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EVIDENCING THE “ROBOT PHASE TRANSITION” IN HUMAN-AGENT EXPERIMENTAL FINANCIAL MARKETS

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Abstract: Johnson, Zhao, Hunsader, Meng, Ravindar, Carran, and Tivnan (2012) recently suggested the existence of a phase transition in the dynamics of financial markets in which there is free interaction between human traders and algorithmic trading systems (‘robots’). Above a particular time-threshold, humans and robots trade with one another; below the threshold all transactions are robot-to-robot. We refer to this abrupt system transition as the ‘robot phase transition’. Here, we conduct controlled experiments where human traders interact with ‘robot’ trading agents in minimal models of electronic financial markets to see if correlates of the two regimes suggested by Johnson et al. (2012) occur in such laboratory conditions. Our results indicate that when trading robots act on a super-human timescale, the market starts to fragment, with statistically lower human-robot interactions than we would expect from a fully mixed market. We tentatively conclude that this is the first empirical evidence for the robot phase transition occurring under controlled experimental conditions.

1 INTRODUCTION

In February 2012, Johnson et al. (2012) published a working paper that immediately received widespread media attention. Having analysed millisecond-by-millisecond stock-price movements, Johnson et al. (2012) argued that there was evidence for a phase transition in the behaviour of financial markets at the sub-second time-scale. At the point of this transition, the market dynamics switch from a domain involving interactions among a mix of human traders and ‘robot’ automated algorithmic trading systems, to a newly-identified domain in which the robots interact only among themselves, with no human traders involved. At sub-second timescales, below the transition, the robot-only market exhibits ‘fractures’ (ultra-fast swings in price) that are undesirable, little understood, and intriguingly appear to be linked to longer-term instability of the market as a whole. This discovery is potentially significant for the global financial markets. If the short term micro-effects can indeed give some indication of longer-term macro-scale behaviour then it is possible that new methods for monitoring the stability of markets could be developed, offering early-warning systems for major crashes.

In March 2012, we were commissioned by the UK Government Office for Science’s Foresight unit to run a series of agent-human experiments exploring the robot transition under controlled laboratory conditions (see Cartlidge & Cliff, 2012, for full details). We did this by varying the speed/reaction of robot-trader agents in OpEx (OpEx SourceForge, 2012), an “artificial stock exchange” that had been developed as an apparatus for evaluating human-robot and robot-robot interaction in electronic markets. Our aim was to test the hypothesis that when robot trader agents are able to act/react on a timescale quicker than the human traders are, we will see a transition from a mixed market (where humans and robots are equally likely to interact with one another) to a more fragmented market where robots are more likely to trade with robots, and humans with humans. Our results support the existence of the robot phase transition, although in our experiments the effects of increasing robot speed seem to give a progressive response rather than a step-change. To our knowledge, this is the first time that the robot phase transition, a newly identified real-world phenomenon, has been synthesised under controlled conditions.

1Our study was one of the 31 background reviews commissioned by the UK Government Office for Science’s Foresight project investigating the future of computer trading in the financial markets. The final report from that investigation was published in October 2012, and is available at: http://bit.ly/UvGE4Q.
laboratory conditions.

The remainder of this paper is structured as follows. In Section 2 we introduce relevant background material, before describing our experimental methods in Section 3. Results are presented in Section 4 and discussed in Section 5. Finally, Section 6 concludes.

2 BACKGROUND

An ‘ideal’ market can be perfectly described by the aggregate quantity supplied by sellers and the aggregate quantity demanded by buyers at every price-point (i.e., the market’s supply and demand schedules). At some price-point, the quantity demanded will equal the quantity supplied. This is the theoretical market equilibrium, with price and quantity \((P_0, Q_0)\) determined by the intersection between the supply and demand schedules. The dynamics of competition in the market will tend to drive transactions toward this equilibrium point. However, in the real world, markets are not ideal. They will always trade away from equilibrium at least some of the time. We use the following metrics to calculate the ‘performance’ of a market by how far from ideal equilibrium it trades:

**Smith’s alpha:** following Smith (1962), we measure the equilibration (equilibrium-finding) behaviour of markets as \(\alpha\), the root mean square difference between each of \(n\) transaction prices, \(p_i\) (for \(i = 1 \ldots n\)) over some period, and the \(P_0\) value for that period, expressed as a percentage of the equilibrium price:

\[
\alpha = \frac{100}{P_0} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (p_i - P_0)^2}
\]

(1)

In essence, \(\alpha\) captures the standard deviation of trade prices about the theoretical equilibrium. A low value of \(\alpha\) is desirable, indicating trading close to \(P_0\).

**Allocative Efficiency:** for each trader, \(i\), the maximum theoretical profit available, \(\pi^*_i\), is the difference between the price they are prepared to pay (their ‘limit price’) and the theoretical market equilibrium price, \(P_0\). Efficiency, \(E\), is used to calculate the performance of a group of \(n\) traders as the mean ratio of realised profit, \(\pi_i\), to theoretical profit, \(\pi^*_i\):

\[
E = \frac{1}{n} \sum_{i=1}^{n} \frac{\pi_i}{\pi^*_i}
\]

(2)

As profit values cannot be negative (traders in these experiments are not allowed to enter into loss-making deals), a value of 1.0 indicates that the group has earned the maximum theoretical profit available, \(\pi^*_i\), on all trades. A value below 1.0 indicates that some opportunities have been missed. Finally, a value above 1.0 means that additional profit has been made by taking advantage of a trading counterparty’s willingness to trade away from \(P_0\).

**Profit Dispersion** is a measure of the extent to which the profit/utility generated by a group of traders in the market differs from the profit that would be expected of them if all transactions took place at the equilibrium price, \(P_0\). For a group of \(n\) traders, profit dispersion is calculated as the root mean square difference between the profit achieved, \(\pi_i\), by each trader, \(i\), and the maximum theoretical profit available, \(\pi^*_i\):

\[
\pi_{disp} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\pi_i - \pi^*_i)^2}
\]

(3)

Low values of \(\pi_{disp}\) indicate that traders are extracting actual profits close to those available if all trades take place at the equilibrium price \(P_0\); while higher values of \(\pi_{disp}\) indicate that traders’ profits differ from those expected at equilibrium. The attraction of this statistic is that it is not masked by zero-sum effects between buyers and sellers.

**Delta Profit** is used to calculate the difference in profit maximising performance between two groups, \(x\) and \(y\), as a percentage difference relative to the mean of the two groups:

\[
\Delta P(x - y) = \frac{2(\pi_x - \pi_y)}{\pi_x + \pi_y}
\]

(4)

Delta profit directly measures the difference in profit gained by two groups. In a perfect market with identically matched groups, delta profit should be zero, since all groups should trade at \(P_0\).

For all the experiments described in this paper, we use the Adaptive-Aggressive (AA) strategy for our robot trader algorithms. AA has previously been demonstrated to be the dominant robot algorithm in the academic literature (De Luca & Cliff, 2011). AA robots have short-term and longer-term learning processes. In the short-term, robots update the aggressiveness of their bidding behaviour; with more aggressiveness meaning an agent will sacrifice profit to improve its chance of transacting. In the longer-term, robots learn how to best combine their aggressiveness with their estimation of the market equilibrium price, calculated by observing transaction prices over a time window, to choose which bids or asks to submit in the market (for full details, see Vytelingum, 2006).

3 METHODOLOGY

Open Exchange (OpEx) is a real-time financial-market simulator specifically designed to enable eco-
Table 1: Permit-schedule timetable. Six permit types are issued to each market participant, depending on their role. For each role, there is one human and one robot participant. Permit values show limit price - $P_0$. Numbers in brackets show the time-step sequence in which permits are allocated.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$P_0$</th>
<th>Cyclic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>272</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>209</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>291</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>241</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>258</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>239</td>
<td>No</td>
</tr>
</tbody>
</table>

2During one run, the user interface of a human participant crashed, causing the participant to achieve very low profit. Hence, we use only results from the other 23 experiments to perform our statistical analysis.
Table 3: Agent configurations used in experiments.

<table>
<thead>
<tr>
<th>Name</th>
<th>Agent</th>
<th>Sleep-Wake</th>
<th>Internal</th>
</tr>
</thead>
<tbody>
<tr>
<td>AA-0.1</td>
<td>AA</td>
<td>0.1</td>
<td>0.05</td>
</tr>
<tr>
<td>AA-1</td>
<td>AA</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>AA-5</td>
<td>AA</td>
<td>5</td>
<td>2.5</td>
</tr>
<tr>
<td>AA-10</td>
<td>AA</td>
<td>10</td>
<td>2.5</td>
</tr>
</tbody>
</table>

results. This lets us infer that if a statistical difference is present in results from cyclical-replenishment markets that is not present in random-replenishment markets, then the difference is an artificial consequence of the artificial experimental constraint of cyclically replenishing the traders’ entitlements to buy and sell (for a lengthy discussion on the importance of incorporating ‘realism’ into experimental design, see De Luca et al., 2011).

For each experiment, all robots were configured with an identical parameter set, detailed in Table 3. Agents were selected from one of four configurations: AA-0.1, AA-1, AA-5, and AA-10. The numerical suffix indicates a robot’s sleep-wake cycle time in seconds. The greater the value, the longer the robot ‘sleeps’ between actions. By varying this sleep parameter, we are able to control the ‘speed’ at which robots act. Johnson et al. (2012) suggest that: “in many areas of human activity, the quickest that someone can notice [a] cue and physically react, is approximately 1000 milliseconds”. Thus, to test the effect of robot speed on the market, we select sleep values that comfortably range from well below human reaction speed (0.1s) to well above human reaction speed (10s). In this way we should be certain that our collection of robot configurations ‘cross the boundary’ of human reaction time, enabling us to compare the dynamics of markets containing robots that act at super-humanly fast speeds, with markets containing robots that act on human time-scales. To ensure that robots are able to act sensibly upon waking from sleep, robots are enabled to observe the market and perform internal calculations during their ‘sleep’ phase. To achieve this, a second ‘internal’ timer is used to control when a robot will observe and calculate. Table 3 shows the internal timers used for each robot configuration; in each case, the internal timer has a shorter period than the sleep-wake cycle. Robots are also configured to wake from sleep upon a new order stimulus and new trade stimulus. Finally, all robots have a ‘maximum spread’ parameter set to 0.01, meaning that if the spread between the best ask price and best bid price is less than 1%, the agent will automatically cross with the best bid/ask on the other side (for further details, see Cartlidge & Cliff, 2012).

For evaluating statistical significance we use the nonparametric Robust Rank Order (RRO) test reported by Feltovich (2003).

4 RESULTS

4.1 Smith’s $\alpha$

In Fig. 1 we can observe the equilibration behaviour of the markets by plotting Smith’s $\alpha$ for each cycle period (on a log-scale). We see that there is no difference between robots. Under all conditions, $\alpha$ rapidly falls from a value close to 10% in the initial period, to $\alpha \approx 2\%$ in period two; $\alpha$ then continuous to fall more gradually over the course of an experiment, tending to $\alpha \approx 1\%$ by market close.

In Fig. 2 we see mean $\alpha$ ($\pm 95\%$ confidence interval) plotted for cyclical and random markets. Under both conditions, $\alpha$ follows a similar pattern, tending to $\alpha \approx 1\%$ by market close. However, in the first period, cyclical markets produce significantly greater $\alpha$ than random markets (RRO, $p < 0.0005$). This is due to the sequential order allocation of permits in cyclical markets, where limit prices furthest from equilibrium are allocated first. This encourages ‘exploratory’ quotes and trades to occur far from equilibrium. In
4.2 Efficiency

Mean efficiency results for each robot type averaged across all experiments are summarised in Table 4. We see that the efficiency of robots is greater than the efficiency of humans under every condition, with robots securing a delta profit gain of 0.4% – 1.8%. Across all experiments, robots achieve a significantly greater efficiency (RRO, \( p < 0.025 \)). Grouping by market type, robots achieve significantly greater efficiency in random markets (RRO, \( p < 0.1 \)) and robots achieve significantly greater efficiency in cyclical markets (RRO, \( p < 0.1 \)). When comparing the efficiencies of robots with the efficiencies of humans across all markets grouped by robot type, robots are still shown to be more efficient but the difference is only significant for robots AA-0.1 and AA-5 (RRO, \( p < 0.104 \)). The difference between robots AA-10 and humans is not significant at the \( p = 0.104 \) level and the difference between robots AA-1 and humans is not significant at the \( p = 0.104 \) level.

Fig. 3 plots mean efficiency (\( \pm 95\% \) confidence interval) of robots grouped by type. As robot sleep time decreases, the efficiency of robots appears to increase, however, across all markets this difference is not significant. However, when comparing data from only cyclical markets, AA-0.1 robots attain a mean efficiency score significantly higher than AA-1 (RRO, \( p = 0.05 \)), AA-5 (RRO, \( p = 0.05 \)) and AA-10 (RRO \( p = 0.1 \)).

4.3 Profit Dispersion

Table 5 summarises profit dispersion by market type. We see that random markets have significantly lower profit dispersion (RRO, \( 0.005 < p < 0.01 \)), significantly lower profit dispersion of humans (RRO, \( 0.025 < p < 0.05 \)), and significantly lower profit dispersion of agents (RRO, \( 0.001 < p < 0.005 \)). However, when comparing profit dispersion by robot types (data not presented), we find no significant difference in profit dispersion of markets, robots, or humans.

4.4 Execution Counterparties

Table 6 shows the mean proportion of counterparty executions grouped by robot type. In a fully mixed market, we expect roughly half of all trades to have homogeneous counterparties (humans trading with humans and robots trading with robots) and the other half to have heterogeneous counterparties (humans trading with agents, or vice versa). Fig. 4 plots the median number of homogeneous counterparties in markets containing each of the four robot types, with error bars showing the range of values. There appears to be an inverse relationship between robot sleep time and proportion of homogeneous counterparties. RRO tests show that the proportion of homogeneous interactions in AA-0.1 markets is significantly higher than AA-1 and AA-5 markets (\( p < 0.051 \)), and AA-10 markets (\( p = 0.0011 \)); and for AA-1 and AA-5 markets the proportion is significantly higher than AA-10 (\( p < 0.104 \)). For AA-10 robots, the proportion of homogeneous counterparties is significantly lower in random markets than cyclical markets (\( p < 0.05 \)).
all other robot types, there is no significant difference in the proportion of homogeneous counterparties between markets.

If we assume a normal distribution for the counterparty distributions, then calculating confidence intervals shows that in all (six) markets containing AA-0.1 robots, the proportion of homogeneous counterparties is significantly greater than 50% ($p < 0.0001$). In contrast, for markets containing AA-1 robots (five), AA-5 robots (six), and AA-10 robots (six), the null hypothesis that the proportion of homogeneous counterparties is 50% is not rejected at the 10% level of significance. This suggests that for the fastest robots (AA-0.1) there is a trend to market fragmentation, with humans trading with humans and robots trading with robots more than would be expected by chance.

5 DISCUSSION

5.1 Evidence for the Phase Transition

Here, we summarize the main results that hold across all our market experiments. In Section 5.2 we discuss results that demonstrate significant differences between cyclical and random markets.

Across all markets, and for all robot types, robots are shown to outperform humans, securing greater allocative efficiency scores under every condition and gaining a delta profit increase of between 0.4%-1.8%. These results are significant across all markets and robot types, except AA-10, the slowest of the robots. For readers familiar with previous papers (such as De Luca et al., 2011; Cartlidge et al., 2012), this result may come as something of a surprise: prima facie, this is the first time that robots have been shown to be more efficient than humans in a continuous replenishment, real-time experimental market with liquidity drip-fed into the market over time. There is weak evidence (not statistically significant) of a trend for the efficiency of agents to decrease as sleep time of agents increases, suggesting that speed is beneficial to agents. This is an intuitively appealing interpretation, but is not confirmed with a reasonable level of statistical significance by our results. Furthermore, the prima facie novelty of this result is primarily due to problems with the earlier results (published in De Luca et al., 2011; Cartlidge et al., 2012). Those earlier results, we learned in the course of analysing the results of the experiments reported here, were affected by a bug in the OpEx implementation of the AA robot-trader algorithm, and fixing that bug was the main cause of the increase in efficiency with respect to those earlier results (for further details, see Cartlidge & Cliff, 2012).

Across all markets, $\alpha$ values start high ($\alpha \approx 10\%$) as traders ‘explore’ the space of prices, and then quickly reduce, with markets tending to an equilibrium level of $\alpha \approx 1\%$. This demonstrates markets trading at a level much closer to equilibrium than previously observed in De Luca et al. (2011) and Cartlidge et al. (2012); and suggests that the market’s price-discovery is readily finding values close to $P_0$. These results demonstrate a well-functioning robot-human market trading near equilibrium, with robots out-competing humans on profit. This is an interesting result, but for our purpose of exploring the robot phase transition described by Johnson et al. (2012) it only serves as demonstrative proof that our experimental markets are performing as we would expect. The real interest lies in whether we can observe a phase transition between two regimes: one dominated by robot-robot interactions, and one dominated by human-robot interactions. We seek evidence of this by observing the proportion of homogeneous counterparties within a market; that is, the number of trade executions that occur between a pair of humans or a pair of robots, as a proportion of all market trades. Theory suggests that in a fully mixed market with no asymmetry in the behaviour of participants, we should expect 50% of trade counterparties to be homogeneous, and 50% to be heterogeneous, as traders execute with counterparties at random. From Section 4.4, our results demonstrate that for markets containing AA-0.1 robots, the proportion of homogeneous counterparties is significantly higher than we would expect in a mixed market, whereas with slower-acting robots, the proportion of homogeneous counterparties cannot be significantly differentiated from 50%. We present this as tentative first evidence for a robot-phase transition in experimental markets with a boundary between 100 milliseconds.
and 1 second, although in our experiments the effects of increasing robot speed seem to give a progressive response rather than a step-change. However, we feel obliged to caveat this result as non-conclusive proof until further experiments have been run, and until our results have been independently replicated.

The careful reader may have noticed that the results presented have not demonstrated ‘fractures’ – ultra-fast series of multiple sequential up-tick or down-tick trades that cause market price to deviate rapidly from equilibrium and then just as quickly return – phenomena that Johnson et al. (2012) revealed in real market data. Since we are constraining market participants to one role (as buyer, or seller) and strictly controlling the flow of orders into the market and limit prices of trades, the simple markets we have constructed do not have the capacity to demonstrate such fractures. For this reason, we use the proportion of homogeneous counterparties as proxy evidence for the robot phase transition.

5.2 Artefacts or Evidence?

As we argued in Section 3, the cyclical-replenishment experimental markets used by De Luca et al. (2011) and Cartlidge et al. (2012) are a poorer approximation to real-world markets than are the random-replenishment markets. For that reason, where results from cyclical markets show a significant effect of agent-speed, that is not also present in our random markets, we interpret as another indication that introducing artificial constraints into experimental markets for ease of analysis runs the risk of also introducing artefacts that, because they are statistically significant, can be misleading. The following relationships were all observed to be statistically significant in cyclical-replenishment markets and not statistically significant in random-replenishment markets; providing further support for the argument for realism in artificial-market experiment design, previously advanced at length by De Luca et al. (2011):

1. Cyclical-replenishment markets produced significantly greater $\alpha$ values in the first period of trade. This is a direct consequence of cyclical-replenishment allocating orders in a monotonically decreasing sequence from most profitable to least profitable. As such, the first orders allocated into the market have limit prices far from equilibrium. Since the market is empty, there is no mechanism for price discovery other than trial-and-error exploration, leading to large $\alpha$. In random-replenishment markets, the initial orders entering the market are drawn at random from the demand and supply schedules. This leads to lower bounds on limit prices and hence lower $\alpha$. Subsequently, price discovery is led by the order book, resulting in lower $\alpha$ over time.

2. In cyclical-replenishment markets, the efficiency of AA-0.1 robots is significantly higher than the efficiency of the other robot types. While there is some evidence of an inverse relationship between robot sleep time and robot efficiency across all markets, we infer that this difference is an artefact of cyclical replenishment until further experimental trials can confirm otherwise.

3. When comparing random and cyclical markets, profit dispersion in cyclical-replenishment markets is significantly higher for agents, humans, and the market as a whole. Since lower profit dispersion is a desirable property of a market, this suggests that the relatively high profit dispersion observed in previous cyclical-replenishment experiments (De Luca et al., 2011; Cartlidge et al., 2012) is an artefact of the experimental design.

5.3 Future Work

We have gathered tentative evidence to support the existence of the robot phase transition. The next step is to see if we can observe market dynamics analogous to the market ‘fractures’ reported by Johnson et al. (2012). To achieve this, it may be necessary to introduce role diversity (e.g., enabling participants to buy and sell and hence act as ‘market makers’). If we are able to achieve this goal, we will then have a controlled method for exploring the relationship between localised ultra-fast mini-crashes and longer-term global instabilities (flash-crashes) observed in real-world markets. Any progress in this area could have significant positive impact on our understanding of the global financial markets and offer potential new regulatory mechanisms to avoid the occurrence of future flash-crash events.

Other questions that have arisen from this research and require further exploration, include:

- What happens if we vary the rate of order replenishment inter-arrival times? When orders start to arrive faster than humans can react, do we see a robot phase transition here?
- What happens if we vary the proportion of robots in the market? Are market dynamics significantly different when the market is dominated by robots? How does this affect the robot phase transition?
- What happens if robots do not wake up on new trade stimuli? Does this make a fairer proxy of agent ‘speed’? How does this change affect the robot phase transition?
While these questions are interesting, progress will necessarily be slow. Unlike many facets of computer science, where variations on a question theme can be easily tweaked by altering the values of some parameters and then pressing ‘run’, experimental economics offers the pragmatic challenge of soliciting and incentivizing human participants, arranging a venue, ensuring participants arrive, and finally, ensuring that the system is ‘correctly’ configured and functioning error-free during the ‘one-shot performance’ of each experiment. For many empirical computer scientists working on artificial intelligence and autonomous software agents, this is an alien landscape.

6 CONCLUSIONS

We have presented results from a series of human-vs.-robot experimental financial markets to test the hypothesis that when robot trader agents in OpEx are able to act/react on a timescale quicker than the human traders are, we will see a transition from a mixed market (where humans and robots are equally likely to interact with one another) to a more fragmented market where robots are more likely to trade with robots, and humans with humans, similar to the robot phase transition that Johnson et al. (2012) argue for the existence of in real financial markets. Our primary conclusion is that our results are supportive of Johnson et al.’s (2012) hypothesis concerning the existence of the robot phase transition, although in our experiments the effects of increasing robot speed seem to give a progressive response rather than a step-change. This result could have potentially profound consequences. By evidencing the robot phase transition under controlled laboratory conditions, we have opened a new pathway for studying this recently observed phenomenon. Hopefully, future work will replicate sub-second ‘fractures’ and subsequent global instabilities (‘crashes’). We will then be in a position to dynamically observe the relationship between these intriguing phenomena, enabling us to design monitoring tools and/or introduce safety mechanisms, in order to avoid, or contain, future ‘flash crash’ events in the global financial markets.

We also explored the effects of increasing the ‘realism’ of the structure of the experiments conducted on OpEx. In doing this, we discovered that some statistically significant effects observed in artificial, constrained experimental set-ups, disappear when the experiments are more realistic and less constrained. This leads us to our second conclusion: that in experiments such as those reported here, the more realistic the set-up of the experiment, the more the results can be trusted.

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