’s optimal order quantity \((q_s^e)\), let \(z^s = G^{-1}\left(\frac{p+g-w-s\delta}{p+g-v}\right)\), then \(q_s^e = \mu + \sigma z^s\), where \(z^s\) is the optimal quantile. For the effect of demand variance \((\sigma)\), the following corollary is obtained.

**Corollary 2**

1. If \(b < \frac{p+g-v-2w}{2e}\), then \(\mu < q_b^e < q_s^e\), both \(q_b^e\) and \(q_s^e\) are increasing functions of \(\sigma\).
2. If \(b > \frac{p+g-v-2w}{2e} > s\), then \(q_b^e < \mu < q_s^e\), \(q_b^e\) is a deceasing function of \(\sigma\) and \(q_s^e\) is an increasing function of \(\sigma\).
3. If \(s > \frac{p+g-v-2w}{2e}\), then \(q_b^e < q_s^e < \mu\), both \(q_b^e\) and \(q_s^e\) are decreasing functions of \(\sigma\).

From this corollary, we know that under the cap-and-trade policy, when the unit price of carbon emission trading with the outside market is low, both the lower limit and the upper limit for the retailer’s optimal order quantity are higher than the mean of the stochastic demand. In this case, if the demand variance \((\sigma)\) of stochastic demand is high, then both the lower limit and the upper limit for the retailer’s optimal order quantity are high, and vice versa. When the unit price of carbon emission trading with the outside market is medium, the lower limit for the retailer’s optimal order quantity is lower than the mean of the stochastic demand and the upper limit for the retailer’s optimal order quantity is higher than the mean of the stochastic demand. In this case, if the demand variance \((\sigma)\) of stochastic demand is high, then the lower limit for the retailer’s optimal order quantity is low and the upper limit for the retailer’s optimal order quantity is high, and vice versa. When the unit price of carbon emission trading with the outside market is high, both the lower limit and the upper limit for the retailer’s optimal order quantity are lower than the mean of the stochastic demand. In this case, if the demand variance \((\sigma)\) of stochastic demand is high, then both the lower limit and
the upper limit for the retailer’s optimal order quantity are low, and vice versa.

5 The model with green technology investment

In this section, we discuss the model with green technology investment. Without the carbon emission policy, the decision problem faced by the retailer is to decide the optimal ordering and green technology investment decisions. The retailer’s profit with green technology investment, denoted by \( \pi_n(q, k) \), is

\[
\pi_n(q, k) = \text{pmin}(q, D) + v(q - D)^+ - g(D - q)^+ - wq - T \tag{4}
\]

The first term is retail revenue, the second term is the salvage value, and the last three terms represent the shortage cost, purchase cost and green technology investment respectively.

The retailer’s expected profit with green technology investment, denoted by \( E[\pi_n(q, k)] \), is

\[
E[\pi_n(q, k)] = (p + g - w)q - (p + g - v)\int_0^q F(x) \, dx - g\int_0^{+\infty} xf(x) \, dx - t(k_0 - k)^2 \tag{5}
\]

As to the retailer’s optimal order quantity \( (q_n^*) \) and optimal warehouse carbon emissions during the retailing period after green technology investment \( (k_n^*) \) in the model with green technology investment, it is clear that \( q_n^* = q_n = F^{-1}\left(\frac{p+g-w}{p+g-v}\right) \) and \( k_n^* = k_0 \), which means that the retailer will not invest on the green technology without the cap-and-trade policy.

Considering the cap-and-trade policy and green technology investment, the decision problem faced by the retailer is to decide the optimal ordering, green technology investment and carbon emission trading decisions to meet the initial carbon emissions cap set by the government and maximize his expected profit. Then, the retailer’s decision making model is

\[
\max_{q} E[\pi(q, k)] = \max_{q} \left\{ E[\pi_n(q, e)] - bE_0^+ + sE_0^- \right\}
\]

s.t. \( eq + k = K + E_0 \) \tag{6}

This constraint means that the retailer’s total carbon emission after green technology investment is equal to the initial carbon emissions cap set by the government and the trading quantity of carbon emission with the outside market. \( E_0 > 0 \) means that the retailer buys carbon emissions quota from the outside market at unit price \( b \). \( E_0 < 0 \) means that the
retailer sells carbon emissions quota to the outside market at unit price $s$. $E_0 = 0$ means that the retailer does not trade with the outside market.

With regard to the retailer’s optimal decisions and carbon emissions in the model considering cap-and-trade and green technology investment, the following proposition is obtained.

**Proposition 2**

1. When $K < eq^t_b + k^t_b$, then $q^t = q^t_b$, $k^t = k^t_b$ and $E^t_0 = eq^t_b + k^t_b - K$.
2. When $eq^t_b + k^t_b \leq K \leq eq^t_s + k^t_s$, then $q^t = q^t_0$, $k^t = k^t_0$ and $E^t_0 = 0$.
3. When $K > eq^t_s + k^t_s$, then $q^t = q^t_s$, $k^t = k^t_s$ and $E^t_0 = eq^t_s + k^t_s - K$.

Where $q^t_b = F^{-1}\left(\frac{p+g-w-v}{p+g-v}\right)$, $q^t_0 = \frac{K+k^t_b}{e}$, $q^t_s = F^{-1}\left(\frac{p+g-w-v}{p+g-v}\right)$, $k^t_b = k_0 - \frac{b}{2t}$, $k^t_0 = k_0 - \frac{s}{2t}$.

This proposition indicates that considering cap-and-trade and green technology investment, the retailer’s optimal order quantity ($q^t$), optimal warehouse carbon emissions during the retailing period after green technology investment ($k^t$) and optimal carbon emissions trading quantities ($E^t_0$) are in existence and unique. This proposition also indicates that there are two important limits: $q^t_b$ and $q^t_s$. $q^t_b$ is the lower limit for the retailer’s optimal order quantity ($q^t$), and $q^t_s$ is the upper limit for the retailer’s optimal order quantity ($q^t$). That is, the retailer’s optimal order quantity ($q^t$) always falls into the interval $[q^t_b, q^t_s]$.

Again, we defined the retailer’s optimal inventory policy as a **two-side limit inventory policy**.

Considering cap-and-trade and green technology investment, the retailer’s profit can be described by Fig. 2. When the initial carbon emissions cap set by the government is low, that is, $K < eq^t_b + k^t_b$, then the retailer’s optimal order quantity is the lower limit ($q^t_b$), he will invest $\frac{b^2}{4t}$ to green technology and buy the shortage carbon emissions quota $eq^t_b + k^t_b - K$ from the outside market. When the initial carbon emissions cap set by the government is medium, that is, $eq^t_b + k^t_b \leq K \leq eq^t_s + k^t_s$, then the retailer’s optimal order quantity is $q^t_s$, which is equal to the case without the cap-and-trade policy, he will not trade with the outside market. When the initial carbon emissions cap set by the government is high, that is, $K > eq^t_s + k^t_s$, then the retailer’s optimal order quantity is the upper limit ($q^t_s$), he will invest $\frac{s^2}{4t}$ in
green technology and sell the surplus carbon emissions quota $K - eq^t_s - k^t_s$ to the outside market.

From proposition 2, the following corollary can be obtained.

**Corollary 3** $k^t_b$, $k^t_0$ and $k^t_s$ all are increasing functions of $t$.

This corollary indicates that under the cap-and-trade policy, if retailer’s green technology investment cost coefficient ($t$) is high, then the retailer’s optimal warehouse carbon emissions during retail period after green technology investment is high and the retailer will invest less on green technology. In contrast, if retailer’s green technology investment cost coefficient ($t$) is low, then the retailer’s optimal warehouse carbon emissions during the retail period after green technology investment is low and the retailer will invest more on green technology.

**6 Discussion**

In this section, the effects of green technology investment ($T$), initial carbon emission allowance from government ($K$), and the unit price of carbon emission trading with the outside market ($b$ and $s$) on the retailer’s decisions and profit are discussed.

**6.1 The effect of green technology investment ($T$)**
As to the effect of green technology investment \((T)\) on the retailer’s decisions and profit, the following proposition is obtained.

**Proposition 3**

1. \(q^e = q^t\).  
2. \(k^t < k_0\).  
3. When \(K < eq^e_b + k_0\), \(E^t_0 < E^e_0\); when \(K \geq eq^e_b + k_0\), \(|E^t_0| > |E^e_0|\).  
4. \(E[\pi(q^t, k^t)] > E[\pi(q^e)]\).

This proposition means that green technology investment has no effect on the retailer’s optimal ordering policy. However, it provides a useful tool for the retailer that faces the challenge of carbon emission quota shortage or surplus to increase the expected profit. When the initial carbon emission allowance from government is low, green technology investment will reduce the warehouse carbon emissions during the retailing period and the carbon emission quota shortage. So the retailer will buy less carbon emission quota from the outside market. In contrast, when the initial carbon emission allowance from government is high, green technology investment will reduce the warehouse carbon emissions during the retailing period and increase the carbon emission quota surplus. So the retailer will sell more carbon emission quota to the outside market.

### 6.2 The effect of initial carbon emission allowance \((K)\) from government

With regard to the effect of initial carbon emission allowance \((K)\) from government on the retailer’s decisions and profit, the following proposition is obtained.

**Proposition 4**

1. When \(K < eq^e_b + k_0\) or \(K > eq^e_s + k_0\), \(q^e\) has no relationship with \(K\). When \(eq^e_b + k_0 \leq K \leq eq^e_s + k_0\), \(q^e\) is an increasing function of \(K\). When \(K < eq^t_b + k^t_b\) or \(K > eq^t_s + k^t_s\), both \(q^t\) and \(k^t\) have no relationship with \(K\). When \(eq^t_b + k^t_b \leq K \leq eq^t_s + k^t_s\), both \(q^t\) and \(k^t\) are increasing functions of \(K\).
2. When \(E^0_0 \neq 0\), \(E^t_0\) is a decreasing function of \(K\); when \(E^t_0 \neq 0\), \(E^t_0\) is a decreasing function of \(K\).
3. Both \(E[\pi(q^e)]\) and \(E[\pi(q^t, k^t)]\) are increasing functions of \(K\).

This proposition means that when the retailer trades with outside market, the initial carbon emission allowance from government \((K)\) has no effect on the retailer’s optimal ordering quantity in the case without/with green technology investment. When the retailer does not trade with the outside market and if the initial carbon emission allowance from government \((K)\) is higher, then the retailer will order more products. If the initial carbon emission
allowance from government \((K)\) is high, the retailer will buy less emissions quota from the outside market \((E^e_0 > 0 \text{ or } E^e_0 > 0)\) when the emissions quota has a shortage. Alternatively, the retailer will sell more carbon emissions quota to the outside market \((E^e_0 < 0 \text{ or } E^e_0 < 0)\) when the carbon emissions quota has a surplus. Then the retailer will gain more profit.

### 6.3 The effect of unit price of carbon emission trading with the outside market \((b\text{ and }s)\)

In relation to the effect of unit price of carbon emission trading with the outside market \((b\text{ and }s)\) on the retailer’s decisions and profit, the following proposition is obtained.

**Proposition 5**

1. Both \(q^e_b\) and \(q^t_b\) are decreasing functions of \(b\), both \(q^e_s\) and \(q^t_s\) are decreasing functions of \(s\).
2. \(k^t_b\) is a decreasing function of \(b\), and \(k^t_s\) is a decreasing function of \(s\).
3. When \(K < eq^e_b + k_0\), then \(E[\pi(q^e)]\) is a decreasing function of \(b\); when \(eq^e_b + k_0 \leq K \leq eq^e_s + k_0\), then \(E[\pi(q^e)]\) has no relationship with \(b\) and \(s\); when \(K > eq^e_s + k_0\), then \(E[\pi(q^e)]\) is an increasing function of \(s\). When \(K < eq^t_b + k^t_b\), then \(E[\pi(q^t_b,k^t)]\) is a decreasing function of \(b\); when \(eq^t_b + k^t_b \leq K \leq eq^t_s + k^t_s\), then \(E[\pi(q^t_b,k^t)]\) has no relationship with \(b\) and \(s\); when \(K > eq^t_s + k^t_s\), then \(E[\pi(q^t_b,k^t)]\) is an increasing function of \(s\).

This proposition means that with high carbon emissions trading price \((b\text{ or } s)\), both the lower limit \((q^e_b\text{ or } q^t_b)\) and the upper limit \((q^e_s\text{ or } q^t_s)\) of the retailer without/with green technology investment will be lower, and the retailer will invest more green technology to reduce the warehouse carbon emissions during the retailing period \((k^t_b\text{ and } k^t_s)\).

When the initial carbon emission allowance from government \((K)\) is low and if its carbon emissions quota is in shortage, the retailer will gain more profit with lower unit price of buying carbon emissions quota from the outside market \((b)\) in the case without/with green technology investment. In contrast, when the initial carbon emission allowance from government \((K)\) is high and if its carbon emissions quota is in surplus, the retailer will gain more profit with higher unit price of selling carbon emission quota to the outside market \((s)\) in the case without/with green technology investment. Therefore, from policy makers’ view, the unit price of carbon emission should be reduced when buying from the outside market \((b)\) and
it should be increased when selling to the outside market (s) should be increased, that is, the difference between the above two unit prices should be minimized.

7 Case study

A case study is presented that focuses on the warehouse management of one distribution depot of a major UK grocery supermarket chain. The UK is one of the participants of European Union Emissions Trading System (EU ETS), the largest multi-country and multi-sector cap-and-trade system in the world. The grocery retail chain is committed to reduce carbon emissions and mitigate the business risks of climate change. The depot studied in this case operates 24 hours a day and serves 96 stores in the region. All products are stored at a multi-temperature controlled warehouse facility. 950,000 cases of chilled products are handled per week. We use the case study to illustrate how our analytical modelling results can be applied to improve the low carbon warehouse management in the real world. In addition, further sensitivity analysis is provided to examine the impact of various factors on the performance of low carbon warehouse management.

The carbon emissions of warehouse operation in the depot are mainly categorized into two parts: the fixed initial warehouse carbon emissions and the operating carbon emissions. The fixed initial emissions are dependent on the warehouse capacity. For instance, all the chilled products are kept in the room with the temperature condition of 2–8 °C. In order to make the chilled warehouse facility functioning, it requires an assumption of certain amount energy based on the floor space covered in the warehouse. In this case, the fixed initial emission can be estimated by the amount of electricity required. The other part of emissions depends on the amount of goods stored in the warehouse. More goods stored in the warehouse require more movement of goods and staff that leads to higher energy consumption and more emissions.

7.1 Effects of demand fluctuation

First, a numerical example is presented to illustrate the effects of risk of demand fluctuation on the retailer’s maximum expected profit under the cap-and-trade policy, and examine the impact green technology investment on the retailer’s maximum profit. Based on the case
scenario, the parameter values displayed in table 2 are used in the analysis.

**Table 2: Parameter values**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D), The stochastic market demand</td>
<td>(\mu = 100,000)</td>
</tr>
<tr>
<td>(p), unit retail price of product</td>
<td>£8</td>
</tr>
<tr>
<td>(w), unit wholesale price of product</td>
<td>£5</td>
</tr>
<tr>
<td>(v), units salvage value of product</td>
<td>£1</td>
</tr>
<tr>
<td>(g), retailer’s unit penalty cost for demand that cannot be filled</td>
<td>£2</td>
</tr>
<tr>
<td>(K), initial carbon emission allowance from government</td>
<td>50</td>
</tr>
<tr>
<td>(k_a), initial warehouse carbon emissions during retail period</td>
<td>30</td>
</tr>
<tr>
<td>(e), unit warehouse carbon emission during retail period</td>
<td>0.2</td>
</tr>
<tr>
<td>(t), retailer’s green technology investment cost coefficient</td>
<td>0.014</td>
</tr>
</tbody>
</table>

According the historical sale data, the stochastic demand for the selected product is normally distributed. Here we use the different values of standard deviation of demand \((\sigma)\) to describe demand fluctuation. In the numerical analysis, we also specify the unit price of carbon emission buying from the outside market \((b)\) as 1 and the unit price of carbon emission selling to the outside market \((s)\) as 0.9. With respect to the effects of demand fluctuation on the retailer’s maximum expected profit, corresponding results are shown in Fig. 3.

![Fig. 3. Effects of demand risk](image)

**Note:** \(E[\pi(q^e)]\) and \(E[\pi(q^t, k^t)]\) represents retailer’s maximum expected profit, with and without green technology investment, respectively.
From Fig. 3, we observe that demand fluctuation does affect the retailer’s maximum expected profit without or with green technology investment. As the risk of demand fluctuation increases, the retailer’s maximum expected profit decreases in both cases. In addition, the retailer’s maximum expected profit with green technology investment ($E[\pi(q^t, k^t)]$) is higher than that without green technology investment ($E[\pi(q^e)]$). That is, with green technology investment, the retailer can reduce the carbon emissions and gain more profit from providing more warehousing service or trading with the outside market.

7.2 Effects of unit price of carbon emissions

Now, we examine how the design of cap-and-trade policy influences firms’ decision on green technology investment. Here, a numerical analysis is presented to illustrate the effects of unit trading prices of carbon emissions with the outside market ($b$ and $s$) on the retailer’s maximum expected profit. The analysis is conducted through a comparison between the maximum expected profits without and with green technology investment. We assume that the standard deviation of demand ($\sigma$) is 70. With respect to the effect of the unit price of carbon emission buying from the outside market ($b$) on the retailer’s maximum expected profit, we specify that $s$ is £0.8, and the corresponding results are shown in Fig. 4 (a). With respect to the effect of the unit price of carbon emission selling to the outside market ($s$) on the retailer’s maximum expected profit, we specify that $b$ is £1, and the corresponding results are shown in Fig. 4 (b).

![Fig. 4 Effects of unit price of carbon emission](image-url)
When the carbon emissions exceed the initial allowance set by the government, then the retailer will buy additional emission quota from the outside market. From Fig. 4 we observe that the unit price of carbon emission buying from the outside market \((b)\) has an effect on the retailer’s maximum expected profit. That is, more expensive trading price will decrease the profit margin for firms buying emission quota from the outside market as illustrated in Fig. 4(a). Since, the retailer has no spare emission quota to sell to the market, the unit price of carbon emission selling to the outside market \((b)\) has no effect on the profit margin. In this case, policy makers should look at the emission cap and the unit price of carbon emission buying from the outside market \((b)\) in order to encourage firms to invest on green technologies and reduce the carbon emissions. When there is a surplus, the retailer will sell the spare emission quota to the outside market. In this case, the unit price of carbon emission selling to the outside market \((s)\) has an effect on retailer’s maximum expected profit. That is, more expensive trading price will increase the profit margin for firms selling emission quota to the outside market as illustrated in Fig. 4(b). Since the retailer does not require buying additional quota, the unit price of carbon emission buying from the outside market \((b)\) has no effect on the profit margin. From the view of government, the unit price of carbon emissions trading with the outside market can be adjusted to induce firms to invest more on green technologies and reduce the carbon emissions.

8 Conclusion and future research

This research investigates the impact of the cap-and-trade emission policy on the warehouse management decisions and explores the role of green technology investment in achieving the sustainability objectives of warehouse operation. Under the cap-and-trade emissions policy, we analyse the optimal decisions on warehouse management and technology investment considering the trade-offs between the economic and environmental objectives.

This research makes several important contributions. Theoretically, the research complements the existing literature on sustainable warehouse management by examining the impacts of the cap-and-trade policy and green technology investment on firms’ warehouse decisions and performances. Practically, we derived the two-side limit inventory policy, which
helps firms make optimal operational and investment decisions on warehouse management to improve both the economic and environmental performances. Finally, from the policy makers’ perspective, our research findings provide interesting insights on how the initial carbon emissions cap and the trading prices of carbon emissions affect the warehouse decisions and performances. The findings also support policy makers to develop effective carbon emissions control policies that can enable the green warehouse management.

There are several fruitful directions for future investigation. First, in our model, the green technology investment is only assumed to reduce the initial warehouse carbon emissions. Although this assumption is reasonable for warehouse operation, one important research extension is to incorporate the technology investment on reducing the unit warehouse carbon emission \( e \) in the modelling. In addition, Fahimnia et al. (2015) argued that instead of choosing between carbon tax and cap-and-trade system, a hybrid regulatory scheme can be investigated, and suggested that developing and comparing deterministic versus stochastic modeling efforts can examine the differences in these types of policies. Another future extension of this could be to analyse the green/low carbon warehouse management under a combination of different carbon emissions control policies, and discuss their effects on warehouse decisions and performances. Finally, this research can be extended to other supply chain processes such as production, logistics, and retailing, as achieving the low carbon objective requires coordinated action of the whole supply chain.

Acknowledgments
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Appendix A

**Proof of Proposition 1**

From (3), we get \( E[\pi(q)] = E[\pi_n(q)] - b(eq + k_0 - K)^+ + s(eq + k_0 - K)^- \).

When \( K < eq + k_0 \), we get \( E[\pi(q)] = E[\pi_n(q)] - b(eq + k_0 - K) \cdot \frac{dE[\pi(q)]}{dq} = p + g - w - \)
\[(p + g - v)F(q) - be\] \[\text{and } \frac{d^2 E[\pi(q)]}{dq^2} = -(p + g - v)f(q) < 0, \text{ that is, } E[\pi(q)] \text{ is a concave function of } q. \text{ Let } \frac{dE[\pi(q)]}{dq} = 0, \text{ we get } q_b^e = F^{-1}\left(\frac{p + g - w - be}{p + g - v}\right).

When \(K > eq + k_0\), we get \(E[\pi_i(q)] = E[\pi_n(q)] + s(K - eq - k_0)\). \(\frac{dE[\pi(q)]}{dq} = p + g - w - (p + g - v)F(q) - se\) \[\text{and } \frac{d^2 E[\pi(q)]}{dq^2} = -(p + g - v)f(q) < 0, \text{ that is, } E[\pi(q)] \text{ is a concave function of } q. \text{ Let } \frac{dE[\pi(q)]}{dq} = 0, \text{ we get } q_s^e = F^{-1}\left(\frac{p + g - w - se}{p + g - v}\right).

\[\text{When } K = eq + k_0, \text{ then } q_0^e = \frac{K - k_0}{e}.
\]

Recalling \(b > s\), then \(q_b^e > q_b^e\). Hence, (1) when \(K < eq_b^e + k_0\), then \(q^e = q_b^e\) and \(E_b^e = eq_b^e + k_0 - K\); (2) when \(eq_b^e + k_0 \leq K \leq eq_s^e + k_0\), then \(q^e = q_0^e\) and \(E_0^e = 0\); (3) when \(K > eq_s^e + k_0\), then \(q^e = q_s^e\) and \(E_0^e = eq_s^e + k_0 - K\), where \(q_b^e = F^{-1}\left(\frac{p + g - w - be}{p + g - v}\right)\), \(q_0^e = \frac{K - k_0}{e}\) and \(q_s^e = F^{-1}\left(\frac{p + g - w - se}{p + g - v}\right)\). This completes the proof.

**Proof of Corollary 1**

From proposition 1, we get that \(F(q_b^e) = \frac{p + g - w - be}{p + g - v}\) and \(F(q_s^e) = \frac{p + g - w - se}{p + g - v}\). Then \(\frac{dq_b^e}{de} = -\frac{b}{f(q_b^e)(p + g - v)} < 0, \frac{dq_0^e}{de} = -\frac{K - k_0}{e^2} < 0\) and \(\frac{dq_s^e}{de} = -\frac{s}{f(q_s^e)(p + g - v)} < 0\). That is, \(q_b^e, q_0^e\) and \(q_s^e\) all are decreasing functions of \(e\). This completes the proof.

**Proof of Corollary 2**

(1) When \(b < \frac{p + g - v - 2w}{2e}\), then \(\frac{p + g - w - be}{p + g - v} > \frac{1}{2}\) and \(\frac{p + g - w - se}{p + g - v} > \frac{1}{2}\), that is, \(0 < z_b < z_s\). So, \(\mu < q_b^e < q_s^e\). \(\frac{dq_b^e}{ds} = z_b > 0\) and \(\frac{dq_s^e}{ds} = z_s > 0\), then both \(q_b^e\) and \(q_s^e\) are increasing functions of \(\sigma\).

(2) When \(b > \frac{p + g - v - 2w}{2e} > s\), then \(\frac{p + g - w - be}{p + g - v} < \frac{1}{2}\) and \(\frac{p + g - w - se}{p + g - v} > \frac{1}{2}\), that is, \(z_b < 0 < z_s\). So, \(q_b^e < \mu < q_s^e\). \(\frac{dq_b^e}{ds} = z_b < 0\) and \(\frac{dq_s^e}{ds} = z_s > 0\), then \(q_b^e\) is a decaying function of \(\sigma\) and \(q_s^e\) is an increasing function of \(\sigma\).

(3) When \(s > \frac{p + g - v - 2w}{2e}\), then \(\frac{p + g - w - be}{p + g - v} < \frac{1}{2}\) and \(\frac{p + g - w - se}{p + g - v} < \frac{1}{2}\), that is, \(z_b < z_s < 0\). So, \(q_b^e < q_s^e < \mu\). \(\frac{dq_b^e}{ds} = z_b < 0\) and \(\frac{dq_s^e}{ds} = z_s < 0\), then both \(q_b^e\) and \(q_s^e\) are decreasing functions of \(\sigma\).
This completes the proof.

Proof of Proposition 2
From (6), we get $E[\pi(q, k)] = E[\pi_n(q, k)] - b(eq + k - K)^+ + s(eq + k - K)^-.

When $K < eq + k$, then $E[\pi(q, k)] = E[\pi_n(q, k)] - b(eq + k - K) \cdot \frac{\partial E[\pi(q, k)]}{\partial q} = p + g - w - (p + g - v)F(q) - be$ and $\frac{\partial^2 E[\pi(q, k)]}{\partial q^2} = -(p + g - v)f(q) < 0 \cdot \frac{\partial E[\pi(q, k)]}{\partial k} = 2t(k_0 - k) - b$ and

$$\frac{\partial^2 E[\pi(q, k)]}{\partial k^2} = -2t \cdot \frac{\partial^2 E[\pi(q, k)]}{\partial q \partial k} = 0.$$ 

So we get that $\frac{\partial^2 E[\pi(q, k)]}{\partial q^2} = \frac{\partial^2 E[\pi(q, k)]}{\partial \partial e} = 2t(p + g - v)f(q) > 0$, that is, $E[\pi(q, k)]$ is joint concave in $q$ and $k$. Let $\frac{\partial E[\pi(q, k)]}{\partial q} = \frac{\partial E[\pi(q, e)]}{\partial k} = 0$, we get $p + g - w - (p + g - v)F(q) - be = 0$ and $2t(k_0 - k) - b = 0$. Then we get $q_b^t = F^{-1}\left(\frac{p + g - w - be}{p + g - v}\right)$ and $k_b^t = k_0 - \frac{b}{2t}$.

When $K > eq + k$, then $E[\pi(q, k)] = E[\pi_n(q, k)] + s(K - eq - k) \cdot \frac{\partial E[\pi(q, k)]}{\partial q} = p + g - w - (p + g - v)F(q) - se$ and $\frac{\partial^2 E[\pi(q, k)]}{\partial q^2} = -(p + g - v)f(q) < 0 \cdot \frac{\partial E[\pi(q, k)]}{\partial k} = 2t(k_0 - k) - s$ and

$$\frac{\partial^2 E[\pi(q, k)]}{\partial k^2} = -2t \cdot \frac{\partial^2 E[\pi(q, k)]}{\partial q \partial k} = 0.$$ 

So we get that $\frac{\partial^2 E[\pi(q, k)]}{\partial q^2} = \frac{\partial^2 E[\pi(q, k)]}{\partial \partial e} = 2t(p + g - v)f(q) > 0$, that is, $E[\pi(q, k)]$ is joint concave in $q$ and $k$. Let $\frac{\partial E[\pi(q, k)]}{\partial q} = \frac{\partial E[\pi(q, e)]}{\partial k} = 0$, we get $p + g - w - (p + g - v)F(q) - se = 0$ and $2t(k_0 - k) - s = 0$. Then we get $q_s^t = F^{-1}\left(\frac{p + g - w - se}{p + g - v}\right)$ and $k_s^t = k_0 - \frac{s}{2t}$.

Where $K = eq + k$, then $E[\pi(q, k)] = E[\pi_n(q, k)]$, that is, $E[\pi(q, k)] = (p + g - w)\frac{K - k}{e} - (p + g - v)\int_0^\infty rF(x)dx - g\int_0^\infty xf(x)dx - t(k_0 - k)^{-2} \cdot \frac{\partial E[\pi(q, k)]}{\partial k} = -\frac{1}{e^2}(p + g - w) + \frac{1}{e}(p + g - v)F\left(\frac{K - k}{e}\right) + 2t(k_0 - k)$ and $\frac{\partial^2 E[\pi(q, k)]}{\partial k^2} = -\frac{1}{e^2}(p + g - v)f\left(\frac{K - k}{e}\right) - 2t < 0$. That is, $E[\pi(q, k)]$ is concave in $k$. Let $\frac{\partial E[\pi(q, k)]}{\partial k} = 0$, we get $q_0^t = \frac{K - k_0^t}{e}$ and $k_0^t = k_0 - \frac{p + g - v}{2t} \left[F(q_n) - F\left(\frac{K - k_0^t}{e}\right)\right]$.

$eq_b^t + k_b^t = e F^{-1}\left(\frac{p + g - w - be}{p + g - v}\right) + k_0 - \frac{b}{2t}, eq_s^t + k_s^t = e F^{-1}\left(\frac{p + g - w - se}{p + g - v}\right) + k_0 - \frac{s}{2t}$ Recalling $b > s$, then $eq_s^t + k_s^t > eq_b^t + k_b^t$.
Hence, (1) when \( K < e q_b^e + k_b^e \), then \( q^t = q_b^e \), \( k^t = k_b^e \) and \( E_0^t = e q_b^e + k_b^e - K \); (2) when \( e q_b^e + k_b^e \leq K \leq e q_s^e + k_s^e \), then \( q^t = q_s^e \), \( k^t = k_s^e \) and \( E_0^t = 0 \); (3) when \( K > e q_s^e + k_s^e \), then \( q^t = q_s^e \), \( k^t = k_s^e \) and \( E_0^t = e q_s^e + k_s^e - K \), where \( q_0^t = F^{-1}\left(\frac{p+g-w-be}{p+g-v}\right) \), \( q_0^t = \frac{K-k_b^e}{e} \) and \( q_s^e = F^{-1}\left(\frac{p+g-w-se}{p+g-v}\right) \), \( k_b^e = k_0 - \frac{b}{2t} \), \( k_0^e = k_0 - \frac{p+g-v}{2et} \left[F(q_n) - F\left(\frac{K-k_b^e}{e}\right)\right] \) and \( k_s^e = k_0 - \frac{s}{2t} \). This completes the proof.

**Proof of Corollary 3**

From proposition 2, we get that \( \frac{dk_b^e}{dt} = \frac{b}{2t^2} > 0 \), \( \frac{dk_s^e}{dt} = \frac{e(p+g-v)}{2et^2+(p+g-v)F(\frac{K-k_b^e}{e})} > 0 \) and \( \frac{dk_s^e}{dt} = \frac{s}{2t^2} > 0 \), that is, \( k_b^e \), \( k_0^e \) and \( k_s^e \) are all increasing functions of \( t \). This completes the proof.

**Proof of Proposition 3**

(1) From proposition 1 and 2, we get \( q_b^e = q_b^e \) and \( q_s^e = q_s^e \). When \( q < q_b^e \), then \( q^e = q_b^e = q_b^e = q^t \).

When \( q_b^e \leq q \leq q_s^e \), then \( E_0^e = E_0^e = 0 \), that is, \( q_0^e = q_0^e \). When \( q > q_s^e \), then \( q^e = q_s^e = q_s^e = q^t \).

So, \( q^e = q^t \).

(2) From proposition 1 and 2, we get \( q_b^e = q_b^e \) and \( q_s^e = q_s^e \). When \( q < q_b^e \), then \( k^t = k_b^e = k_0 - \frac{b}{2t} < k_0 \). When \( q_b^e \leq q \leq q_s^e \), then \( k^t = k_b^e = k_0 - \frac{p+g-v}{2et} \left[F(q_n) - F\left(\frac{K-k_b^e}{e}\right)\right] \) \( < k_0 \). When \( q > q_s^e \), then \( k^t = k_s^e = k_0 - \frac{s}{2t} < k_0 \). So, \( k^t < k_b^e \).

(3) When \( K < e q_b^e + k_b^e \), then \( E_0^e = e q_b^e + k_b^e - K \) and \( E_0^e = e q_b^e + k_b^e - K \), so \( E_0^e - E_0^e = k_b^e - k_0 < 0 \), that is, \( E_0^e < E_0^e \). When \( e q_b^e + k_b^e \leq K < e q_s^e + k_b^e \), then \( E_0^e = 0 \) and \( E_0^e = e q_b^e + k_b^e - K \), that is, \( E_0^e < E_0^e \). So, when \( K < e q_b^e + k_b^e \), \( E_0^e < E_0^e \).

When \( K > e q_s^e + k_b^e \), then \( |E_0^e| = K - e q_s^e - k_0 \) and \( |E_0^e| = K - e q_s^e - k_s^e \). Recalling \( q_s^e = q_s^e \) and \( k_b^e < k_0 \), then \( |E_0^e| > |E_0^e| \). When \( e q_s^e + k_b^e < K < e q_s^e + k_0 \), then \( E_0^e = 0 \) and \( |E_0^e| = K - e q_s^e - k_s^e \). When \( e q_s^e + k_s^e > K > e q_s^e + k_0 \), then \( |E_0^e| > |E_0^e| \). So, when \( K \geq e q_s^e + k_0 \), \( |E_0^e| > |E_0^e| \).

(4) Since \( E[\pi(q^e, k^0)] = E[\pi(q^e)] \), considering the maximality of \( E[\pi(q, k)] \), we get \( E[\pi(q^t, k^t)] > E[\pi(q^t, k^t)] \), that is, \( E[\pi(q^t, k^t)] > E[\pi(q^e)] \). This completes the proof.

**Proof of Proposition 4**

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(1) For the case without green technology investment, from proposition 1, we get that when \( K < e q^e_b + k_0 \) or \( K > e q^e_s + k_0 \), \( q^e \) has no relationship with \( K \). When \( e q^e_b + k_0 \leq K \leq e q^e_s + k_0 \), \( \frac{dq^e}{dK} = \frac{1}{e} > 0 \), that is, \( q^e \) is an increasing function of \( K \).

For the case with green technology investment, from proposition 2, we get that when \( K < e q^t_b + k^t_b \) or \( K > e q^t_s + k^t_s \), both \( q^t \) and \( k^t \) have no relationship with \( K \). When \( e q^t_b + k^t_b \leq K \leq e q^t_s + k^t_s \), \( \frac{dk^t}{dK} = \frac{p+g-v}{2e^2t} \left( 1 - \frac{dk^t}{dK} \right) \), that is, \( \frac{dk^t}{dK} = \frac{(p+g-v)f(K-k^t)}{2e^2t+(p+g-v)f(e)} > 0 \), which means that \( k^t \) is an increasing function of \( K \). When \( e q^t_b + k^t_b \leq K \leq e q^t_s + k^t_s \), \( \frac{dq^t}{dK} = \frac{1}{e} \left( 1 - \frac{dk^t}{dK} \right) = \frac{2et}{2e^2t+(p+g-v)f(e)} > 0 \), that is, \( q^t \) is an increasing function of \( K \).

(2) From proposition 1 and 2, we get that when \( E^s_0 \neq 0 \), \( \frac{dE^s_b}{dK} = -1 < 0 \), that is, \( E^s_0 \) is an decreasing function of \( K \). Similarly, when \( E^t_0 \neq 0 \), \( \frac{dE^t_b}{dK} = -1 < 0 \), that is, \( E^t_0 \) is an decreasing function of \( K \).

(3) When \( K < e q^e_b + k_0 \), then \( E[\pi(q^e)] = E[\pi_n(q^e)] - b(e q^e_b + k_0 - K) \). So \( \frac{dE[\pi(q^e)]}{dK} = b > 0 \).

When \( e q^e_b + k_0 \leq K \leq e q^e_s + k_0 \), then \( q^e = \frac{K-k_0}{e} < q^e_s < q_n \), and \( E[\pi(q^e)] = E[\pi_n(q^e)] \). So, \( \frac{dE[\pi(q^e)]}{dK} = \frac{dE[\pi_n(q^e)]}{dK} > 0 \). When \( K > e q^e_s + k_0 \), then \( E[\pi_e(q^e)] = E[\pi_n(q^e)] + s(K - e q^e_s - k_0) \). So \( \frac{dE[\pi(q^e)]}{dK} = s > 0 \). Hence, \( E[\pi(q^e)] \) is an increasing function of \( K \). Similarly, we can get that \( E[\pi(q^t,k^t)] \) is an increasing function of \( K \). This completes the proof.

**Proof of Proposition 5**

(1) From proposition 1 and 2, we get that both \( q^e_b \) and \( q^e_s \) are decreasing functions of \( b \), both \( q^e_s \) and \( q^e_s \) are decreasing function of \( s \).

(2) From proposition 1 and 2, we get that \( k^t_b \) is an decreasing function of \( b \), and \( k^t_s \) is an decreasing function of \( s \).

(3) From proposition 1, we get that when \( K < e q^e_b + k_0 \), \( q^e = q^e_b = F^{-1} \left( \frac{p+g-v-b e}{p+g-v} \right) \), then \( \frac{dE[\pi(q^e)]}{db} = -b < 0 \), that is, \( E[\pi(q^e)] \) is an decreasing function of \( b \). When \( e q^e_b + k_0 \leq K \leq e q^e_s + k_0 \), \( q^e = q^e_s \) and \( E^e = 0 \), then \( \frac{dE[\pi(q^e)]}{db} = \frac{dE[\pi(q^e)]}{ds} = 0 \), that is, \( E[\pi(q^e)] \) has no relationship with
and \( s \). When \( K > \text{eq}_b^e + k_0 \), \( q^e = q_s^e = F^{-1}(\frac{p+g-w-se}{p+g-v}) \), then \( \frac{dE[\pi(q^e)]}{db} = s > 0 \), that is, \( E[\pi(q_b^e)] \) is an increasing function of \( s \).

Similarly, from proposition 2, we get that when \( K < \text{eq}_b^t + k_b^t \), then \( E[\pi(q^t,k^t)] \) is an decreasing function of \( b \); when \( \text{eq}_b^t + k_b^t \leq K \leq \text{eq}_s^t + k_s^t \), then \( E[\pi(q^t,k^t)] \) has no relationship with \( b \) and \( s \); when \( K > \text{eq}_s^t + k_s^t \), then \( E[\pi(q^t,k^t)] \) is an increasing function of \( s \). This completes the proof.

References


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